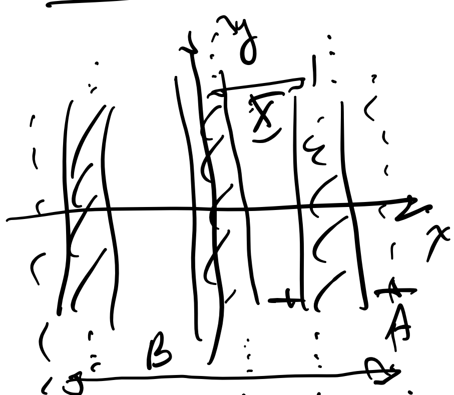
ECE 438 Lecture April 21, 2023

Announcements

1. HW 10 is posted
2. solutions will provided on Monday
3. No coverage of computed tomography on final exam

Module 2.1.4 2D Signals & Systems – Periodic structures

Example 3



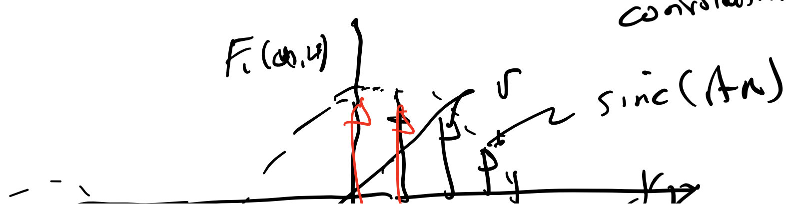
Spatial domain

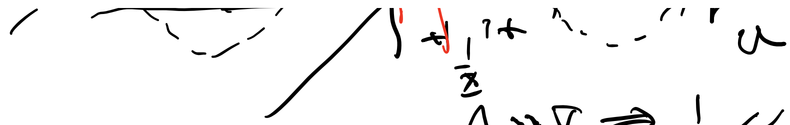
The analysis will NOT follow posted notes.
I am taking a simpler approach using separability

$$f_3(x,y) = \overbrace{rep_X\left[rect(x/A)\right] rect(x/B)}\cdot \underline{1(y)}$$

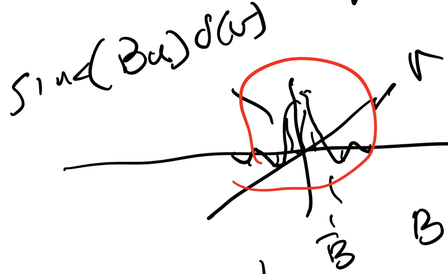
$$F_3(u,v) = \left[\frac{AB}{X} comb_{\frac{1}{X}}\left[sinc(Au)\right] * sinc(Bu)\right]\cdot \delta(v)$$

1D convolution

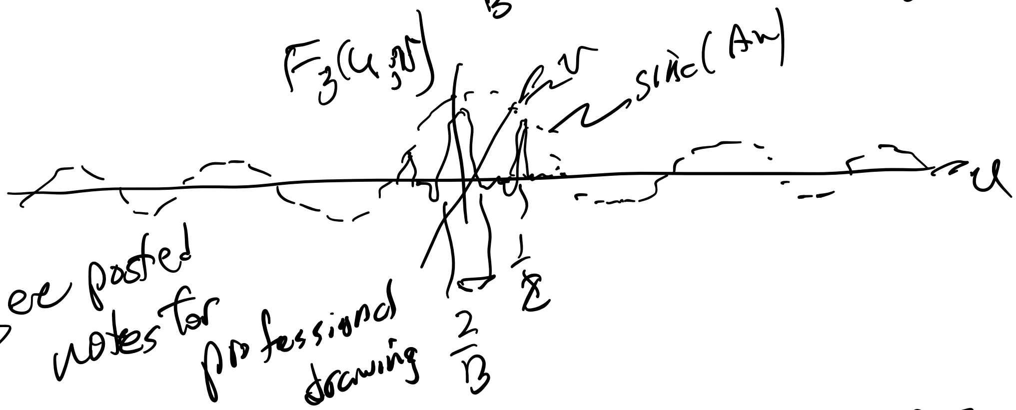




$A \gg X \Rightarrow \frac{1}{X} \ll \frac{1}{A}$



$B \gg A \Rightarrow \frac{1}{A} \gg \frac{1}{B}$



See posted notes for professional drawing

Continue with Module 2.1.4

2D rep & comb operators

definitions

$$\text{comb}_{XY}\left[f(x,y)\right] = \sum_{k}\sum_{l} f(kX, lY)\,\delta(x-kX, y-lY)$$

