ECE 438 Lecture — 8 April 2019

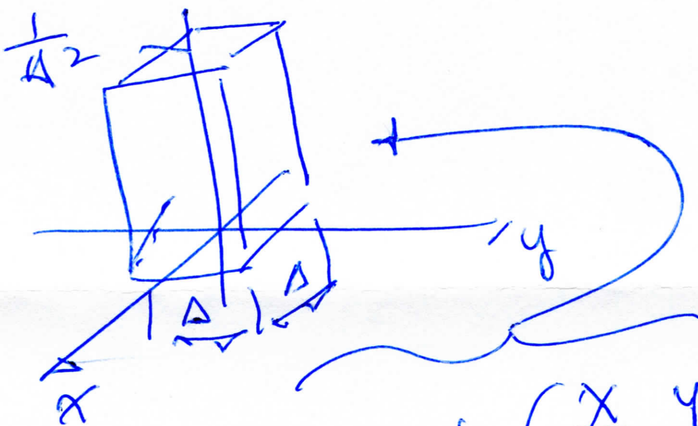
Help Session TODAY 3:30 – 5:00p EE222

↑ note delayed start

Module 2.1.1

$$\mathrm{rect}(x, y) = \begin{cases} 1, & |x|, |y| < 1/2 \\ 0, & \text{else} \end{cases}$$

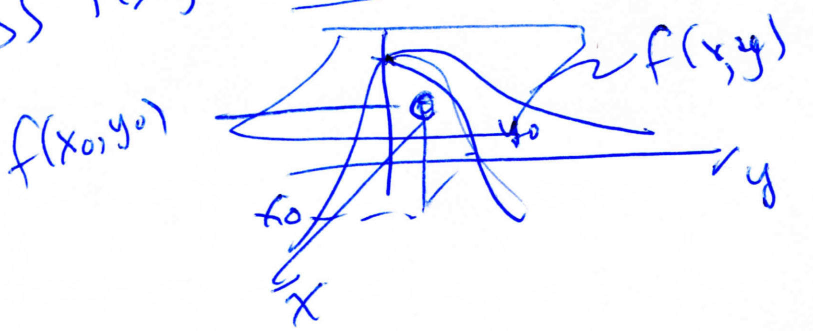
2-D Impulse Function



$$\delta(x, y) = \lim_{\Delta \to 0} \frac{1}{\Delta^2} \mathrm{rect}\left(\frac{x}{\Delta}, \frac{y}{\Delta}\right)$$

Properties

1. $$\iint f(x,y)\, \delta(x - x_0, y - y_0)\, dx\, dy = f(x_0, y_0)$$

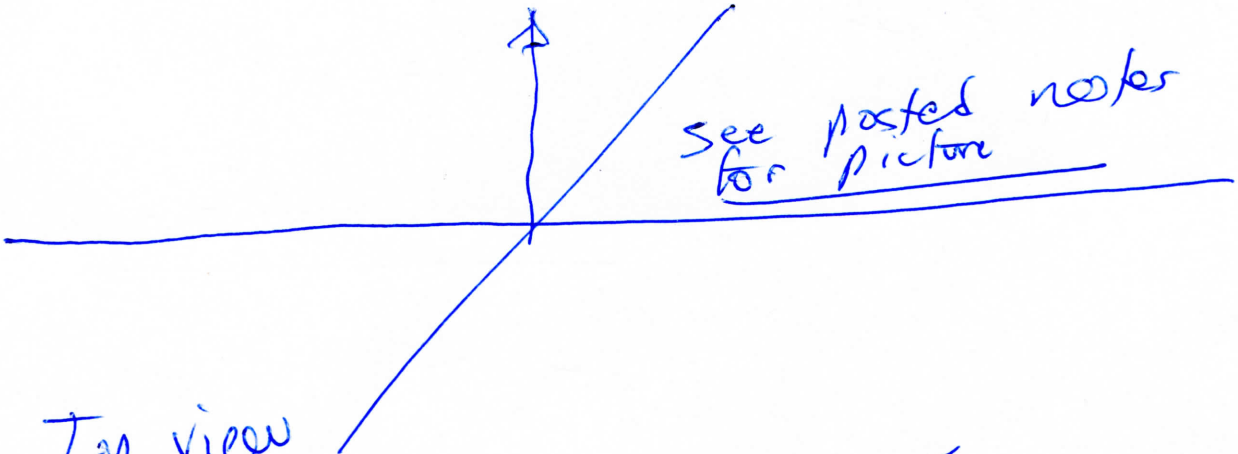


2. $$f(x,y)\, \delta(x - x_0, y - y_0) = f(x_0, y_0)\, \delta(x - x_0, y - y_0)$$

