ECE 438 Lecture Wednesday 12 April 2023

Announcements:

- Office Hours today 3:30p EDT
- HW #9 due today 11:59p EDT
- Exam 3 — 10:30a Wed. 19 April 2023 in MSEE 012
  - covering:
    - * HW's 7, 8, 9
    - * and associated lecture
  - Bring photo-ID
  - You will be given updated Formulas

2D Signals & Systems & Image Processing
Module 2.0

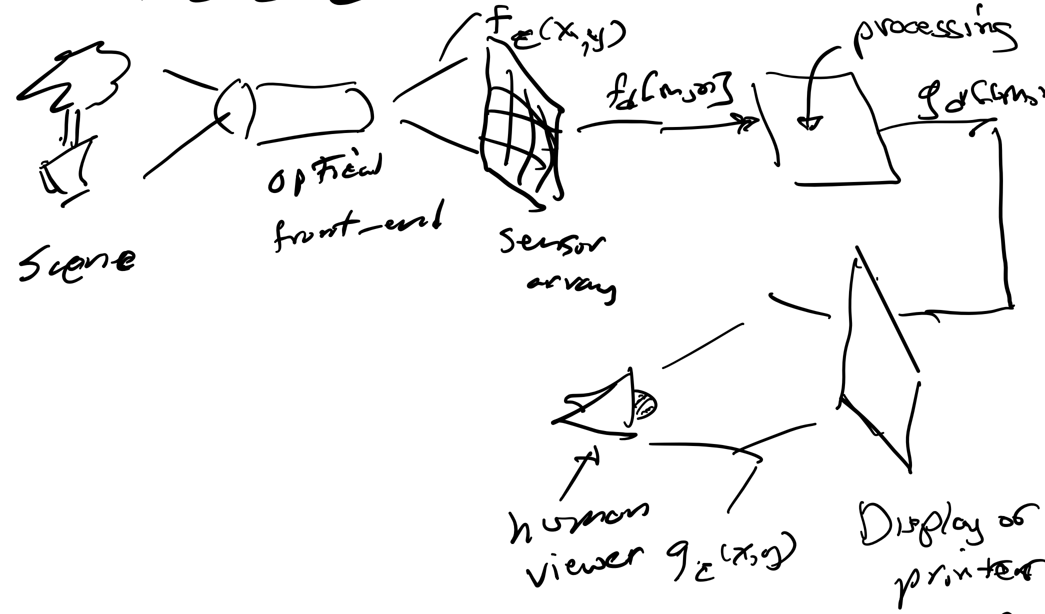
Module		Topic — cover?
2.1	1	2D Linear Systems & Spectral Analysis (Yes)
2.2	2	Sampling & scanning (some)
—		Human Visual System (no)
2.3	3	Printing & Display of Images (some)
2.4	4	Image Enhancement (Yes)
2.5		Image Compression (no)

Formally prepared

hand written 2.6 (5) Image Reconstruction (yes)

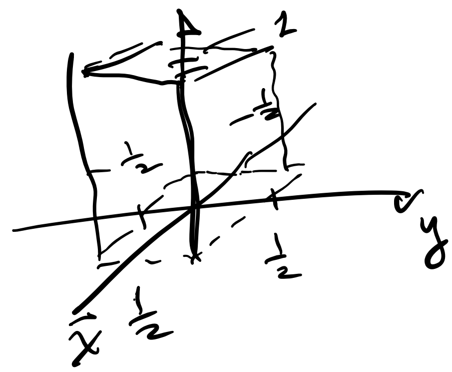
computer-aided tomography (CAT)

Imaging Systems Overview

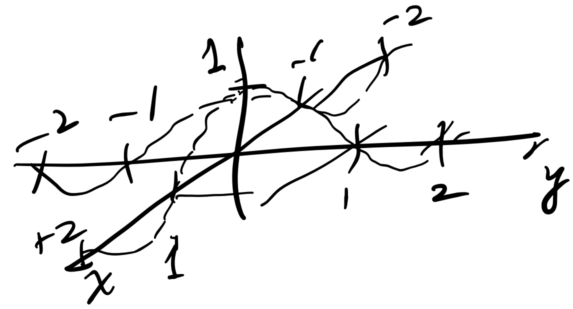


Module 2.1.1 Special 2D Signals

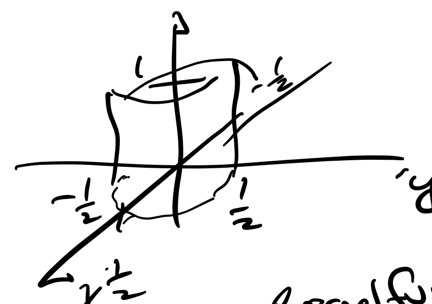
(1) $\mathrm{rect}(x,y) = \begin{cases} 1, & |x|, \& \ |y| \le 0.5 \\ 0, & \text{else} \end{cases}$



