ECE 438 Lecture April 14, 2023

Exam on Wednesday 19 April

Continue discussion of 2-D signals & systems

Module 2.1.2

Continuous-space Fourier transform (CSFT)

extension of CTFT to 2D

Forward $F(u,v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\,dy$

(u,v) - spatial frequencies

Inverse $f(x,y) = \iint_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\,dv$

5. Parseval's relation

$$\iint |f(x,y)|^2\, dx\,dy = \iint |F(u,v)|^2\, du\,dv$$

Also, there is a 2D Discrete-Space Fourier Transform (DSFT) — will not cover it in detail

6. Initial value

ⓐ $\iint f(x,y)\, dx\,dy = F(0,0)$

ⓑ $\iint F(u,v)\, du\,dv = f(0,0)$

7. Separability

A function $f(x,y)$ is separable if it can be written as $f(x,y) = g(x)h(y)$

$$f(x,y) = g(x)h(y) \xrightarrow[\nu]{CSFT} F(u,v) = G(u)H(v)$$

very useful!

<u>examples</u>

$$\mathrm{rect}(x,y) = \mathrm{rect}(x)\,\mathrm{rect}(y)$$

$$\mathrm{sinc}(x,y) = \mathrm{sinc}(x)\,\mathrm{sinc}(y)$$

depending on how we define it:

$$\delta(x,y) = \delta(x)\delta(y)$$

Separability is unique to 2D functions!

8. Preservation of separability by convolutions

Consider $\underbrace{f(x,y) ** g(x,y)}_{\text{2D convolution}}$ where $f(x,y) = f_1(x) f_2(y)$

and $g(x,y) = g_1(x) g_2(y)$

then $f(x,y) ** g(x,y) = \overbrace{(f_1(x) * g_1(x))}^{\text{1D convolution}}(f_2(y) * g_2(y))$

$\therefore$ by separability

$$f(x,y) ** g(x,y) \xrightarrow{CSFT} [F_1(u)G_1(u)][F_2(v)G_2(v)]$$

Important transform pairs:

① $\mathrm{rect}(x,y) \xrightarrow{CSFT} \mathrm{sinc}(u,v)$

② $\mathrm{circ}(x,y) \xrightarrow{CSFT} \mathrm{jinc}(u,v)$

③ $\delta(x,y) \xrightarrow{CSFT} 1$

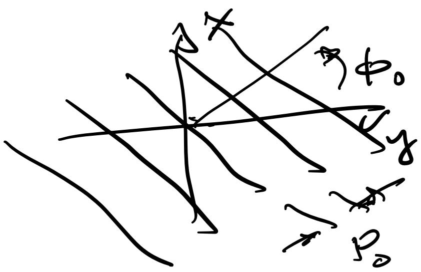
④ $1 \xleftrightarrow{CSFT} \delta(u,v)$

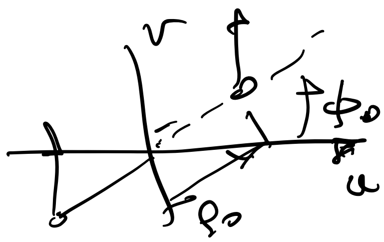
⑤ $e^{j2\pi(u_0 x + v_0 y)} \xleftrightarrow{CSFT} \delta(u-u_0, v-v_0)$

from modulation property

⑥ $\cos(2\pi(u_0 x + v_0 y)) \xrightarrow{CSFT}$

$$\frac{1}{2}\delta(u-u_0, v-v_0) + \frac{1}{2}\delta(u+u_0, v+v_0)$$

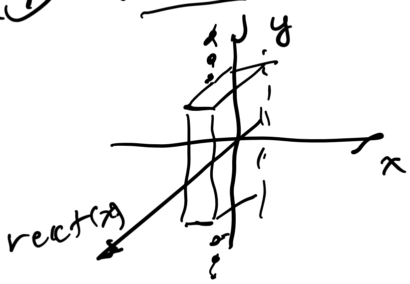




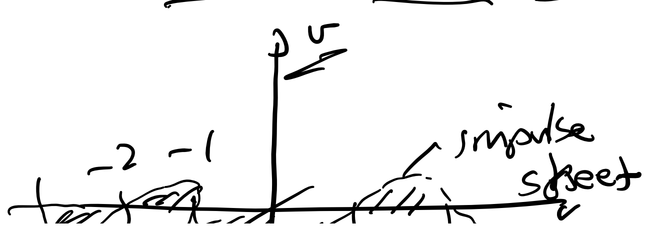
$\rho_0 = \dfrac{1}{P_0}$ $\qquad \phi_0 = \arctan\left(\dfrac{v_0}{u_0}\right)$