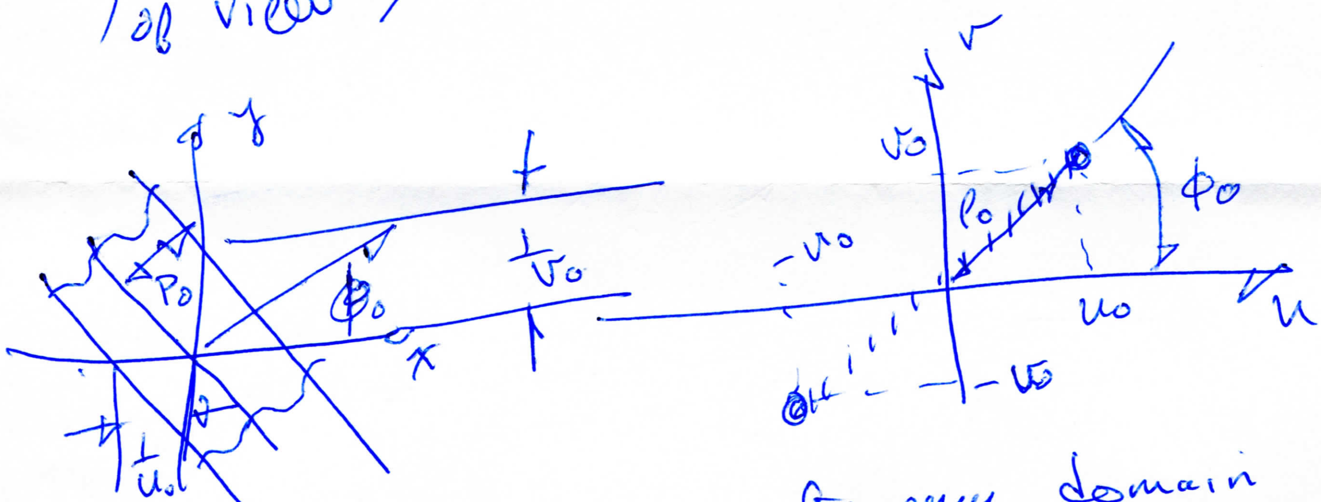
(3) $\delta(ax+b, cy-d) \equiv \frac{1}{|ac|}\delta(x-b/a, y-c/d)$

Spatial frequency component

$A\cos(2\pi(u_0 x + v_0 y) + \theta)$



Top view



can show that

$\phi_0 = \arctan\left(\frac{v_0}{u_0}\right)$ $\qquad P_0 = \frac{1}{\rho_0}$

$\rho_0 = \sqrt{u_0^2+v_0^2}$

2D Continuous-Space Fourier Transform
CSFT (analogous to CTFT)

Forward transform

$$F(u,v) = \iint f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

Inverse transform

$$f(x,y) = \iint F(u,v)\, e^{j2\pi(ux+vy)}\, du\, dv$$

units of u & v — cycles/unit distance

2-D Transform Relations

1. linearity
$$a_1 f_1(x,y) + a_2 f_2(x,y) \overset{CSFT}{\longleftrightarrow} a_1 F_1(u,v) + a_2 F_2(u,v)$$

2. scaling & shifting
$$f\left(\frac{x-x_0}{a}, \frac{y-y_0}{b}\right) \overset{CSFT}{\longleftrightarrow} |ab|\, F(au, bv)\, e^{-j2\pi(ux_0+vy_0)}$$

3. modulation
$$f(x,y)\, e^{j2\pi(u_0x+v_0y)} \overset{CSFT}{\longleftrightarrow} F(u-u_0, v-v_0)$$

4. reciprocity
Given $\underline{f(x,y)} \overset{CSFT}{\longleftrightarrow} \underline{F(u,v)}$
then $F(x,y) \overset{CSFT}{\longleftrightarrow} f(-u,-v)$