$$\text{rep}_{XY}\left[f(x,y)\right] = \sum_{k}\sum_{l} f(x-kX, y-lY)$$

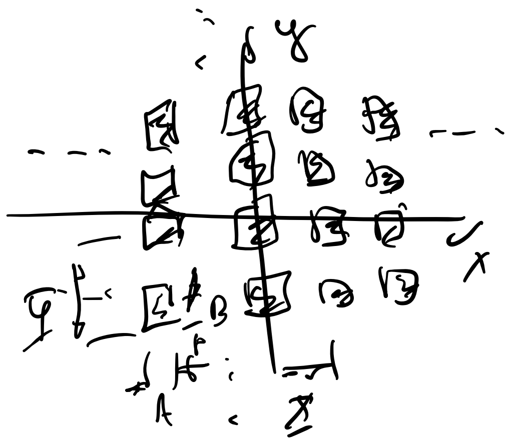
transform pairs

$$\text{rep}_{XY}\left[f(x,y)\right] \xrightarrow{\text{2D CSFT}} \frac{1}{X\cdot Y}\,\text{comb}_{\frac{1}{X}\frac{1}{Y}}\left[F(u,v)\right]$$

by reciprocity, also have

$$\text{comb}_{XY}[f(x,y)] \xleftrightarrow{\text{CSFT}} \frac{1}{XY}\,\text{rep}_{\frac{1}{X}\frac{1}{Y}}[F(u,v)]$$

Example 4



$$f_4(x,y) = \begin{cases} 1, & \boxed{} \\ 0, & \text{else} \end{cases}$$

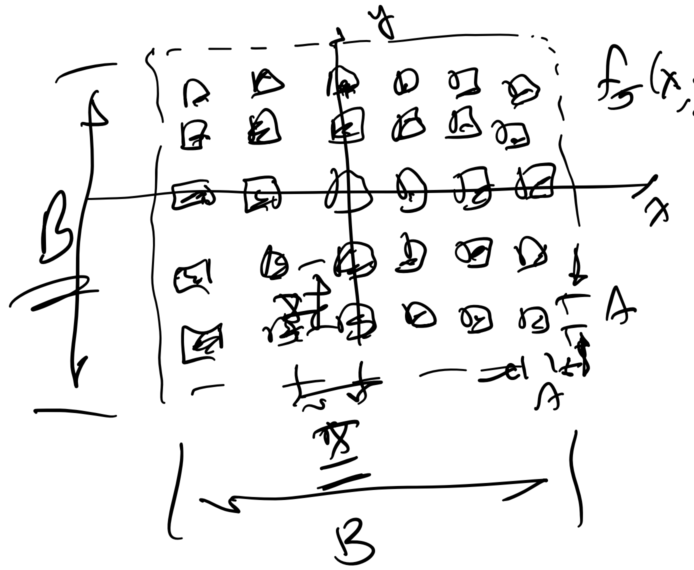
$$f_4(x,y) = \text{rep}_{XY}\left[\text{rect}\left(\frac{x}{A}, \frac{y}{B}\right)\right]$$

$$F_4(u,v) = \frac{1}{XY}\,\text{comb}_{\frac{1}{X}\frac{1}{Y}}\left\{AB\,\text{sinc}(Au, Bv)\right\}$$

$$= \frac{AB}{XY}\sum_k\sum_l \text{sinc}\left(\frac{A}{X}k, \frac{B}{Y}l\right)\delta\left(u-\frac{k}{X}, v-\frac{l}{Y}\right)$$

