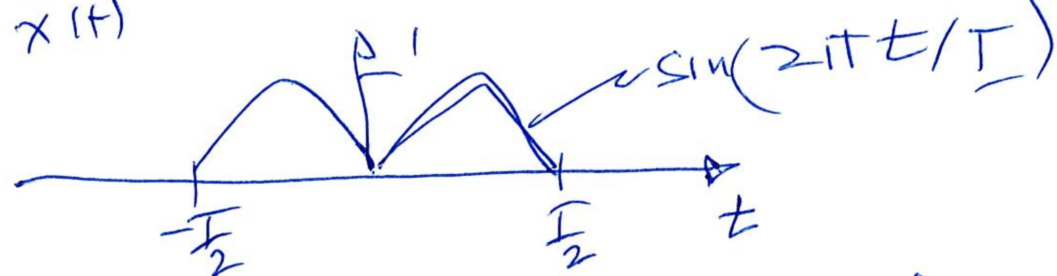
ECE 438 Lecture 1 1/28/2019

CTFT

$$X(f) = \int x(t) e^{-j2\pi ft} dt$$

$$x(t) = \int X(f) e^{j2\pi ft} df$$

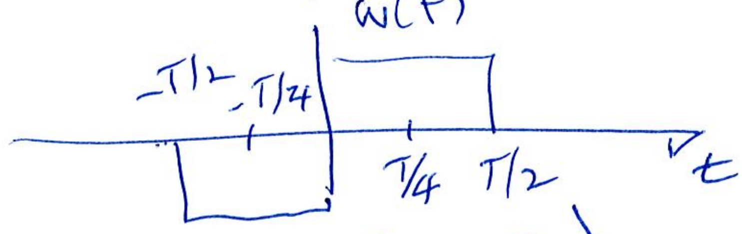
Example of CTFT computation



Find $X(f)$

$$X(f) = -\int_{-T/2}^{0} \sin(2\pi t/T) e^{-j2\pi ft} dt + \int_{0}^{T/2} \sin(2\pi t/T) e^{-j2\pi ft} df$$

Recommended Approach



$$w(t) = \text{rect}\left(\frac{t-T/4}{T/2}\right) - \text{rect}\left(\frac{t+T/4}{T/2}\right)$$

$$\text{rect}(t) \xrightarrow{CTFT} \text{sinc}(f)$$

$$x\left(\frac{t-b}{a}\right) \xleftrightarrow{CTFT} |a| X(af) e^{-j2\pi b f}$$

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$$W(f) = \frac{T}{2} \operatorname{sinc}\left(\frac{T}{2} f\right)\left\{e^{-j2\pi \frac{T}{4} f} - e^{+j2\pi \frac{T}{4} f}\right\}$$

$$x(t) = w(t) \sin(2\pi t/T)$$

Convolution Property

$$x(t) * y(t) \xleftrightarrow{CTFT} X(f) Y(f)$$

Product Relation

$$x(t)\, y(t) \xleftrightarrow{CTFT} X(f) * Y(f)$$

$$X(f) = W(f) * CTFT\{\sin(2\pi t/T)\}$$

$$\operatorname{sinc}\left(\frac{2\pi t}{T}\right) = \frac{1}{j2}\left(e^{+j2\pi \frac{t}{T}} - e^{-j2\pi \frac{t}{T}}\right)$$

$$e^{j\frac{2\pi t}{T}} \xleftrightarrow{CTFT} \delta\left(f + \frac{1}{T}\right)$$

$$e^{-j\frac{2\pi t}{T}} \xleftrightarrow{CTFT} \delta\left(f - \frac{1}{T}\right)$$

$$X(f) = W(f) * \left\{\frac{1}{j2} \delta\left(f + \frac{1}{T}\right)\right.$$

$$\cancel{+}\, W(f) * \frac{1}{j2\pi} \delta\left(f - \frac{1}{T}\right)$$

What is $y(t) * \delta(t - t_0) =$

$y(t - t_0)$

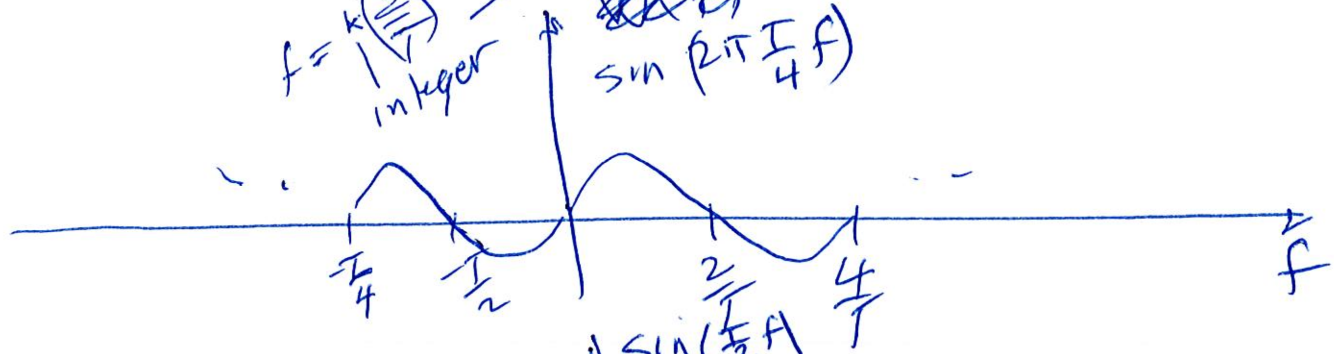
$$X(f) = \frac{1}{j2} W\left(f + \frac{1}{T}\right) - \frac{1}{j2} W\left(f - \frac{1}{T}\right)$$

