ECE 438 15 April 2019

(1)

Help Sessions

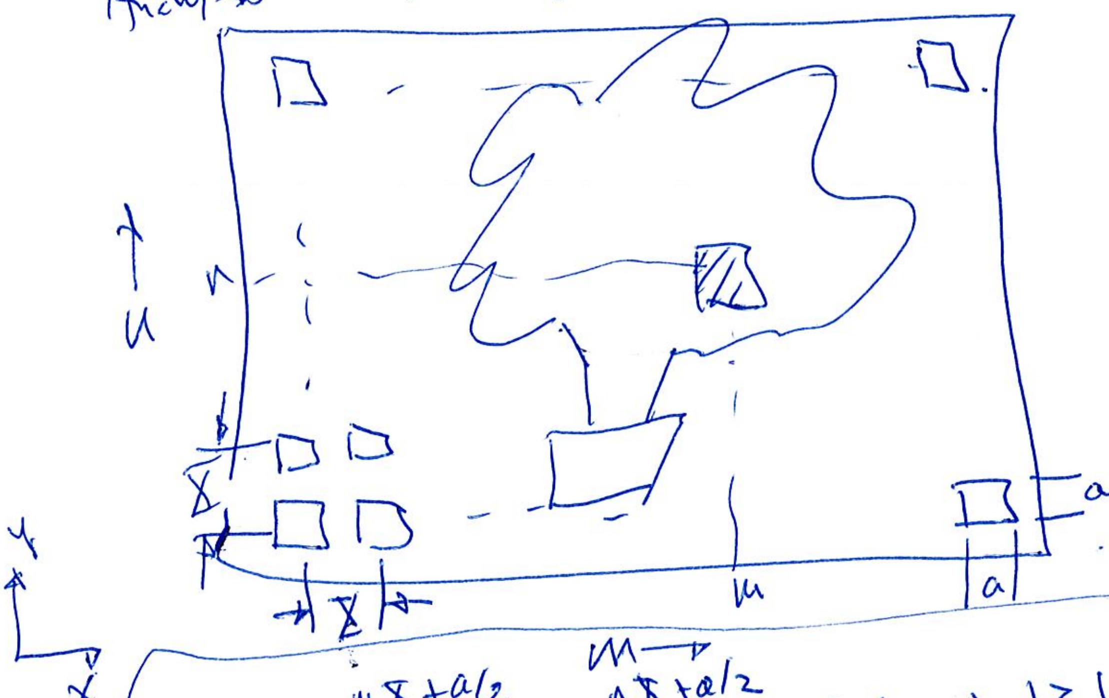
2:00 – 3:00p } EE 222
5:00 – 6:00p }

Exam 3 – Wed. 17 April

Analysis of 2-D signals & systems (Continuous-parameter) (CS)

Analysis of focal plane array

Module 2.2.2



$$\text{sampled signal} \rightarrow \boxed{g[m,n] = \int_{m\Delta - a/2}^{m\Delta + a/2} \int_{n\Delta - a/2}^{n\Delta + a/2} f(\xi, \eta)\, d\xi\, d\eta}$$

Representations $g[m,n]$ (DS)

CS $g_s(x,y) = \sum_m \sum_n g[m,n]\, \delta(x - m\Delta, y - n\Delta)$

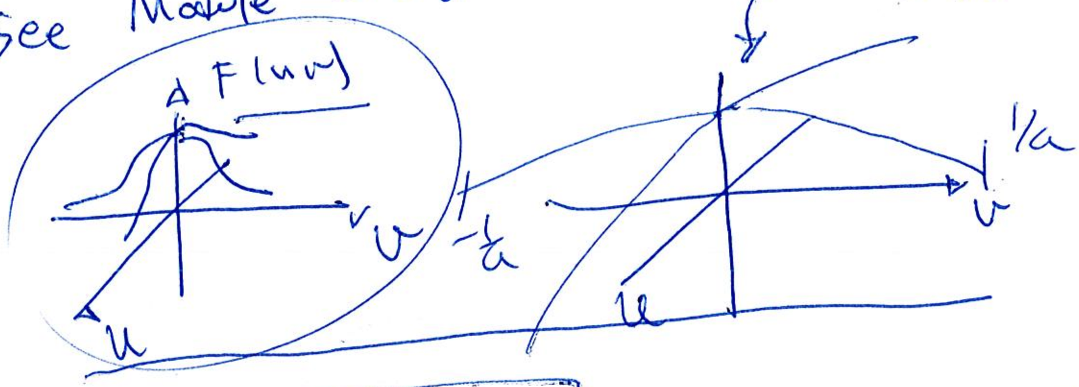
recall development of relation between CTFT & DTFT

