ECE 438 Lecture Monday April 17, 2023

Announcements

1. No quiz today – no HW due this week
2. Exam 3 will be given on Wednesday 19 April
3. Office Hours today: 2:30p EDT
   4:00p EDT

Optical front-end

Imaging an extended object

$$f(x,y) \longrightarrow \boxed{\text{Imaging System}} \longrightarrow g(x,y)$$

system is homogeneous

$$f(\xi,\eta)\,\delta(x-x_0, y-y_0) \longrightarrow f(\xi,\eta)\,\underline{h(x-Mx_0, y-My_0)}$$

For an extended object

$$f(x,y) = \iint_{-\infty}^{\infty} \underline{f(\xi,\eta)}\,\delta(x-\xi, y-\eta)\,d\xi\,d\eta$$

Since system obey superposition:

$$g(x,y) = \iint_{-\infty}^{\infty} f(\xi,\eta)\,h(x-M\xi, y-M\eta)\,d\xi\,d\eta$$

let $\xi' = M\xi$, $\eta' = M\eta$ $\quad d\xi\,d\eta = \frac{1}{M^2}\,d\xi'\,d\eta'$

$$g(x,y) = \frac{1}{M^2}\iint_{-\infty}^{\infty} \underbrace{f\left(\frac{\xi'}{M},\frac{\eta'}{M}\right)}_{\substack{\text{image predicted}\\ \text{by ray tracing}\\ \text{or geometrical}\\ \text{optics}}} \underbrace{h(x-\xi', y-\eta')\,d\xi'\,d\eta'}_{\substack{\text{convolution}\\ \text{with}\\ \text{PSF yields}\\ \text{diffraction-limited}\\ \text{image}}}$$

Model applies to a wide range of optical imaging system

Also, the PSF shape & size typically varies across the field of view (FOV) of the imaging system

Can account for this by partitioning the FOV into "isoplanatic" patches within which the shape and size of the PSF does not vary significantly

For simplicity, we will assume that $M=1$ going forward

Now consider the image of a complex exponential

$$e^{j2\pi(u_0 x + v_0 y)} \longrightarrow \boxed{\text{Imaging system}} \longrightarrow ?$$

$$g(x,y) = \iint_{-\infty}^{\infty} e^{j2\pi(u_0\xi + v_0\eta)}\, h(x-\xi, y-\eta)\,d\xi\,d\eta$$

$$= \iint_{-\infty}^{\infty} e^{j2\pi((x-\xi)u_0 + (y-\eta)v_0)}\, h(\xi,\eta)\,d\xi\,d\eta$$

alternate form of convolution

$$= e^{j2\pi(xu_0 + yv_0)} \iint_{-\infty}^{\infty} h(\xi,\eta) e^{-j2\pi(u_0\xi + v_0\eta)} d\xi d\eta$$

1. complex exponentials are eigenfunctions of the imaging system
2. Constant of proportionality is CSFT of PSF $H(u_0, v_0)$

Decompose any input $f(x,y)$ into its constituent frequency components:

$$f(x,y) = \iint F(u,v) e^{j2\pi(ux+vy)} du dv$$

By linearity:

$$g(x,y) = \iint_{-\infty}^{\infty} H(u,v) F(u,v) e^{j2\pi(ux+vy)} du dv$$

$$\Rightarrow G(u,v) = H(u,v) F(u,v)$$

This is a proof of the convolution theorem (for CSFT)

i.e. if

$$g(x,y) = \iint f(\xi,\eta) h(x-\xi, y-\eta) d\xi d\eta$$

then

$$G(u,v) = H(u,v) F(u,v)$$

By reciprocity, we have

product theorem

2D convolution

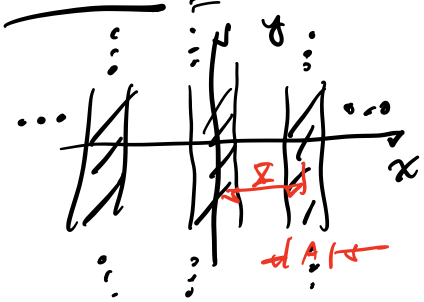
$$f(x,y)\,h(x,y) \xrightarrow{\text{CSFT}} F(u,v) ** G(u,v)$$

This concludes Module 2.1.3

Now consider Module 2.1.4

Periodic structures (see posted notes for better pictures)

Example 1 (top view)



Solution uses separability & different from posted notes

$$f_1(x,y) = \begin{cases} 1, & \boxed{} \\ 0, & \text{else} \end{cases}$$

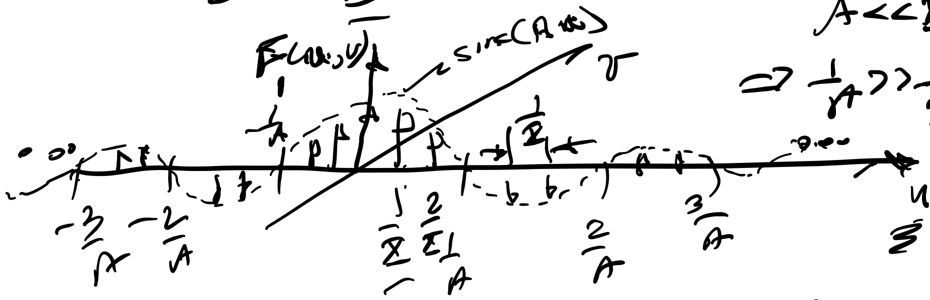
$$f_1(x,y) = \text{rep}_X\left\{\text{rect}\left(\frac{x}{A}\right)\right\} \cdot 1(y)$$

to use separability

$$\Rightarrow F_1(u,v) = \frac{A}{X}\,\text{comb}_{\frac{1}{X}}\left[\text{sinc}(Au)\right]\delta(v)$$

