

## 1.1.2 SIGNAL CHARACTERISTICS

### 1. deterministic

random

### 2. periodic: $x(t) = x(t + nT)$ , for some fixed $T$ , and for all integers $n$ and times $t$ .

aperiodic

### 3. right-sided: $x(t) \neq 0$ , only for $t \geq t_0$ . If $t_0 \geq 0$ , signal is causal.

left-sided:  $x(t) \neq 0$ , only for  $t \leq t_0$  for some fixed  $t_0$ .  
If  $t_0 \leq 0$ , signal is anticausal.

2-sided or mixed causal:

$x(t) \neq 0$  for some  $t < 0$ , and  $x(t) \neq 0$  for some  $t > 0$ .

### 4. finite duration: $x(t) \neq 0$ , only for $-\infty < t_1 \leq t \leq t_2 < \infty$

infinite duration.

## Metrics

### 1. Energy

$$E_x = \int |x(t)|^2 dt \quad \text{CT}$$

$$E_x = \sum_n |x(n)|^2 \quad \text{DT}$$

## 2. Power

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad \text{CT}$$

$$P_x = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2 \quad \text{DT}$$

## Comments

- a. If signal is periodic, we need only average over one period, *e.g.* if  $x(n) = x(n + N)$  for all  $n$ ,

$$P_x = \frac{1}{N} \sum_{n=n_0}^{n_0+N-1} |x(n)|^2 \quad \text{for any } n_0$$

- b. root-mean-square (rms) value

$$x_{\text{rms}} = \sqrt{P_x}$$

### 3. Magnitude

$$M_x = \max_t |x(t)| \quad \text{CT}$$

$$M_x = \max_n |x(n)| \quad \text{DT}$$

If  $M_x < \infty$ , signal is *bounded*.

### 4. Area

$$A_x = \int x(t)dt \quad \text{CT}$$

$$A_x = \sum_n x(n) \quad \text{DT}$$

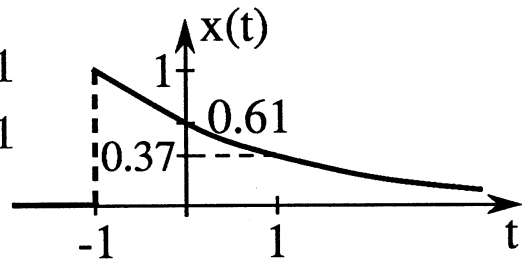
## 5. Average Value

$$x_{\text{avg}} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x(t) dt \quad \text{CT}$$

$$x_{\text{avg}} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N x(n) \quad \text{DT}$$

## Examples

$$\text{A. } x(t) = \begin{cases} e^{-(t+1)/2}, & t > -1 \\ 0, & t < -1 \end{cases}$$



1. deterministic
2. aperiodic
3. right-sided, but not causal
4. infinite duration

## Metrics

$$1. \quad E_x = \int_{-1}^{\infty} |e^{-(t+1)/2}|^2 dt, \quad \text{let } s = t + 1$$

$$= \int_0^{\infty} e^{-s} ds = -e^{-s} \Big|_0^{\infty} = 1$$

$$2. \quad P_x = 0$$

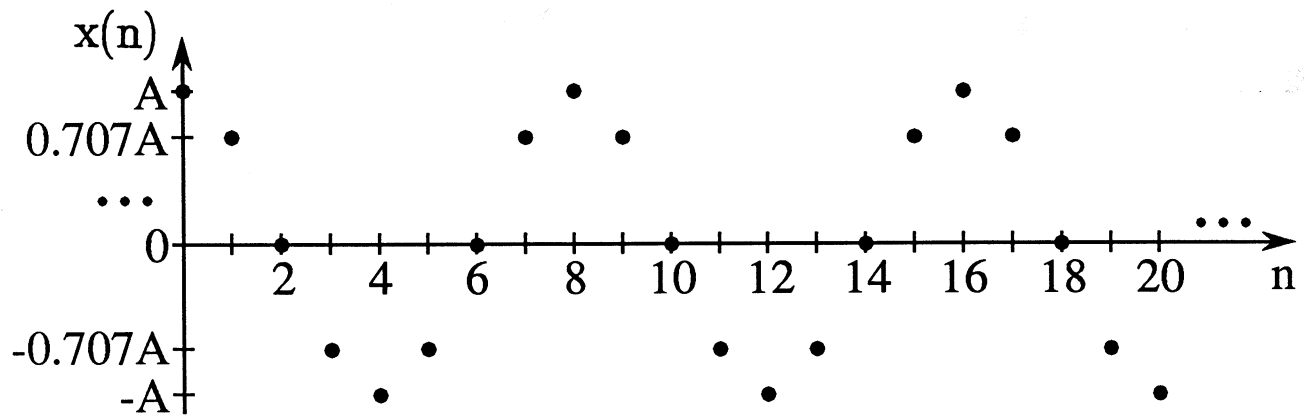
$$4. \quad A_x = 2$$

$$3. \quad M_x = 1$$

$$5. \quad x_{\text{avg}} = 0$$



B.  $x(n) = A \cos(\pi n/4)$



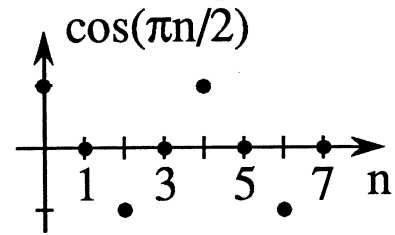
1. deterministic
2. periodic ( $N=8$ )
3. 2-sided
4. infinite duration

## Metrics

1.  $E_x = \infty$

2.  $P_x = \frac{1}{8} \sum_{n=0}^7 |A \cos(\pi n/4)|^2, \quad \cos^2 \theta = \frac{1}{2} + \frac{1}{2} \cos 2\theta$

$$= \frac{A^2}{16} \sum_{n=0}^7 [1 + \cos(\pi n/2)],$$



$$= \frac{A^2}{2}$$

$$x_{\text{rms}} = \frac{A}{\sqrt{2}},$$

3.  $M_x = A,$

4.  $A_x$  is undefined.

5.  $x_{\text{avg}} = 0$