ECE 438 Lecture 4/28/2023

Announcements

1. Supplementary Office today at 4p EDT
2. Supplementary Office on Monday 5/1/2023 TBD
3. Final Exam 7:00p – 9:00p on Monday 5/1/2023 in MSEE B012
4. Exam will consist of 5 problems
   3 from HWs 1-9 2 from HW #10 and associated lecture
   nothing on computed tomography
5. Today is last day to do course evaluation and get 10 pts. credit toward quiz

Spatial filtering

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1

f[m,n]

1/16	1/8	1/16
1/8	1/4	1/8
1/...	1/8	1/16

0	0	0	0	0	0
0	1/16	3/16	4/16	4/16	4/16
0	3/16	9/16	12/16	12/16	12/16
0	4/16	12/16	1	1	1
0	4/16	12/16	1	1	1
0	4/16	12/16	1	1	1

g[m,n]

$-1 \quad 1/16 \quad 0 \quad 1/16$

$-1 \quad 0 \quad 1$

$h[k, l]$

Frequency domain:

definition (DSFT) Discrete Space Fourier Transform

(μ, ν) – (mu, nu) radians/pixel

forward transform

$$F(\mu,\nu) = \sum_m \sum_n f[m,n] e^{-j(m\mu + n\nu)}$$

2π radians = 1 cycle

Nyquist limit: π rad/pixel = $\frac{1}{2}$ cycle/pixel

Inverse transform

$$f(m,n) = \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} F(\mu,\nu) e^{j(m\mu + n\nu)}$$

Convolution for DSFT

$$f[m,n] * g[m,n] \xleftrightarrow{DSFT} F(\mu,\nu) G(\mu,\nu)$$

Separability:

$$f(m) g(n) \longleftrightarrow F(\mu) G(\nu)$$

Return to example:

1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16

$H(\mu,\nu)$ is frequency response of our filter

$h[k,l]$ — filter kernel

filter is separable:

	$l = -1$	0	1	$h_1[k]$
$k = 1$	$1/16$	$1/8$	$1/16$	$1/4$
0	$1/8$	$1/4$	$1/8$	$1/2$
-1	$1/16$	$1/8$	$1/16$	$1/4$
$h_1[l]$	$1/4$	$1/2$	$1/4$	

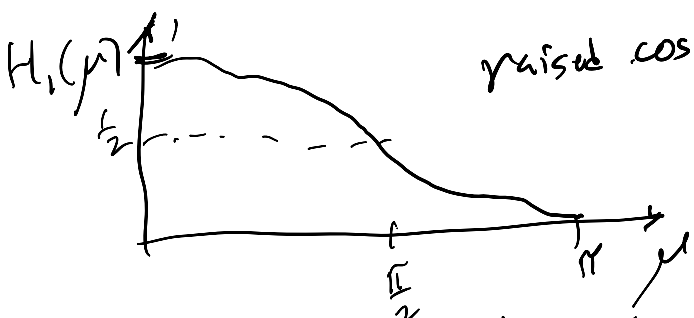
$$h[k,l] = h_1[k]\,h_1[l]$$

$$H(\mu,\nu) = H_1(\mu)\,H_1(\nu)$$

$$H_1(\mu) = \sum_{k=-1}^{1} h_1[k]\, e^{-jk\mu}$$

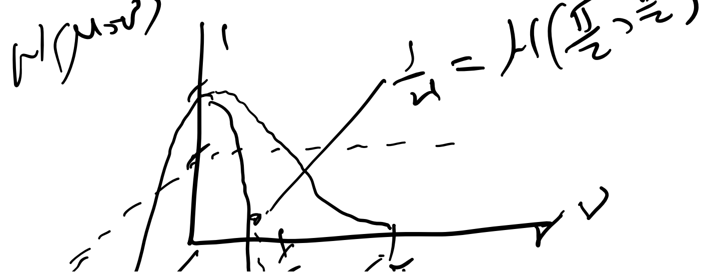
$$= \frac{1}{4} e^{j\mu} + \frac{1}{2} + \frac{1}{4} e^{-j\mu}$$

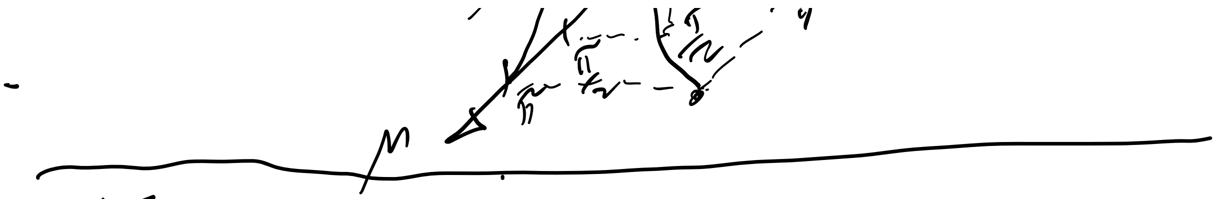
$$= \frac{1}{2}\left[1 + \cos(\mu)\right]$$



raised cosine

DC preserving + low pass





Example 2

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1

$f[m,n]$

$-\frac{1}{9}$	$-1/9$	$-1/9$
$-1/9$	$8/9$	$-1/9$
$-\frac{1}{9}$	$-\frac{1}{9}$	$-1/9$