5. Parseval
$$\iint |f(x,y)|^2\, dx\, dy = \iint |F(u,v)|^2\, du\, dv$$

6. Initial value

$$\iint f(x,y)\,dx\,dy = \underline{F(0,0)}$$

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7. <u>Separability</u>

A function $f(x,y)$ is separable if it satisfies $f(x,y) = g(x)h(y)$

<u>Some examples are:</u>

$$\Rightarrow \text{rect}(x,y) = \text{rect}(x)\,\text{rect}(y)$$

$$\text{sinc}(x,y) = \text{sinc}(x)\,\text{sinc}(y)$$

$$\delta(x,y) = \delta(x)\,\delta(y)$$

Transform relation for separability

let $g(x) \xleftrightarrow{\text{1-D CSFT}} G(u)$

$h(y) \xleftrightarrow{\text{1-D CSFT}} H(v)$

then $f(x,y) = g(x)h(y) \xleftrightarrow{\text{2D-CSFT}} G(u)H(v)$

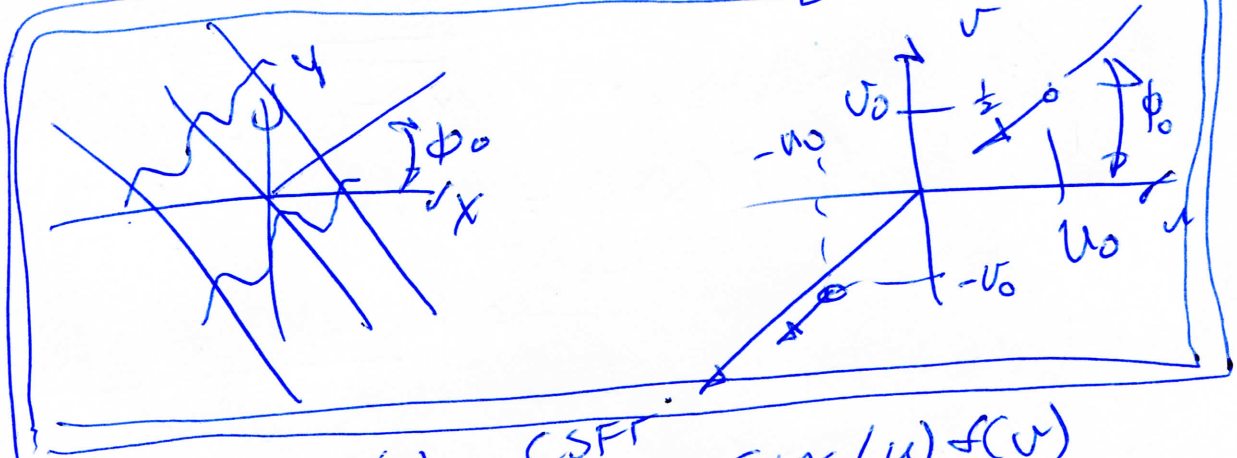
<u>Important Transform Pairs</u>

1. $\text{rect}(x,y) \xleftrightarrow{\text{CSFT}} \text{sinc}(u,v)$
2. $\text{circ}(x,y) \xleftrightarrow{\text{CSFT}} \text{jinc}(u,v)$
3. $\delta(x,y) \xleftrightarrow{\text{CSFT}} 1$
4. $1 \xleftrightarrow{\text{CSFT}} \delta(u,v)$ by reciprocity

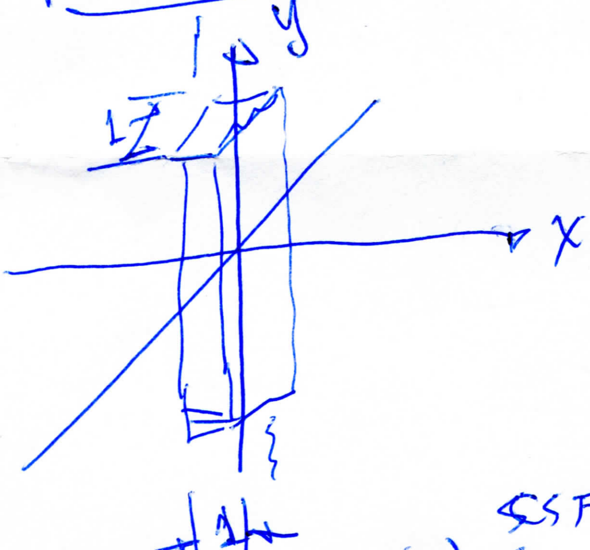
5. $e^{j2\pi(u_0x+v_0y)} \xleftrightarrow{CSFT} \delta(u-u_0, v-v_0)$