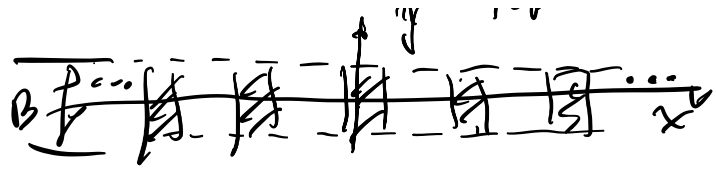
note that $A << X$

$\Rightarrow \frac{1}{A} >> \frac{1}{X}$



Example 2

top view



$$f_2(x,y) = \begin{cases} 1, & \\ 0, & \text{else} \end{cases}$$

$$f_2(x,y) = f_1(x,y)\,\text{rect}(y/B)$$

$$= \underbrace{\text{rep}_X\left[\text{rect}\left(\tfrac{x}{A}\right)\right]}_{f_1(x,y)} \text{rect}\left(\tfrac{y}{B}\right)$$

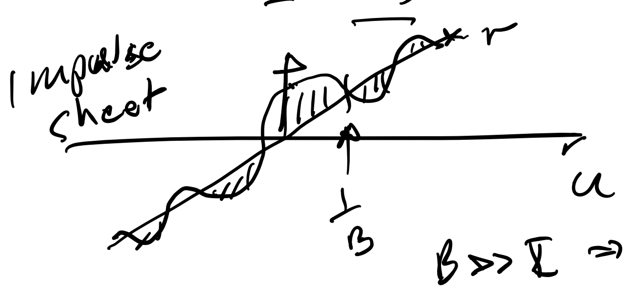
By convolution theorem, we have

$$F_2(u,v) = \underline{F_1(u,v)} ** \text{CSFT}\left\{\text{rect}\left(\tfrac{y}{B}\right)\right\}$$

$$\text{CSFT}\left\{\text{rect}\left(\tfrac{y}{B}\right)\right\} = \text{CSFT}\left\{\underline{1(x)}\cdot \underline{\text{rect}\left(\tfrac{y}{B}\right)}\right\}$$

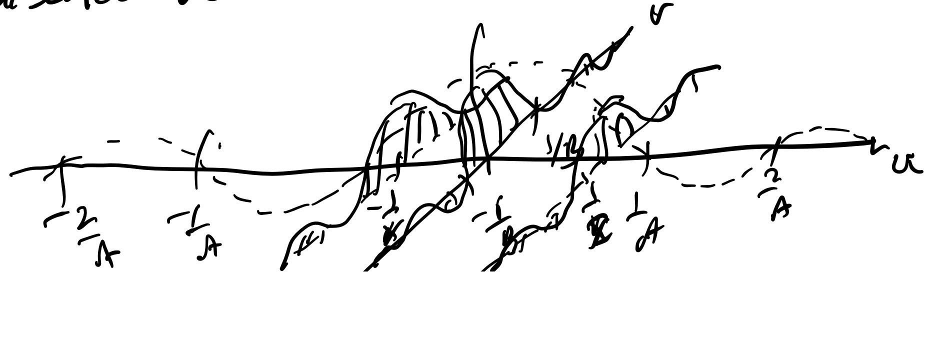
by separability

$$= \underline{\delta(u)}\, B\,\text{sinc}(Bv)$$

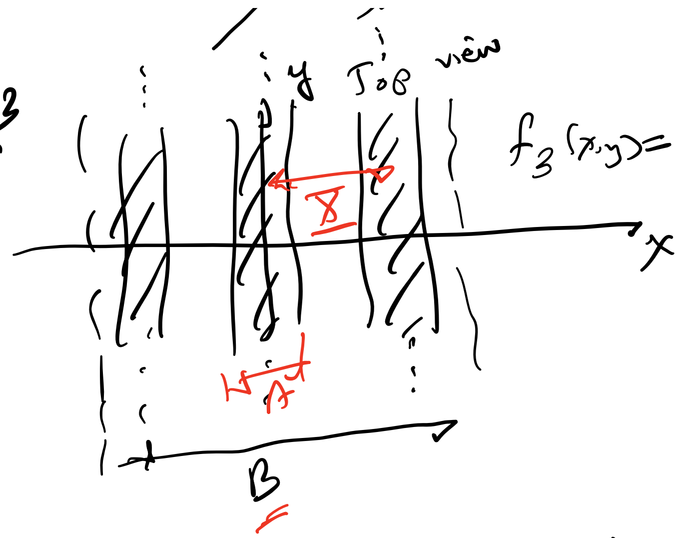


$$B \gg X \Rightarrow \tfrac{1}{B} < \tfrac{1}{X}$$

When convolving $\delta(u) B\,\text{sinc}(Bv)$ with $F_1(u,v)$, each impulse in $F_1(u,v)$ is replaced by a shifted and scaled version of $\delta(u) B\,\text{sinc}(Bv)$



Example 3



$$f_3(x,y) = \begin{cases} 1, & \\ 0, & \text{else} \end{cases}$$

$$f_3(x,y) = f_1(x,y)\,\mathrm{rect}\left(\frac{x}{B}\right)$$

$$f_3(x,y) = \mathrm{rep}_X\left[\mathrm{rect}\left(\frac{x}{A}\right)\right]\mathrm{rect}\left(\frac{x}{B}\right)\cdot 1(y)$$

$$F_3(u,v) = \frac{AB}{X}\,\mathrm{comb}_{\frac{1}{X}}\left[\mathrm{sinc}(Au)\right] \underset{P}{*} \mathrm{sinc}(Bu)\cdot\delta(v)$$

1 D convolution

by separability