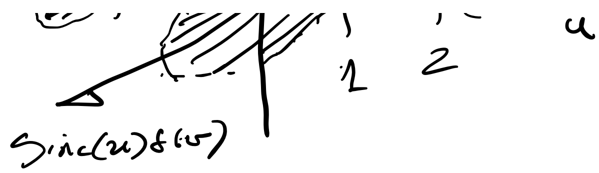
$\rho_0 = \sqrt{u_0^2 + v_0^2}$

⑦ $\mathrm{rect}(x) \xleftrightarrow{CSFT} ??$

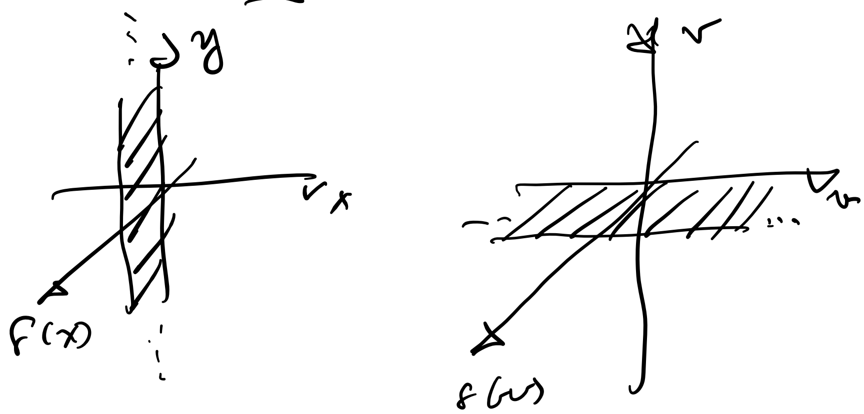


$\mathrm{rect}(x) \cdot 1(y) \xrightarrow{CSFT} \mathrm{sinc}(u) \cdot \delta(v)$

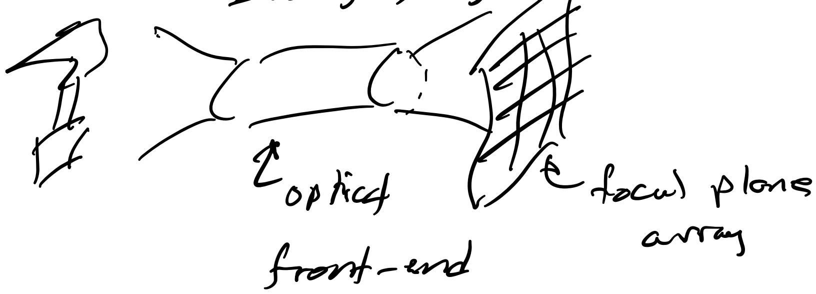




(8) $\delta(x) = \underline{\delta(x)} \cdot \underline{1(y)} \xrightarrow{CSFT} 1 \cdot \delta(v)$

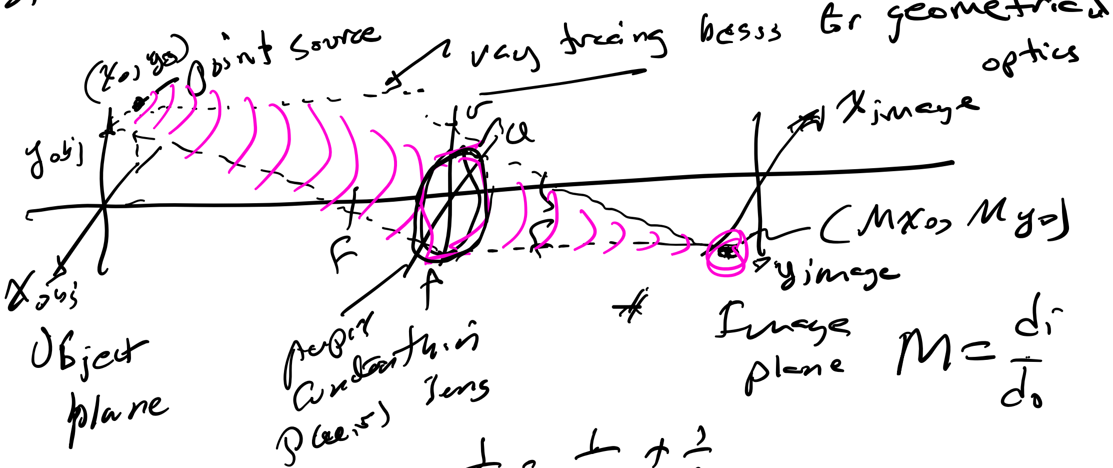


Imaging system



Now we want to analyze the optical front-end

Start with a single thin lens:

