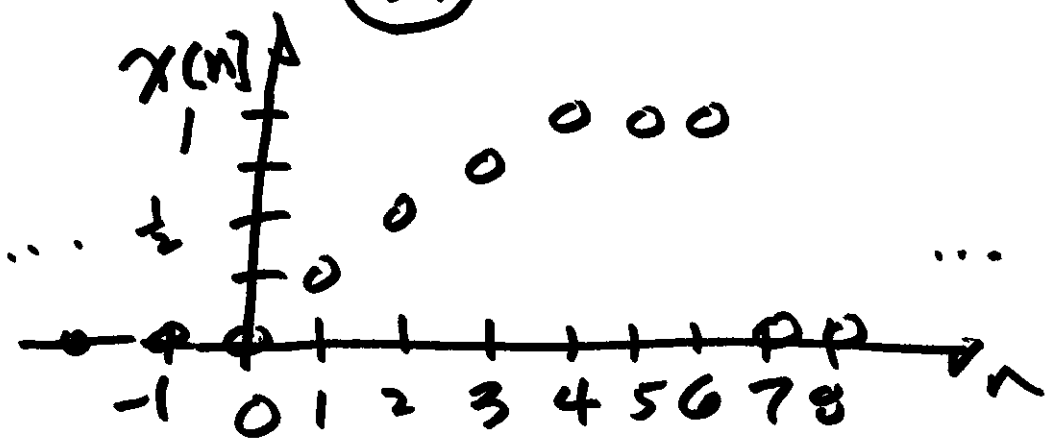
ECE 438 Lecture 1/11/2019

Upsampling / Downsampling

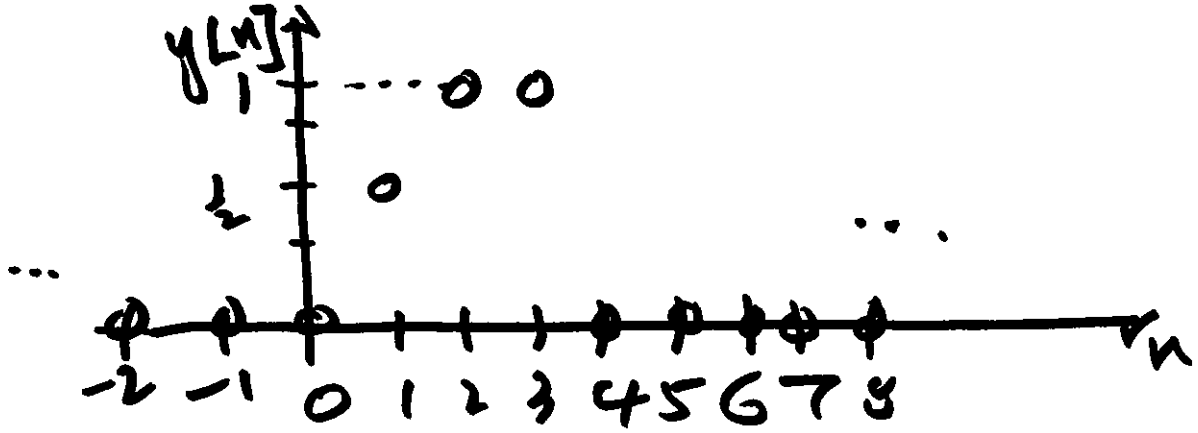
DT

Downsampling

- $y[n] = x[Dn]$ D - integer



