ECE 438 Lecture

4/1/2019 (1)

Linear Predictive Coding (LPC) of Speech (Module 4.3)

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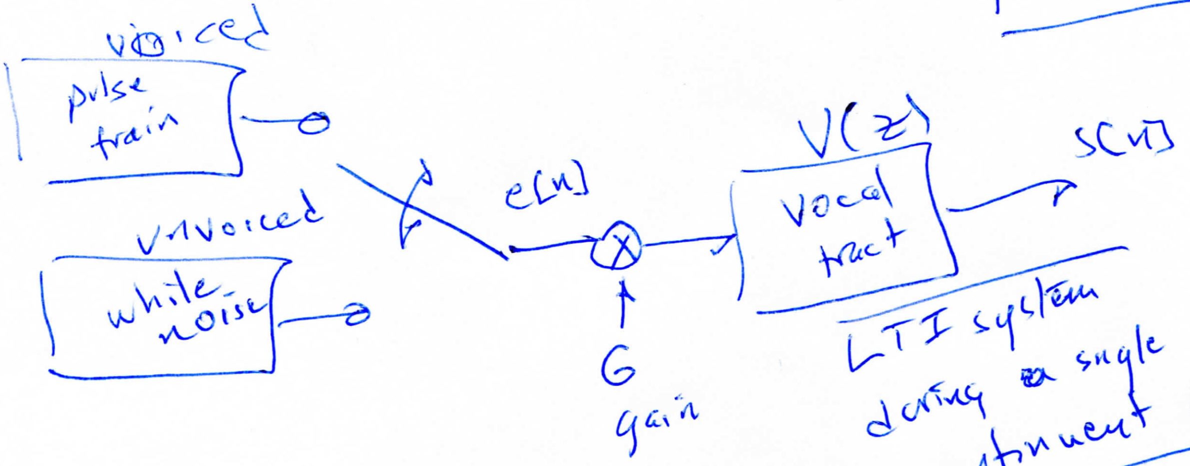

LTI system during a single continuant phoneme

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short-time DTFT (STDTFT)
formant frequencies

} frequency domain

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time-domain

develop a time-domain model for speech waveform

All-pole model

$$V(z) = \frac{G}{1 - \sum_{k=1}^{p} a_k z^{-k}}$$

Model parameters:

a_k, $k = 1, \ldots, p$ - no. of poles

G - gain

Time-domain model:

$$s[n] = \sum_{k=1}^{p} \underbrace{a_k}_{\text{unknown}} s[n-k] + G \underbrace{e[n]}_{\text{excitation}}$$

linear predictor

$$\hat{s}[n] = \sum_{k=1}^{p} \underbrace{\alpha_k}_{\text{predictor coefficients}} s[n-k]$$

how do we judge performance of predictor?

$f[n] = s[n] - \hat{s}[n]$ would like $f[n]$ to be small?

Choose α_k's $k = 1, \dots, p$ to minimize

$$E = \sum_{n} f^2[n]$$

Suppose $\alpha_k = a_k$, $k = 1, \dots, p$

let $A(z) = 1 - \sum_{k=1}^{p} \alpha_k z^{-k}$

$$s[n] \longrightarrow \boxed{A(z)} \longrightarrow f(n)$$

If model is correct & we estimate $k = 1, \dots, p$ then

$\alpha_k = a_k$,

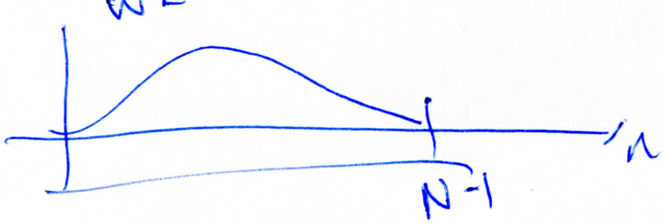
$$f[n] = G e[n]$$

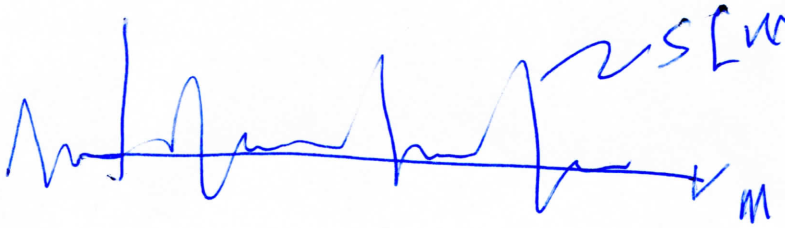
$\Rightarrow$ we can estimate G & the pitch period from $f[n]$