(2) $\mathrm{sinc}(x,y) = \frac{\sin(\pi x)}{(\pi x)} \frac{\sin(\pi y)}{(\pi y)}$

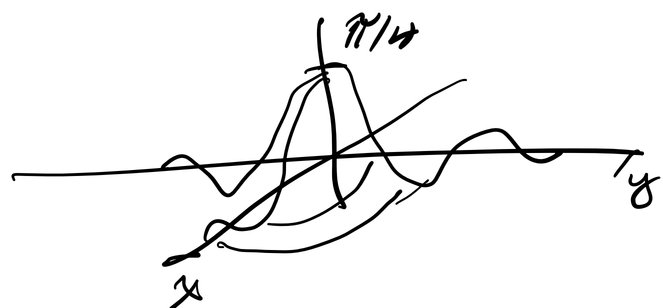


③ $\mathrm{circ}(x,y) = \begin{cases} 1, & \sqrt{x^2+y^2} \le 0.5 \\ 0, & \text{else} \end{cases}$



Bessel function of 1st kind order 1

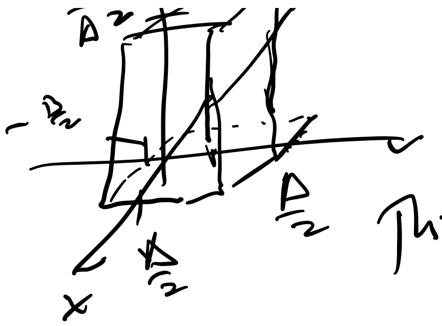
④ $\mathrm{jinc}(x,y) = \dfrac{J_1(\pi\sqrt{x^2+y^2})}{2\sqrt{x^2+y^2}}$



⑤ 2D $\delta(x,y) = \lim_{\Delta \to 0} \dfrac{1}{\Delta^2} \mathrm{rect}\left(\dfrac{x}{\Delta}, \dfrac{y}{\Delta}\right)$

1 |—n.

Volume is unity for all Δ

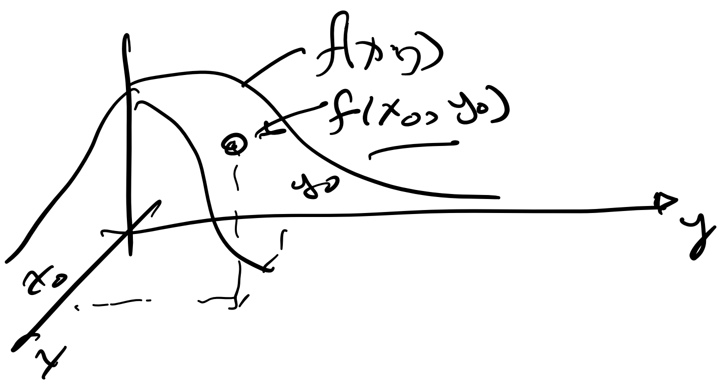


This is only one way to get a 2D impulse

properties of 2D impulse:

(a) $\delta(ax+b, cy+d) \equiv \frac{1}{|ac|}\,\delta\left(x-\frac{b}{a}, y-\frac{d}{c}\right)$

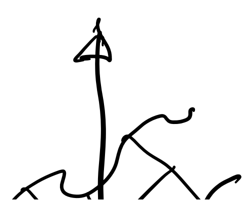
(b) $\iint f(x,y)\,\delta(x-x_0, y-y_0)\,dx\,dy = f(x_0, y_0)$

