EE 438 — 1/14/2019 (1)

Frequency Response

If $\boxed{x[n] = e^{j\omega n}}$

then for a certain class of systems

$$y[n] = H(\omega)\, x[n]$$

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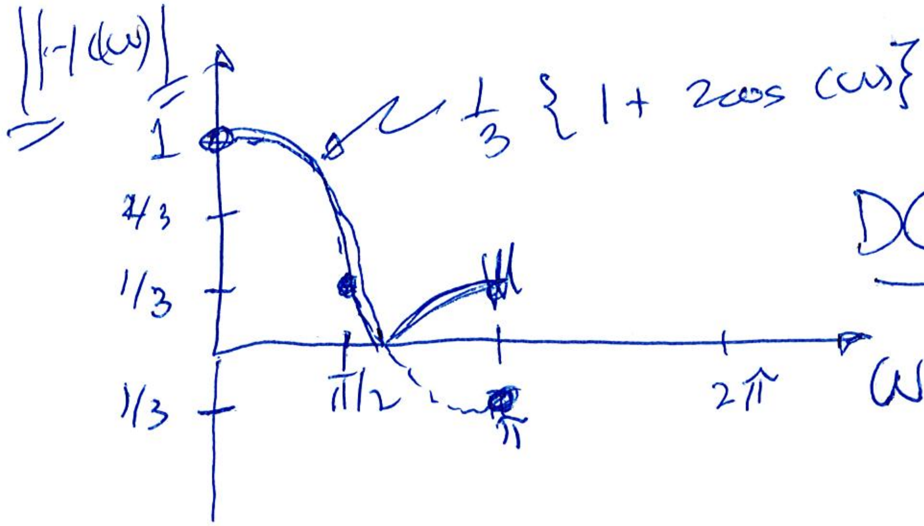
example $y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$

$$H(\omega) = \frac{1}{3}\{1 + 2\cos(\omega)\}e^{-j\omega}$$

$$|H(\omega)| = \Big|\underbrace{\tfrac{1}{3}\{1 + 2\cos(\omega)\}}_{\text{real-valued}}\Big|\ \cancelto{1}{|e^{-j\omega}|}$$



DC-preserving

low-pass filter

2 $y[n] = |x[n]|$

let $x[n] = e^{j\omega n}$

$y[n] = |e^{j\omega n}|$

$\neq H(\omega) e^{j\omega n}$

If the system has the desired property

then

$$x[n] = e^{j\omega n} \longrightarrow \boxed{\text{SYS}} \longrightarrow H(\omega) e^{j\omega n}$$

systems that are linear and time-invariant have this property !!

Linearity

If $x_1[n] \longrightarrow \boxed{\text{SYS}} \longrightarrow y_1[n]$

$x_2[n] \longrightarrow \boxed{\text{SYS}} \longrightarrow y_2[n]$

then $x_3[n] = a_1 x_1[n] + a_2 x_2[n]$

then

$$x_3[n] \longrightarrow \boxed{\text{SYS}} \longrightarrow y_3[n] = a_1 y_1[n] + a_2 y_2[n]$$

Time invariance (TI)

If $x_1[n] \xrightarrow{sys} y_1[n]$

let $x_2[n] = x_1[n-n_0]$ n_0 - integer

then $y_2[n] = y_1[n-n_0]$

Ex 1

$y(n) = \frac{1}{2}\{x[n] + x[n-1]\}$

let $y_1[n] = \frac{1}{2}\{x_1[n] + x_1[n-1]\}$

$y_2[n] = \frac{1}{2}\{x_2[n] + x_2[n-1]\}$

let $x_3[n] = a_1 x_1[n] + a_2 x_2[n]$

$y_3[n] = \frac{1}{2}\{x_3[n] + x_3[n-1]\}$

$= \frac{1}{2}\{(a_1 x_1[n] + a_2 x_2[n]) + (a_1 x_1[n-1] + a_2 x_2[n-1])\}$

$= a_1 \frac{1}{2}\{x_1[n] + x_1[n-1]\} + a_2 \frac{1}{2}\{x_2[n] + x_2[n-1]\}$

$= a_1 y_1[n] + a_2 y_2[n]$ ✓ is L

let $x_3[n] = x_1[n-n_0]$

$y_3[n] = \frac{1}{2}\{x_3[n] + x_3[n-1]\}$

$= \frac{1}{2}\{x_1[n-n_0] + x_1[n-n_0-1]\}$

$= y_1[n-n_0]$ ✓ TI

Ex. 2

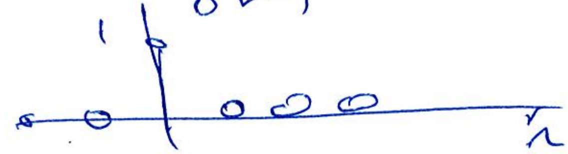
$$y[n] = |x[n]|$$

Special Signal

DT impulse or unit sample

$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & \text{else} \end{cases}$$