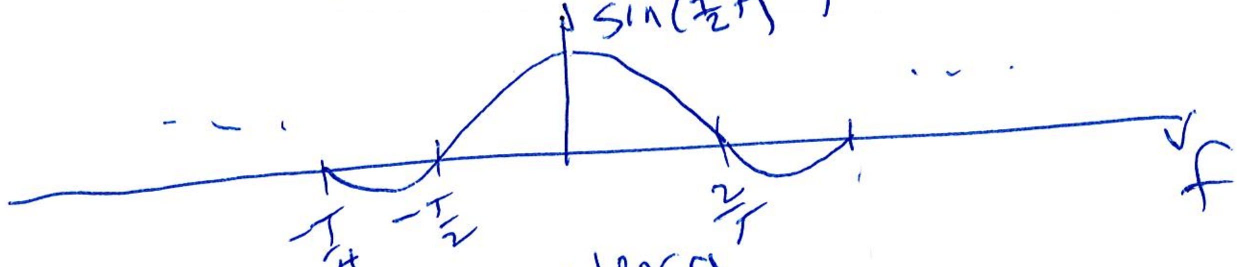
$$W(f) = j\frac{T}{2} \operatorname{sinc}\left(\frac{T}{2} f\right) \sin\left(2\pi \frac{T}{4} f\right)$$