Looking for a relationship for $g_s(x,y)$ or $G_s(u,v)$ in terms of $f(x,y)$ or $F(u,v)$ (2)

$$g_s(x,y) = \text{comb}_{XX}\left[\text{rect}\left(\frac{x}{a},\frac{y}{a}\right) ** f(x,y)\right]$$

$$\underline{G_s(u,v)} = \frac{a^2}{X^2}\,\text{rep}_{\frac{1}{X}\frac{1}{X}}\Big[\overbrace{\underbrace{\cancel{a}\,\text{sinc}(au,av)}_{\text{sinc}\left(\frac{u}{1/a},\frac{v}{1/a}\right)}F(u,v)}^{\hat{F}(u,v)}\Big]$$

See Module 2.2.3

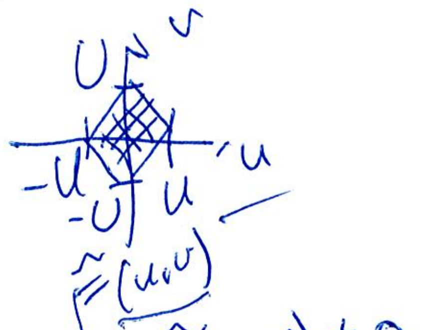


$\boxed{\hat{F}(u,v) = \text{sinc}\left(\frac{u}{1/a},\frac{v}{1/a}\right)F(u,v)}$ rolls off a little faster as $(u,v) \nearrow$ relative to $F(u,v)$ [aperture effect]

∴ How does $\hat{f}(x,y)$ compare $f(x,y)$??

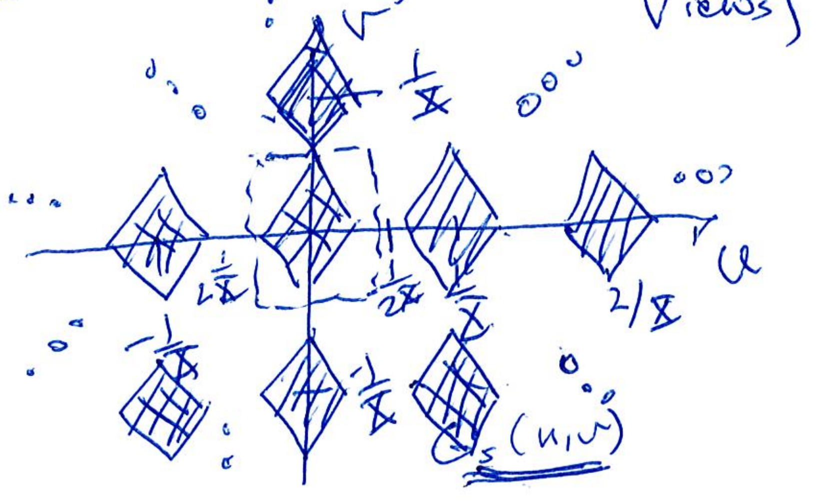
$\hat{f}(x,y)$ will look a bit blurred (unsharp) relative to $f(x,y)$

Now consider effect of sampling (look at top views)



$\hat{F}(u,v)$

$\boxed{} - \hat{F}(u,v) \neq 0$



To recover $f(x,y)$ from $\tilde{f}(x,y)$ apply a LSIV filter with frequency response

$$H_c(u,v) = \begin{cases} \dfrac{1}{\mathrm{sinc}(au, av)}, & |u|, |v| < \underline{U} \\ 0, & \text{else} \end{cases}$$

$\uparrow$ correction

Then $F_c(u,v) = H_c(u,v)\,\tilde{F}(u,v)$

or $f_c(x,y) = h_c(x,y) ** \tilde{f}(x,y)$

Let's assume that this correction has been done so: $g_s(x,y) = \mathrm{comb}_{XX}[f(x,y)]$

$\Rightarrow G_s(u,v) = \frac{1}{X^2}\,\mathrm{rep}_{\frac{1}{X}\frac{1}{X}}[F(u,v)]$