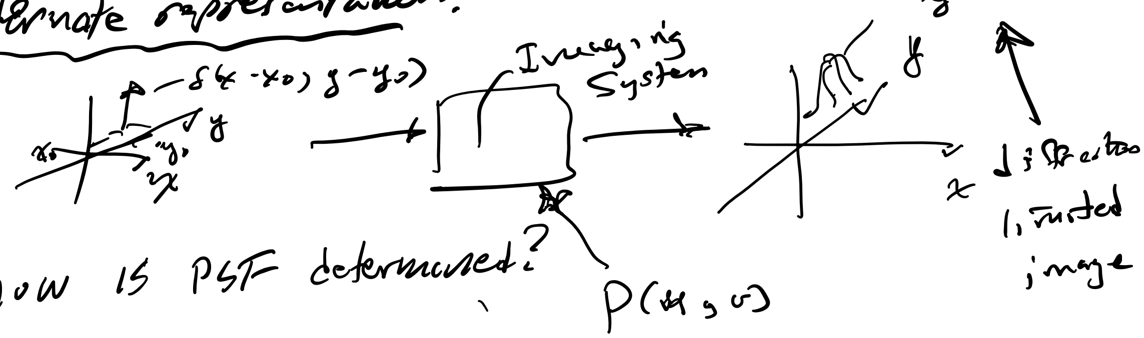


$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$$

What they did not tell you in Freshman Physics:
Image is not a point source — it is a blur spot

Lens is supposed to convert a spherically diverging wave to a spherically converging wave.
But the lens only captures a finite portion of the spherically diverging wave
So the image is a blur spot — called the point spread function (PSF)

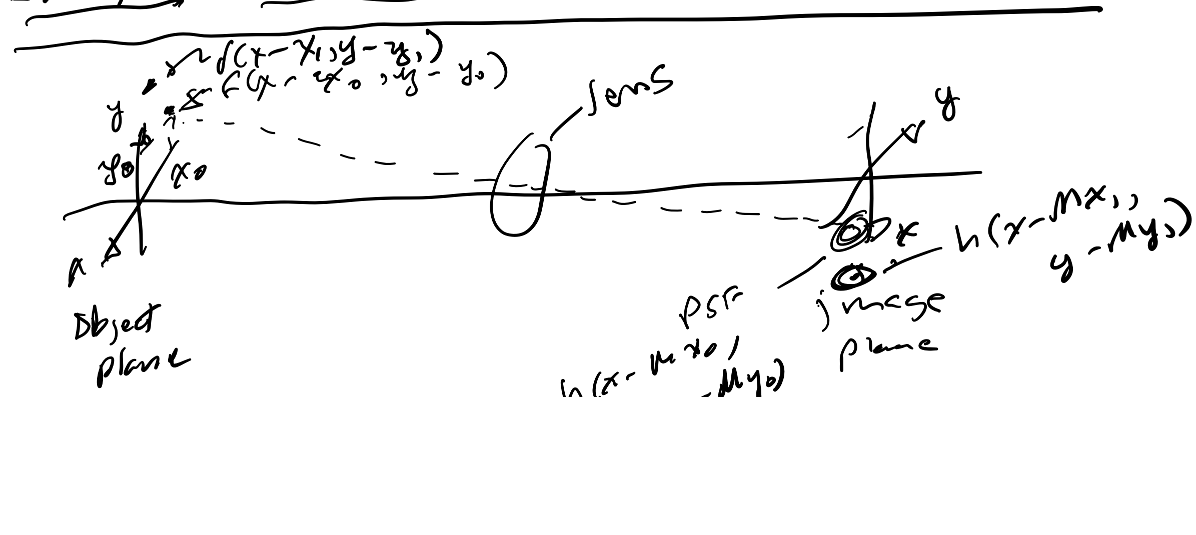
Alternate representation:



how is PSF determined?

will not discuss relation between $P(u,v)$ & $h(x,y)$
See J. W. Goodman, Fourier optics for details

Imaging An Extended Object



Total image is sum of $h(x - Mx_0, y - My_0)$ & $h(x - Mx_1, y - My_1)$

i.e. System obeys superposition

$$f(\xi,\eta)\,\delta(x-\xi, y-\eta) \longrightarrow \boxed{\text{Imaging system}} \longrightarrow f(\xi,\eta)\,h(x - M\xi, y - M\eta)$$

System is also homogeneous