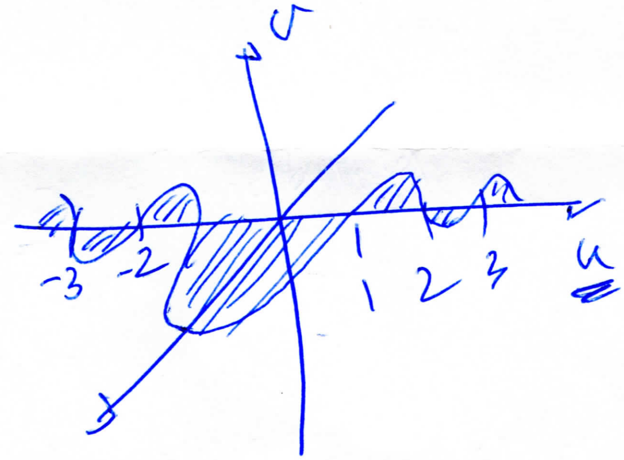
by modulation

6. $\cos(2\pi(u_0x+v_0y)) \xleftrightarrow{CSFT} \frac{1}{2}\delta(u-u_0, v-v_0) + \frac{1}{2}\delta(u+u_0, v+v_0)$



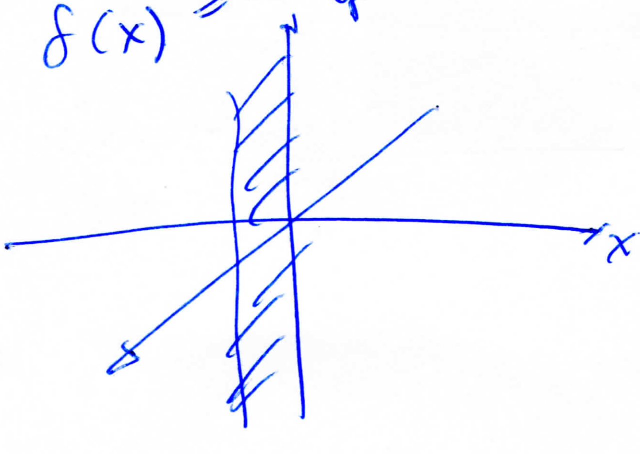
7. $\underline{rect(x)\cdot 1(y)} \xleftrightarrow{CSFT} sinc(u)\delta(v)$