To recover $f(x,y)$ from $g_s(x,y)$, apply an ideal low pass filter: $H_r(u,v) = X^2\,\mathrm{rect}\left(\frac{u}{1/X}, \frac{v}{1/X}\right)$

$= X^2\,\mathrm{rect}(Xu, Xv)$

$\Rightarrow \boxed{h_r(x,y) = \mathrm{sinc}\left(\frac{x}{X}, \frac{y}{X}\right)}$

so $f_r(x,y) = h_r(x,y) ** g_s(x,y)$

so $f_r(x,y) = h_r(x,y) ** \mathrm{comb}_{XX}[f(x,y)]$

$= h_r(x,y) ** \left[\sum_m \sum_n \underline{f(mX, nX)}\,\delta(x - mX, y - nX)\right]$

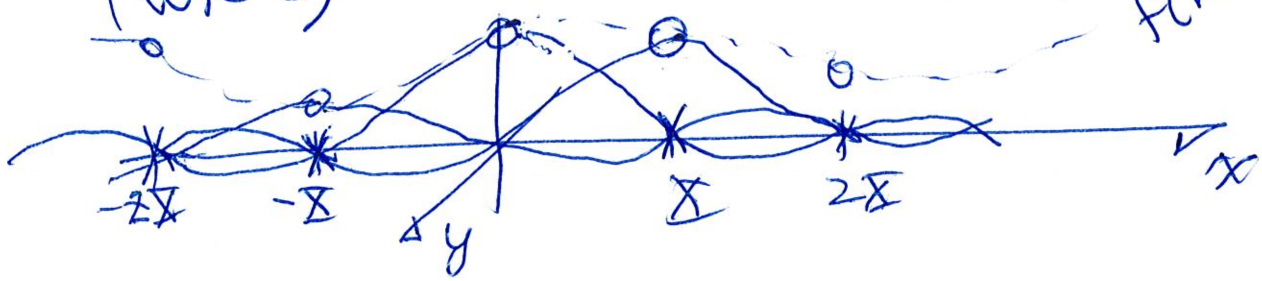
$$f_r(x,y) = \sum_m \sum_n f[mX, nX]\, h_r(x,y) ** \delta(x-mX, y-nX)$$

$$= \sum_m \sum_n f[mX, nX]\, h_r(x-mX, y-nX)$$

$$\boxed{f_r(x,y) = \sum_m \sum_n f[mX, nX]\, \mathrm{sinc}\left(\frac{x-mX}{X}, \frac{y-nX}{X}\right)}$$

Shannon sampling expansion (WKS)

$\Rightarrow f_r(mX, nX) \equiv f(mX, nX)$

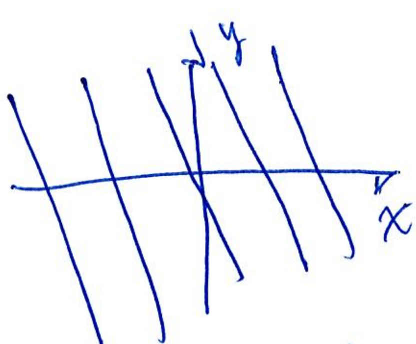


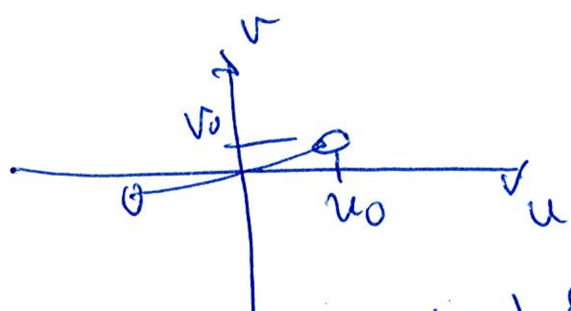
Reconstruction is perfect only if

$F(u,v) = 0, \; |u|, |v| > \frac{1}{2X}$

$\Rightarrow U < \frac{1}{2X}$ Nyquist sampling condition

otherwise we have aliasing





$f(x,y) = \cos(2\pi(u_0 x + v_0 y))$

$F(u,v) = \frac{1}{2}\delta(u-u_0, v-v_0) + \frac{1}{2}\delta(u+u_0, v+v_0)$