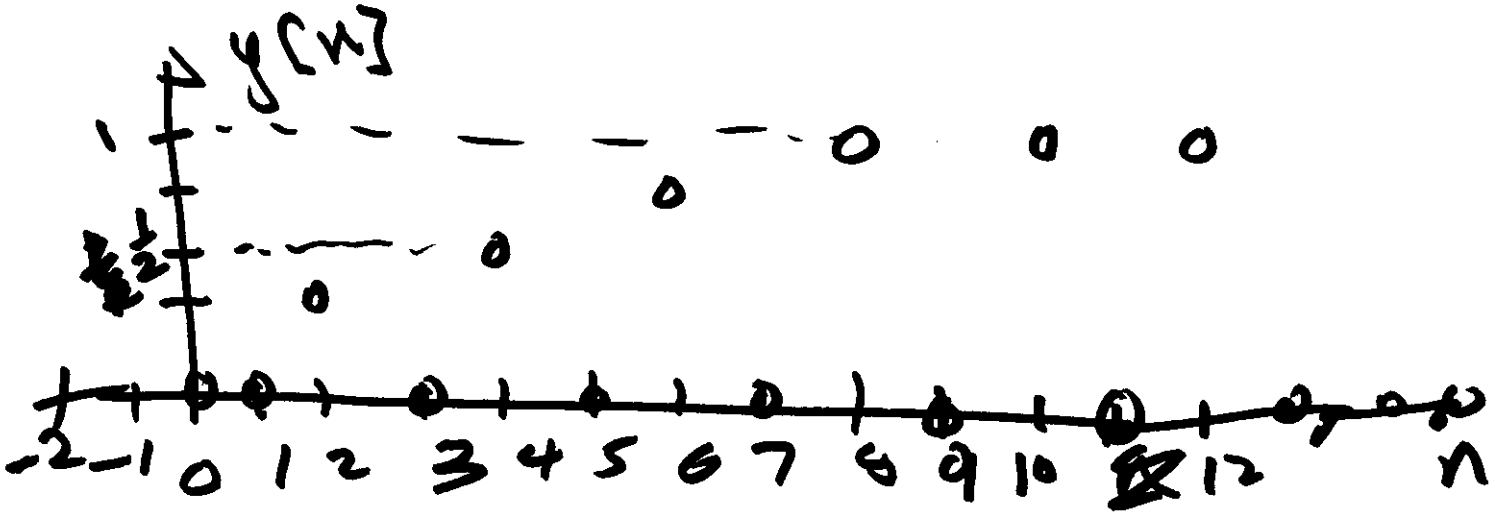
D = 2



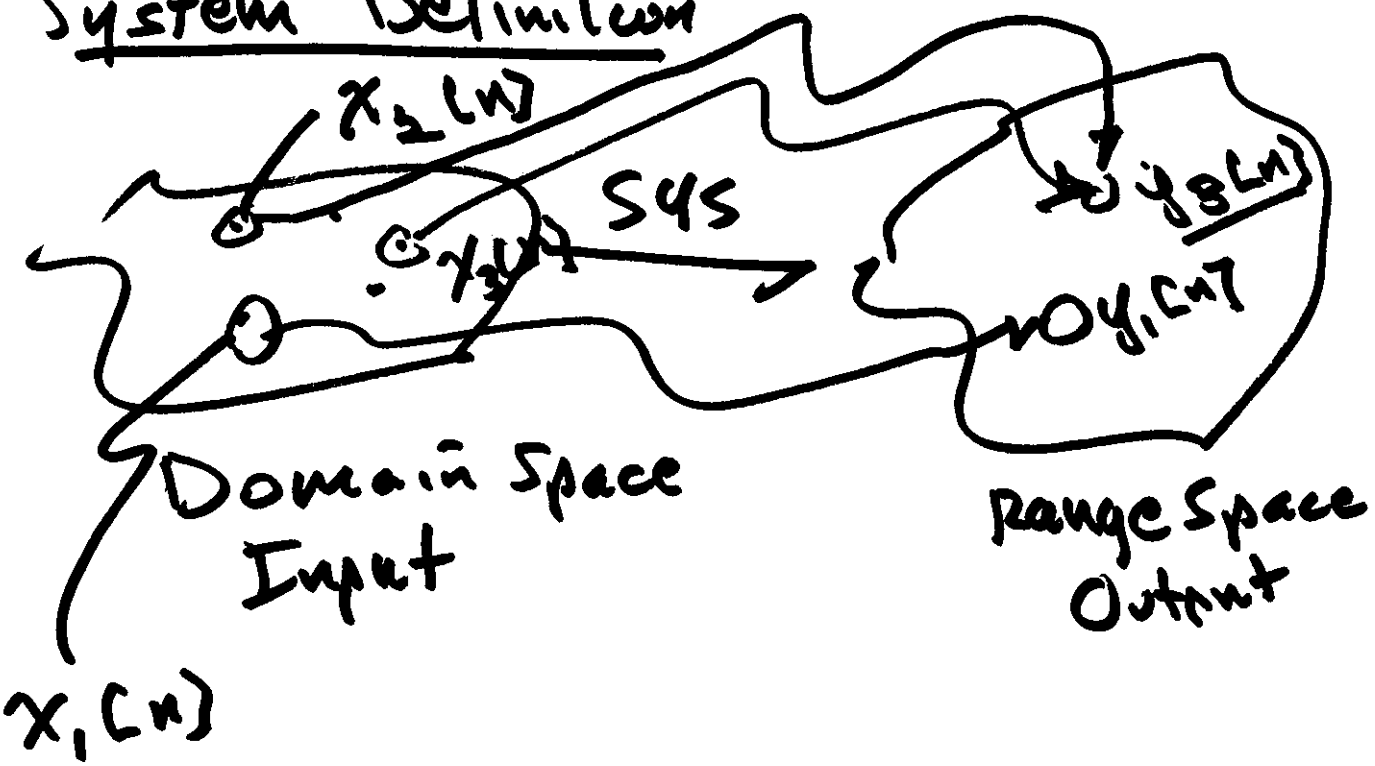
Upsampling

$$y[n] = \begin{cases} x[n/D], & n/D \text{ - integer} \\ 0, & \text{else} \end{cases}$$

Example $D=2$



System Definition