$\delta[n]$



let $x_1[n] = \delta[n] \longrightarrow y_1[n] = \delta[n]$

$x_2[n] = -2\delta[n] \qquad y_2[n] = 2\delta[n]$

$\boxed{a_1 = a_2 = 1}$

$x_3[n] = -\delta[n]$

$\boxed{y_3[n] = \delta[n]}$

$\neq a_1 y_1[n] + a_2 y_2[n]$

$= 3\delta[n]$ not L

$y[n] = |x[n]|$

let $x_2[n] = x_1[n-n_0]$

$y_2[n] = |x_1[n-n_0]|$

$= y_1[n-n_0]$ is TI

Example 3

$$\boxed{y[n] = x[n] - y[n-1]}$$

Is LTI

let $x[n] = e^{j\omega n}$

Assume $y[n] = H(\omega) e^{j\omega n}$

Then $H(\omega) e^{j\omega n} = e^{j\omega n} - H(\omega) e^{j\omega(n-1)}$

$$H(\omega) \cancel{e^{j\omega n}} = \cancel{e^{j\omega n}} \left[1 - H(\omega) e^{-j\omega}\right]$$

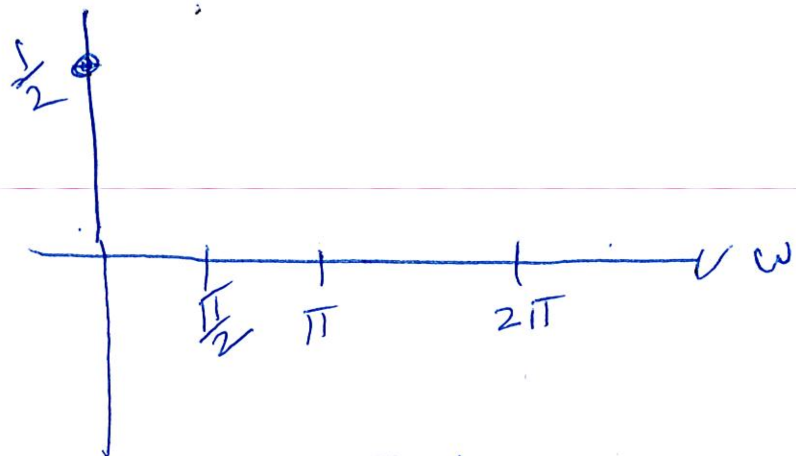
$$H(\omega)\left[1 + e^{-j\omega}\right] = 1$$

$$H(\omega) = \frac{1}{1 + e^{-j\omega}}$$

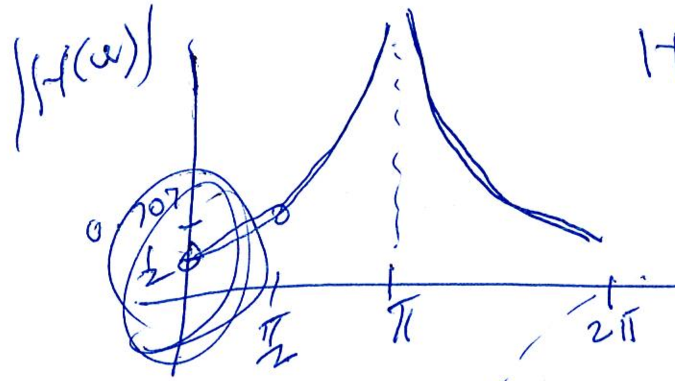
$$= \frac{1}{\left(e^{+j\omega/2} + e^{-j\omega/2}\right) e^{-j\omega/2}}$$ factor out $\frac{1}{2}$ angle

$$= \frac{1}{2\cos(\omega/2)} e^{j\omega/2}$$

$$|H(\omega)| = \left|\frac{1}{2\cos(\omega/2)}\right| \left|\cancel{e^{j\omega/2}}\right|^{1}$$



$$|H(\tfrac{\pi}{2})| = \left|\frac{1}{2(1/\sqrt{2})}\right| = \frac{1}{\sqrt{2}} \cong 0.707$$



High pass
or
more properly
high-emphasis

$$H(\omega) = \frac{1}{2\cos(\frac{\omega}{2})}\left[e^{j\omega/2}\right]$$

$$\angle H(\omega) = \arctan\left(\frac{H_{Im}(\omega)}{H_{Re}(\omega)}\right)$$

let $z_1 = A_1 e^{j\theta_1}$, $z_2 = A_2 e^{j\theta_2}$

then $\angle z_1 z_2 = \angle A_1 e^{j\theta_1} A_2 e^{j\theta_2}$

$$= \angle A_1 A_2 e^{j(\theta_1+\theta_2)}$$

$$= \theta_1 + \theta_2$$

$$H(\omega) = \frac{1}{2\cos(\omega/2)} e^{j\omega/2}$$

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$$\angle H(\omega) = \angle \frac{1}{2\cos(\omega/2)} + \angle e^{j\omega/2}$$

$$\angle e^{j\omega/2} = \omega/2$$