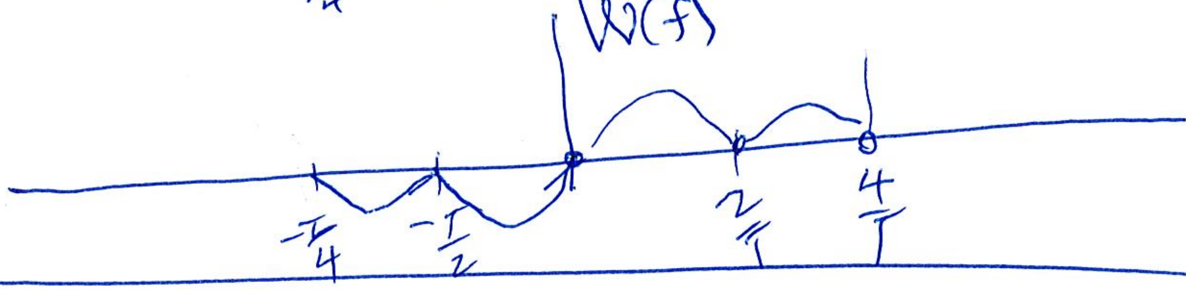
↳ period $\frac{4}{T}$

$f = k\left(\frac{2}{T}\right)$

↳ integer



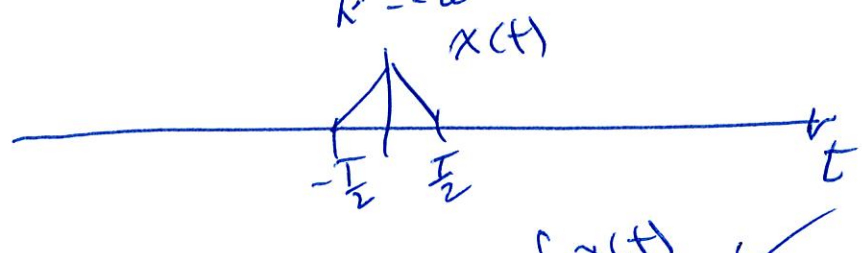


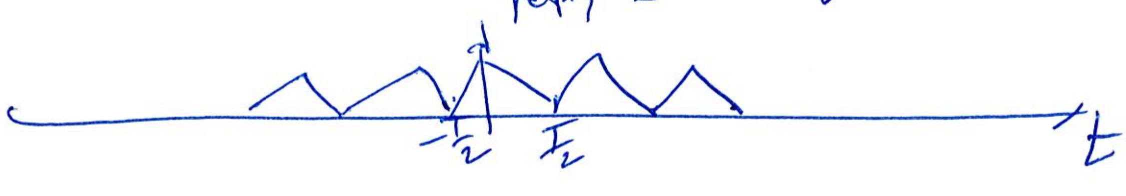


Two new CTFT relationships

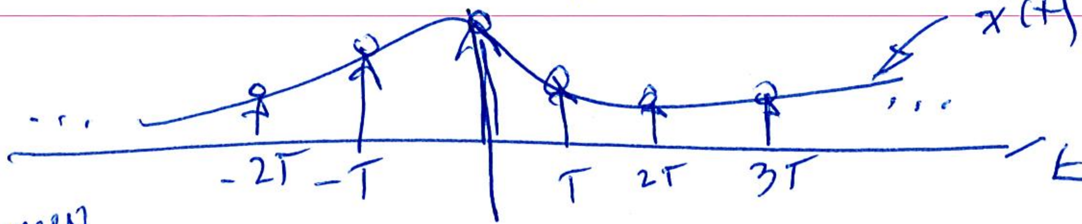
Define two new operators

(1) $\operatorname{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t - kT)$





$$② \quad comb_T[x(t)] = \sum_{k=-\infty}^{\infty} x(kT)\,\delta(t-kT)$$



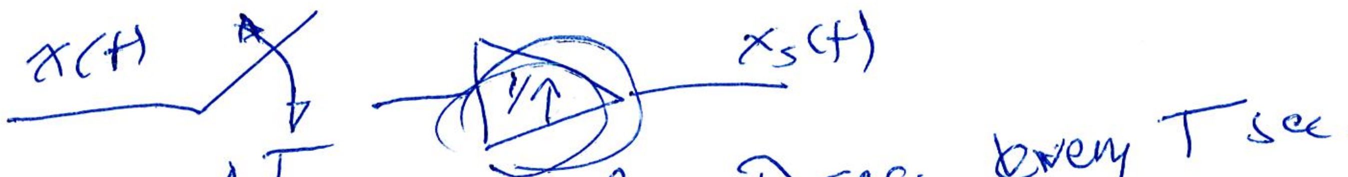
Two new transform relationships

Module 1.3.2

① $rep_T[x(t)] \xleftrightarrow{CTFT} \frac{1}{T}\, comb_{\frac{1}{T}}[X(f)]$

② $comb_T[x(t)] \xleftrightarrow{CTFT} \frac{1}{T}\, rep_{\frac{1}{T}}[X(f)]$

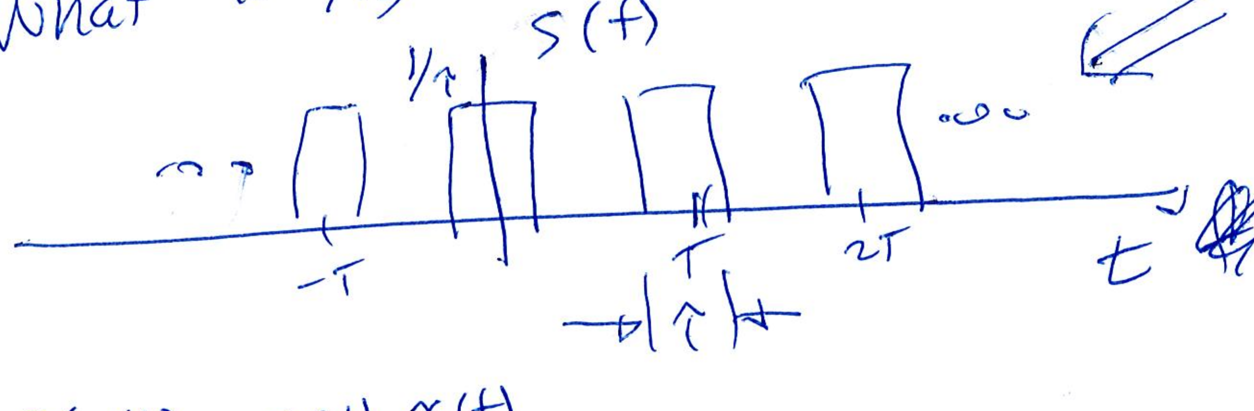
Module 1.4.1 Analysis of sampling



switch is closed for τ sec. every T sec.

Given $X(f)$?

What is $X_s(f)$?



$$x_s(t) = s(t)\,x(t)$$

$$X_s(f) = S(f) * X(f)$$

$$S(t) = \mathrm{rep}_T\left[\mathrm{rect}(t/\tau)\right]$$

$$S(f) = \frac{1}{T}\,\mathrm{comb}_{\frac{1}{T}}\left[\mathrm{sinc}(\tau f)\right]$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\tau \frac{k}{T}\right)\delta\left(f - \frac{k}{T}\right)$$

$$X_s(f) = \frac{1}{T}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\tau \frac{k}{T}\right) X(f) * \delta\left(f - \frac{k}{T}\right)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} \mathrm{sinc}\left(\tau \frac{k}{T}\right) X\left(f - \frac{k}{T}\right)$$

Assume $W < \frac{1}{2T}$