$h[k,l]$

0	0	0	0	0	0
0	$-1/9$	$-2/9$	$-3/9$	$-3/9$	$-3/9$
0	$-2/9$	$5/9$	$3/9$	$3/9$	$3/9$
0	$-3/9$	$3/9$	0	0	0
0	$-3/9$	$3/9$	0	0	0
0	$-3/9$	$3/9$	0	0	0

$g[m,n]$

edge detectors are widely used in computer vision and machine learning

what did this filter do?

edge detector with undershoot followed by overshoot

Note that $\sum_k \sum_l h[k,l] = 0$

Not separable!

But:

$$\begin{bmatrix} -1/9 & -1/9 & -1/9 \\ -1/9 & 8/9 & -1/9 \\ -1/9 & -1/9 & -1/9 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

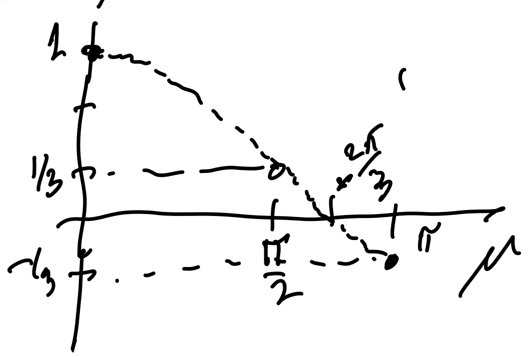
$$\underbrace{\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}}_{h_2[l]} \qquad \underbrace{\begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix}}_{h_2[k]}$$

$$H[k,l] = \delta[k,l] - h_2[k]\,h_2[l]$$

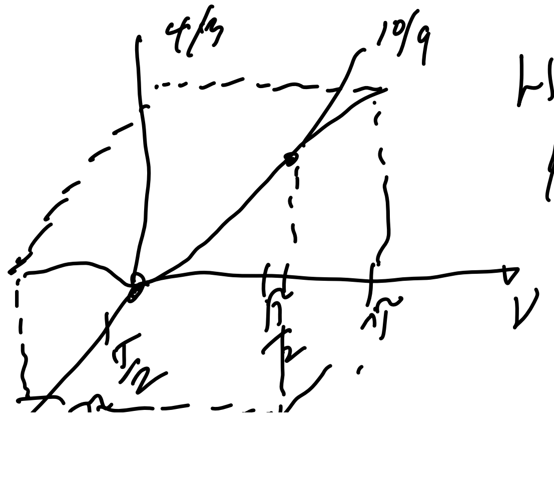
$$H_2(\mu) = \sum_{k=-1} h_2[k]\, e^{-j\mu k}$$

$$= \frac{1}{3} e^{j\mu} + \frac{1}{3} + \frac{1}{3} e^{-j\mu}$$

$$H_2(\mu) = \frac{1}{3}\left[1 + 2\cos(\mu)\right]$$



$$H(\mu,\nu) = 1 - H_2(\mu)\,H_2(\nu)$$



High pass filter

$H(0,0) = 0$ because

$$\sum_k \sum_l h[k,l] = 0$$

[illegible]

Example 3

0	0	0	0	0	0
0	0	0	0	0	0
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1
0	0	1	1	1	1

$f[m,n]$

0	0	0	0	0	0
0	$-\lambda/9$	$-2\lambda/9$	$-3\lambda/9$	$-3\lambda/9$	$-3\lambda/9$
0	$-2\lambda/9$	$1+5\lambda/9$	$1+\lambda/3$	$1+\lambda/3$	$1+\lambda/3$
0	$-3\lambda/9$	$1+\lambda/3$	1	1	1
0	$-3\lambda/9$	$1+\lambda/3$	1	1	1
0	$-3\lambda/9$	$1+\lambda/3$	1	1	1

λ is a parameter

$-\lambda/9$

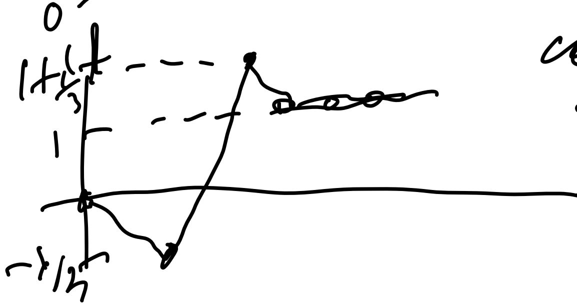
$-\lambda/9$	$-\lambda/9$	$-\lambda/9$
$-\lambda/9$		$-\lambda/9$
$-\lambda/9$	$-\lambda/9$	$-\lambda/9$

$h[k,l]$

$1+\frac{8}{9}\lambda$

$\sum_{k,l}\sum h[k,l]=1$

Sharpening filter for $0\le\lambda$



excites
center-on-surround
off receptor field

Very common sharpening tool