Example System

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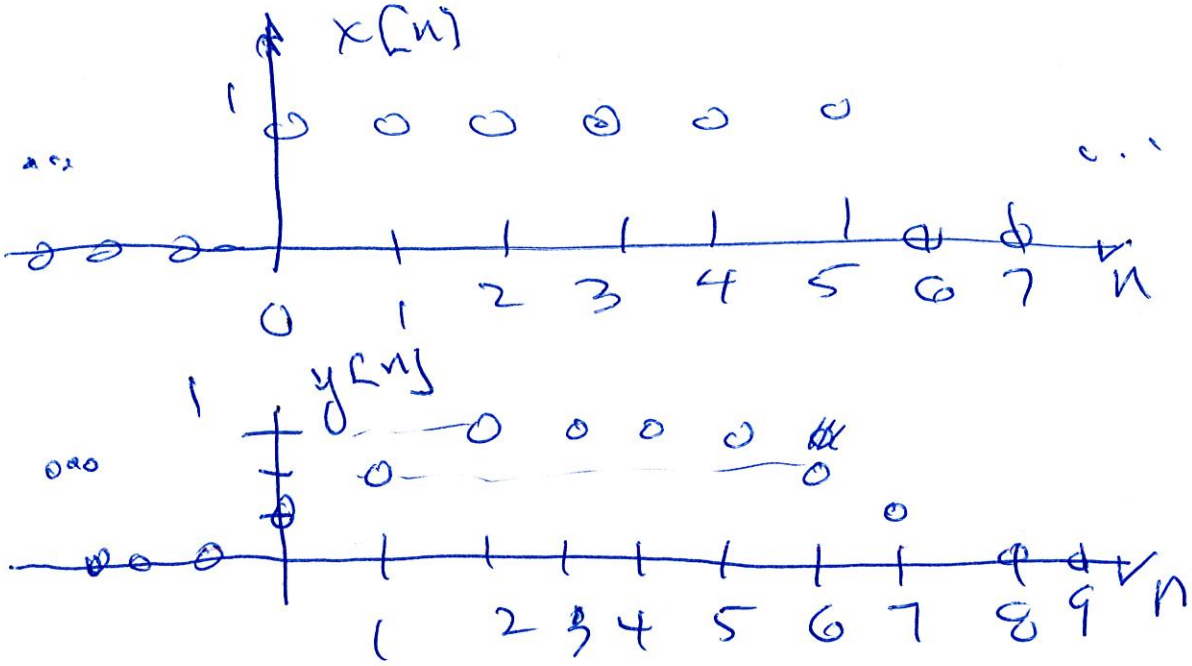
$$y[n] = \frac{1}{3}\left[x[n] + x[n-1] + x[n-2]\right]$$

3-pt. Moving Average (MA)