See posted notes for a better drawing

Now consider Example 5:



$$f_5(x,y) = \begin{cases} 1, & \boxed{1} \\ 0, & \text{else} \end{cases}$$

$$\underline{F_4(u,v)} = \frac{A^2}{X^2} \,\mathrm{rep}_{\frac{1}{X},\frac{1}{X}} \left[\mathrm{sinc}(Au, Av) \right]$$

$$= \frac{A^2}{X^2} \sum_k \sum_l \mathrm{sinc}\left(\frac{Ak}{X}, \frac{Al}{X}\right) \delta\left(u - \frac{k}{X}, v - \frac{l}{X}\right)$$

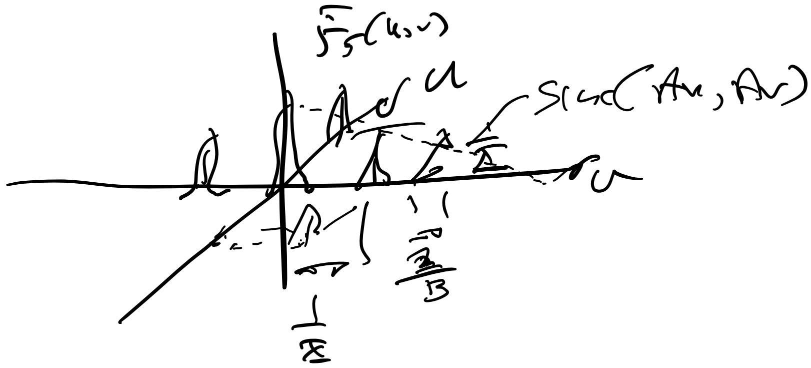
$$f_5(x,y) = f_4(x,y) * \mathrm{rect}\left(\frac{x}{B}, \frac{y}{B}\right)$$

$$F_5(u,v) = \underline{F_4(u,v)} \; B^2 \,\mathrm{sinc}(Bu, Bv)$$

$$B \gg X \Rightarrow \frac{1}{B} \ll \frac{1}{X}$$

$$F_5(u,v) = \frac{A^2 B^2}{X^2} \sum_k \sum_l \mathrm{sinc}\left(\frac{Ak}{X}, \frac{Al}{X}\right) \mathrm{rect}\left[B\left(u - \frac{k}{X}\right), B\left(v - \frac{l}{X}\right)\right]$$

see posted notes



Comments:

General relationship between spatial & frequency domain

spatial	frequency
spatial lattice	reciprocal lattice
microscopic	macroscopic
macroscopic	microscopic

end of module 2.1.4

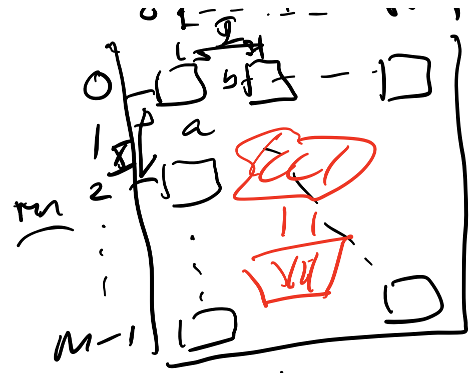
What's next!

Module 2.2.1 – Overview of scanning technologies (2 pages)

2.2.2 – general model ~~for~~ scanning processes (will only cover focal plane arrays)

Start on p.5 of Module 2.2.2:

focal plane array capture of images



$$\underline{f_d[m,n]} = \int_{m\Delta - a/2}^{m\Delta + a/2} \int_{n\Delta - b/2}^{n\Delta + b/2} f_a(x,y)\,dx\,dy$$

digital   analog

Want to use our tools to analyze this system

Answer: $\underline{\tilde{f}_a^{(a,b)}(x,y) = f_a(x,y) ** \mathrm{rect}\left(\frac{x}{a}, \frac{y}{b}\right)}$ blurred

As aperture size increases, capture more photons

⇒ camera is "faster"! BUT the captured image is blurrier