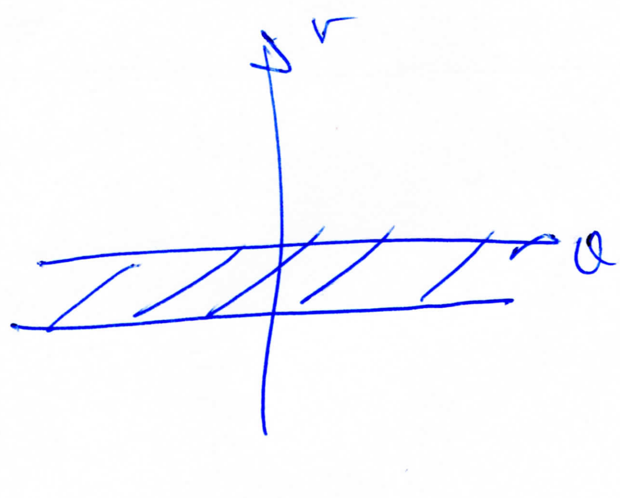




$\delta(x) = \delta(x)\cdot 1(y) \xleftrightarrow{CSFT} 1\cdot\delta(v)$

8. $\delta(x)$

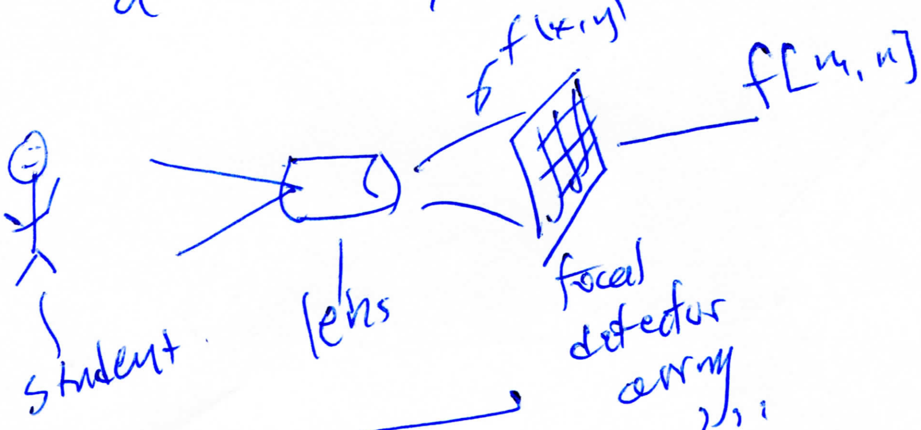




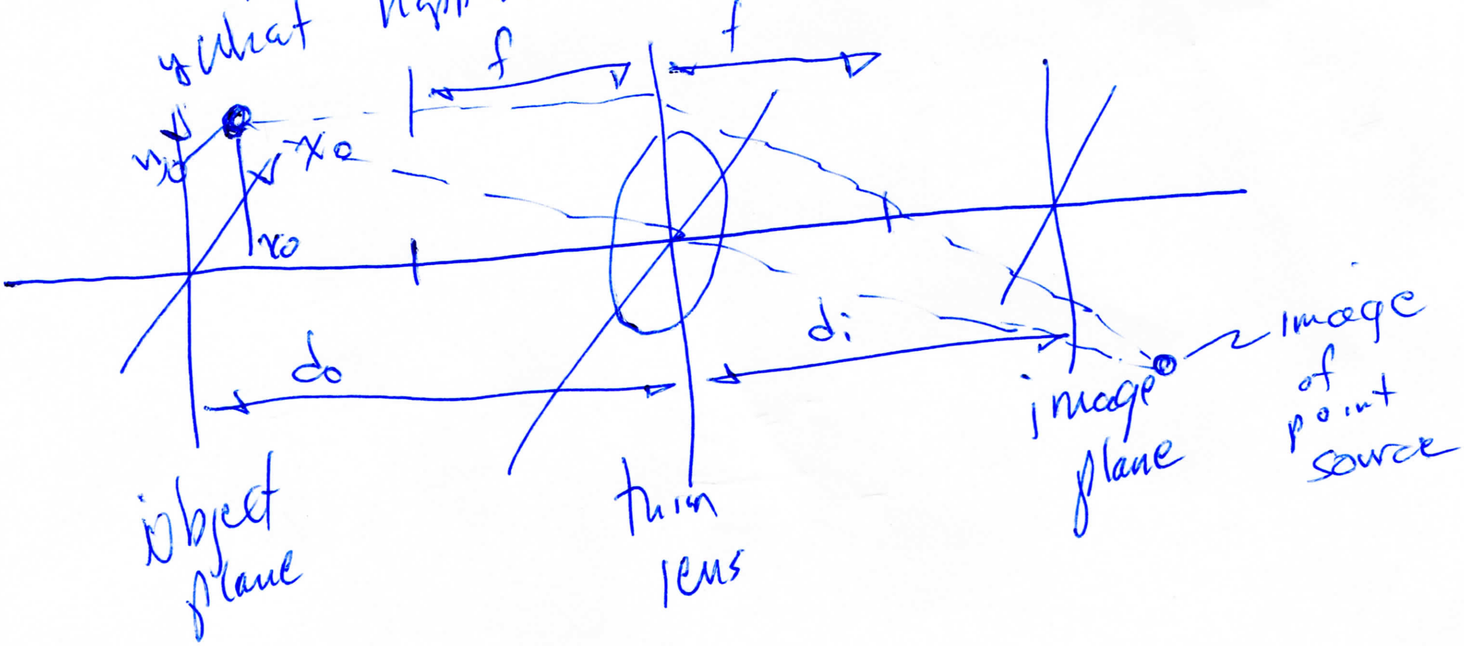
Properties

1. Rotating an image $f(x,y)$ by ϕ_0 degrees also rotates its CSFT $F(u,v)$ by ϕ_0 degrees.

2. A circularly symmetric image has a circularly symmetric CSFT



what happens here???



f - focal length

ray tracing - geometrical optics