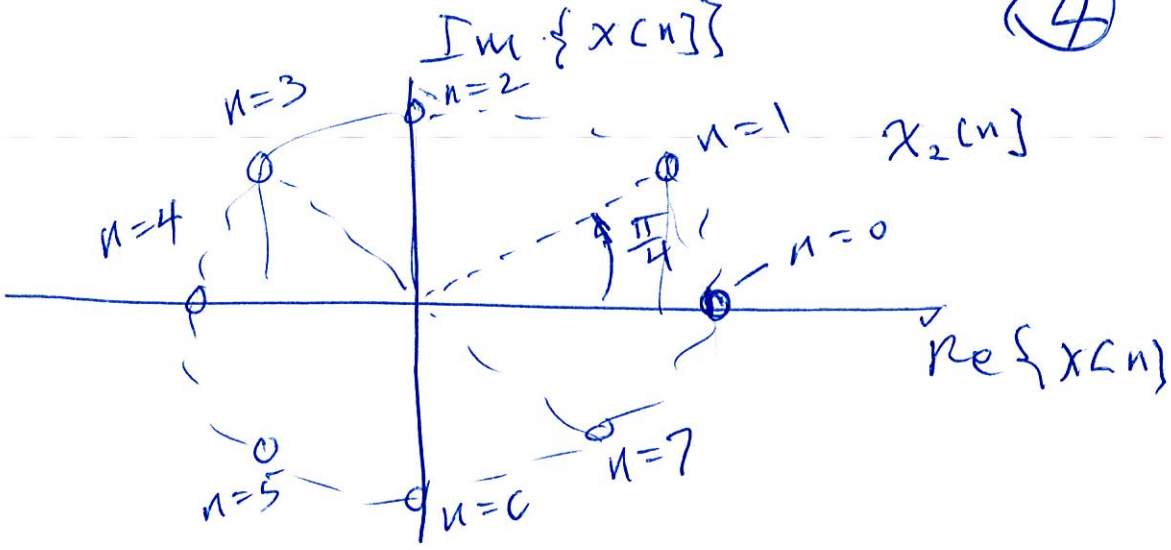


$$x_1[n] = \cos\left(\frac{\pi}{4}n\right)$$

$$y[n] = \frac{1}{3}\left\{\cos\left(\frac{\pi}{4}n\right) + \cos\left[\frac{\pi}{4}(n-1)\right] + \cos\left[\frac{\pi}{4}(n-2)\right]\right\}$$

Consider complex exponential signal

$$x_2[n] = e^{j\left(\frac{\pi}{4}n\right)}$$



$e^{j\theta} = \cos\theta + j\sin\theta$ Euler's identity

$\Rightarrow \quad x_1[n] = Re\{x_2[n]\}$

Review of complex variables:

$x = x_R + j x_I$ Cartesian coordinate form

$x^* = x_R - j x_I$ complex conjugate

$x_R = \frac{1}{2}[x + x^*]$

$x_I = \frac{1}{j2}[x - x^*]$

$x = Ae^{j\theta}$ polar coordinate form

$|x| = A = \sqrt{x x^*} = \sqrt{x_R^2 + x_I^2}$

$\angle x = \arctan\left(\frac{x_I}{x_R}\right) = \theta$

Frequency Response

definition:

If $x[n] = e^{jwn}$ — w = constant (radians/sample)

Let $x[n] \rightarrow$ [sys] $\rightarrow y[n]$

then $H(w) = \frac{y[n]}{x[n]}$

$H(w)$: frequency response

Example

$$y[n] = \frac{1}{3}\{x[n] + x[n-1] + x[n-2]\}$$

let $x[n] = e^{jwn}$

then $y[n] = \frac{1}{3}\{e^{jwn} + e^{jw(n-1)} + e^{jw(n-2)}\}$

Property: let $z \in \mathbb{C}$ be a complex variable

then $z^{(a+b)} = z^a z^b$

$$y[n] = \frac{1}{3}\left[1 + e^{-jw} + e^{-jw2}\right] e^{jwn}$$

$$H(\omega) = \frac{y[n]}{x[n]} = \boxed{\frac{1}{3}\left[1 + e^{-j\omega} + e^{-j\omega 2}\right]} \quad (6)$$

$$|H(\omega)| = \sqrt{\left(\frac{1}{3}\left[1 + \cos(-\omega) + \cos(-2\omega)\right]\right)^2 + \left[\frac{1}{3}\left[\sin(-\omega) + \sin(-2\omega)\right]\right]^2}$$

Factor out the half-angle

$$H(\omega) = \frac{1}{3}\left\{e^{j\omega} + 1 + e^{-j\omega}\right\} e^{-j\omega}$$

$$= \frac{1}{3}\left\{1 + \left(e^{j\omega} + e^{-j\omega}\right)\right\} e^{-j\omega}$$

$$= \frac{1}{3}\left\{1 + 2\cos(\omega)\right\} e^{-j\omega}$$

$$|H(\omega)| = \left|\frac{1}{3}\left\{1 + 2\cos(\omega)\right\}\right| \left|e^{-j\omega}\right|$$

Property: Let z_1 & z_2 be complex

Then $|z_1 z_2| = |z_1||z_2|$