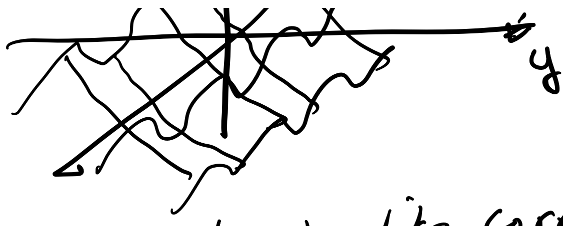


(c) 2D Sinusoid

$A\cos(2\pi(u_0x + v_0y) + \theta)$

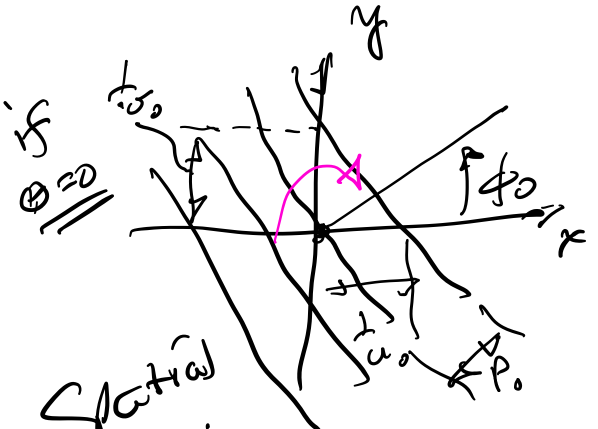
What does this look like?





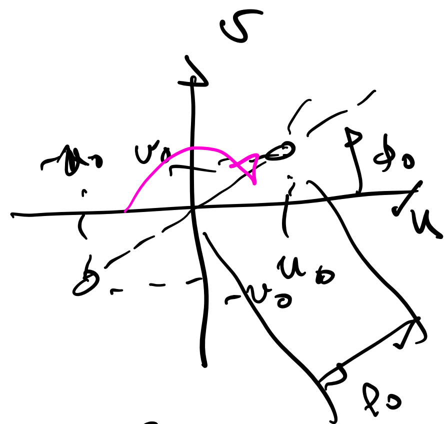
looks like corrugated roofing

Top view



Spatial domain

lines of constant amplitude



Frequency domain

$\rho_0 = \sqrt{u_0^2 + v_0^2}$

$\phi_0 = \arctan\left(\frac{v_0}{u_0}\right) \quad l_0 = \frac{1}{\rho_0}$

Module 2.1.2 - Continuous space Fourier transform (CSFT)

Forward transform

$$F(u,v) = \iint_{-\infty}^{\infty} f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\,dy$$

Inverse transform

$$f(x,y) = \iint_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)}\, du\,dv$$

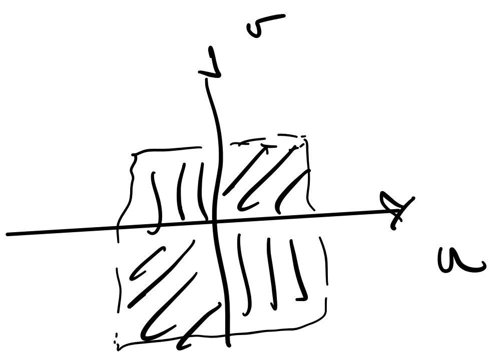
$f(x,y) = \iint F(u,v) e$ ...

Extension of 1D CT FT

(u,v) - spatial frequencies units are cycles/unit distance

Hermetian symmetry

For a real-valued $f(x,y)$, $F(u,v)$ has Hermetian symmetry



only two unique quadrants, since

$$F(-u,-v) = F^*(u,v)$$

complex conjugate

Transform relations

1. linearity

$$a g_1(x,y) + b g_2(x,y) \xleftrightarrow{CSFT} a G_1(u,v) + b G_2(u,v)$$

2. scaling & shifting

$$f(a(x-x_0), b(y-y_0)) \xrightarrow{CSFT} \frac{1}{|ab|} F\left(\frac{u}{a}, \frac{v}{b}\right) e^{-j2\pi(u x_0 + v y_0)}$$

3. Modulation

$$f(x,y) e^{j2\pi(u_0 x + v_0 y)} \xleftrightarrow{CSFT} F(u-u_0, v-v_0)$$

4. Reciprocity

Suppose $f(x,y) \xrightarrow{CSFT} F(u,v)$

then $F(x,y) \xleftrightarrow{CSFT} f(-u, -v)$

5. Parseval's relation

$$\int\int_{-\infty}^{\infty} |f(x,y)|^2 dx dy = \int\int_{-\infty}^{\infty} |F(u,v)|^2 du dv$$