ECE 438 Lecture

3/18/2019 (1)

(1) Estimating statistics
(2) Filtering of random signals } Today

- Wednesday - Exam 2
  HWs 4, 5, 6

- Friday - Speech!

Estimating statistics

Observe i.i.d. sequence mean μ_X, variance σ_X^2

$$X_1, X_2, \ldots, X_N$$

Want to ~~estimate~~ estimate

$$E\{X_i\} = \hat{\mu}_X \leftarrow \text{estimate}$$

$$\text{let } \hat{\mu}_X = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$\hat{\mu}_X$ is a random variable - call it Y

showed that

$$\mathcal{E}\{Y\} = \mu_X \quad \text{— unbiased}$$

$$\sigma_Y^2 = \frac{1}{N}\sigma_X^2 \quad \text{— consistent}$$

$$\lim_{N\to\infty} \sigma_Y^2 = 0$$

Filtering of random signals

define

$$\mu_X[n] = \mathcal{E}\{X_n\}$$

$$r_{XX}[m,n] = E\{X_m X_n\} \quad \text{autocorrelation}$$

Special case

A process is wide-sense stationary (w.s.s)

if (1) $\mu_X[n] = \mu_X$ mean is constant

(2) $r_{XX}[m,n] = r_{XX}[n-m, 0]$

$$\triangleq r_{XX}[n-m]$$

Consider

$$Y[n] = \sum_m h[n-m] \cancel{X}[m] \; X_m$$

Y_n

$$\mu_Y[n] = \mathcal{E}\{Y_n\} = \mathcal{E}\{\sum_m h[n-m]\,X_m\}$$

$$= \sum_m h[n-m]\,\mathcal{E}\{X_m\}$$

$$= \sum_m h[n-m]\,\mu_X[m]$$

filter impulse response

If $\mu_X[m] = \mu_X$ (constant)

$$\mu_Y[n] = \left(\sum_m h[n-m]\right)\mu_X$$

$$= \left(\sum_l h[l]\right)\mu_X$$

$$= \mu_Y \quad \text{(constant)}$$

Find $r_Y[n,m] = \mathcal{E}\{Y_n Y_m\}$

Start with

$$r_{XY}[m,n] = \mathcal{E}\{X_m \underline{\underline{Y_n}}\}$$

$$= \mathcal{E}\{X_m \underbrace{\sum_k h[n-k]\,X_k}_{Y_n}\}$$

$$= \mathcal{E}\{\sum_k h[n-k]\,X_m X_k\}$$

(4)

$$\underline{r_{XY}(m,n)} = \sum_k h[n-k]\,\mathcal{E}\{X_m X_k\}$$

$$= \sum_k h[n-k]\, \underline{r_{XX}[m,k]}$$

Suppose X_k is w.s.s., then

$$r_{XY}(m,n] = \sum_k h[n-k]\, r_{XX}[k-m]$$

let $l = k-m \Rightarrow k = l+m$

then

$$r_{XY}[m,n] = \sum_l h[n-(l+m)]\, r_{XX}[l]$$

or

$$r_{XY}[m,n] = \underbrace{\sum_l h[(n-m)-l]\, r_{XX}[l]}_{\text{convolution with output at time } (n-m)}$$

let $p = n-m$

then $r_{XY}(m,n] = r_{XY}[p] = \sum_l h[p-l]\, r_{XX}[l]$

Now for $\underline{r_{YY}}$!!

$$r_{YY}[m,n] = \mathcal{E}\{Y_m Y_n\}$$

$$= \mathcal{E}\{Y_m \sum_{\ell} h[n-\ell] X_\ell\}$$

$$\mathcal{E}\{X_\ell Y_m\}$$

$$= \sum_{\ell} h[n-\ell] \mathcal{E}\{Y_m X_\ell\}$$

$$= \sum_{\ell} h[n-\ell]\, r_{XY}[m-\ell]$$

let $p = m-\ell \Rightarrow \ell = m-p$

$$r_{YY}[m,n] = \sum_{p} h[n-(m-p)]\, r_{XY}[p]$$

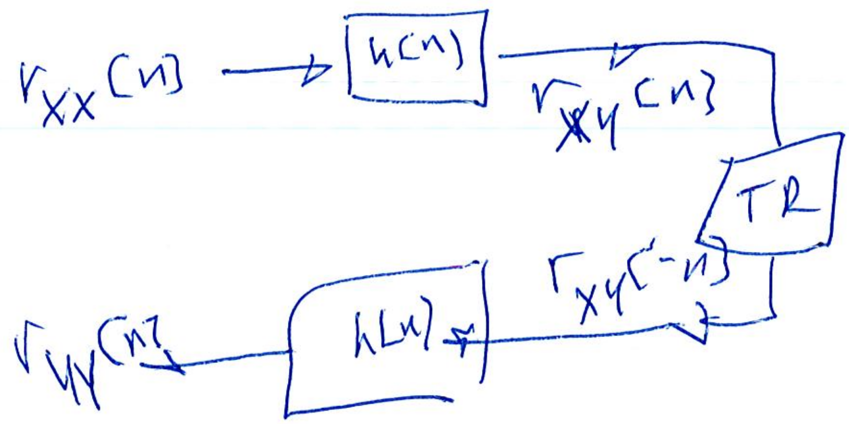
$$= \sum_{p} h[n-m]+p]\, r_{XY}[p]$$

$$\boxed{\text{let } k = -p}$$

$$r_{YY}[n-m] = \sum_{k} h[[n-m]-k]\, r_{XY}[-k]$$

or $$r_{YY}[\ell] = \sum_{k} h[\ell-k]\, r_{XY}[-k]$$

Summarizing



$\therefore$ If input is w.s.s. output is w.s.s.

Result is same if we filter with $h[n]$, then filter with $h[-n]$ and don't flip $r_{xy}[n]$