Consider only short-time intervals of $s[n]$

let $s_n[m] = w[m]\, \underline{s[m+n]}$

$w[m] \neq 0$ for $0 \le m \le N-1$





shift back in time by n samples

let $f_n[m] = \underline{s_n[m]} - \underline{\hat{s}_n[m]}$

recall
$$\boxed{\hat{s}_n[m] = \sum_{k=1}^{p} \alpha_k\, s_n[m-k]}$$

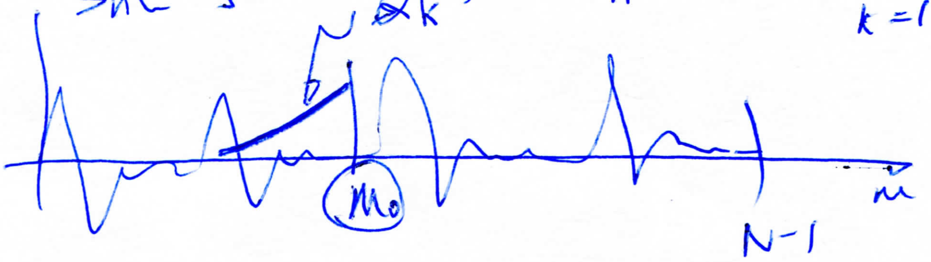
$\neq 0$ only for

$\underline{0 \le m \le N+p-1}$

$s_n[m] \neq 0$ only $\underline{0 \le m \le N-1}$

$\therefore$ $f_n[m] \neq 0$ only for $0 \le m \le N+p-1$

$s_n[m_0] = \sum_{k=1}^{p} \alpha_k s_n[m_0-k]$

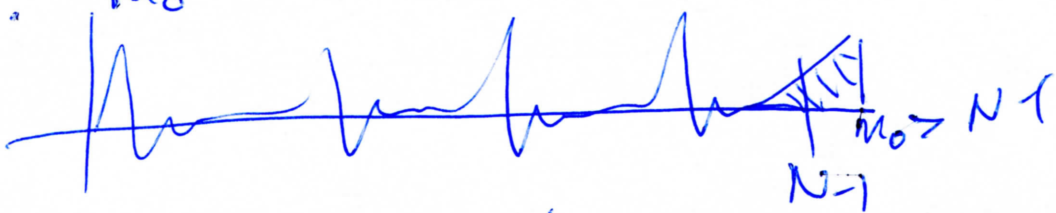


<u>Epochs</u> in prediction error signal

I. $0 \le m_0 \le p$ $\Rightarrow$ <u>large error</u> because I don't have complete data

II. $p \le m_0 \le N-1$ $\Rightarrow$ small error because I <u>have</u> complete data

III. $m_0 > N-1$



prediction should be zero, but we have nonzero data on which prediction based $\Rightarrow$ large error

$\Rightarrow$ should choose following limits for my error:

$$\Phi = \sum_{m=p}^{N-1} (f_n[m])^2$$

But $f_n[m] \neq 0$ for $0 \le m \le N-1+p$

Two solutions:

1. let $E_n = \sum_{m=p}^{N-1} (f_n[m])^2$

   and solve for unknown α_k's

   $\Rightarrow$ covariance method

2. let $E_n = \sum_{m=-\infty}^{\infty} (f_n[m])^2$

   and solve for unknown α_k's

   $\Rightarrow$ autocorrelation method

We will derive optimal α_k's for this case, i.e. minimize E_n

Consider

$$E_n \doteq \sum_{m=-\infty}^{\infty} \left\{ s_n[m] - \sum_{k=1}^{p} \alpha_k s_n[m-k] \right\}^2 \qquad (2)$$

pick a particular α_ℓ to optimize

$$\frac{\partial I_n}{\partial \alpha_\ell} = \sum_{m=-\infty}^{\infty} 2 \left\{ s_n[m] - \sum_{k=1}^{p} \alpha_k s_n[m-k] \right\} \left\{ 0 - s_n[m-\ell] \right\}$$

$$= -2 \sum_m s_n[m] s_n[m-\ell] + 2 \sum_m \sum_{k=1}^{p} \alpha_k s_n[m-k] s_n[m-\ell]$$

let $m - \ell = m'$ in second sum
$m = m' + \ell$

$$\frac{\partial E_n}{\partial \alpha_\ell} = -2 \sum_m s_n[m] s_n[m-\ell] + 2 \sum_{k=1}^{p} \alpha_k \sum_{m'=-\infty}^{\infty} s_n[m' - (k+\ell)] s_n[m']$$

define $R_n[\ell] = \sum_m s_n[m] s_n[m+\ell]$

$$= R_n[-\ell]$$

$$= \sum_m s_n[m] s_n[m-\ell]$$

let $\frac{\partial E_n}{\partial \alpha_\ell} = 0$

unknown

$R_n[k-\ell]$

$$\Rightarrow \boxed{R_n[\ell] = \sum_{k=1}^{p} \alpha_k R_n[\ell - k] \qquad \ell = 1, \ldots, p}$$

known data

Consider matrix-vector formulation:
rows are ℓ

$$\begin{bmatrix} R_n[0] & R_n[1] & \cdots & R_n[p-1] \\ & \ddots & & \\ & & \ddots & \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{bmatrix} = \begin{bmatrix} R_n[0] \\ R_n[1] \\ \vdots \\ R_n[p] \end{bmatrix}$$