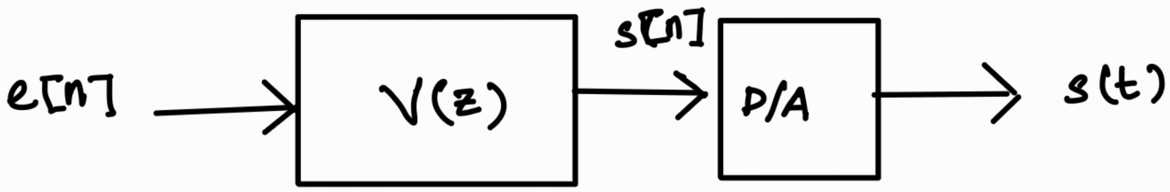
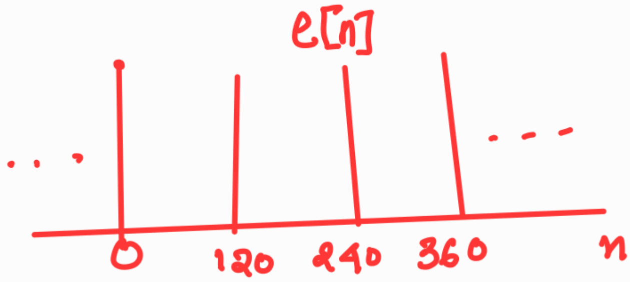
ECE 438 HW No. 8 Solution

Spring - 2023

Problem-1



(a) $e[n] = \sum_{k=-\infty}^{\infty} \delta[n-120k]$



Sample period, $N = 120$

Pitch period $= NT_s = \dfrac{N}{f_s} = \dfrac{120}{12000} = 0.01$ sec $= 10$ ms

Pitch frequency $= \dfrac{1}{\text{pitch period}} = \dfrac{1}{0.01} = 100$ Hz

(b) $\text{pole}_1 : \omega_1 = \dfrac{\pi}{6}$, $\text{pole}_2 : \omega_2 = \dfrac{\pi}{2}$

[poles come in conjugate pairs, so only consider ones which correspond to positive frequency]

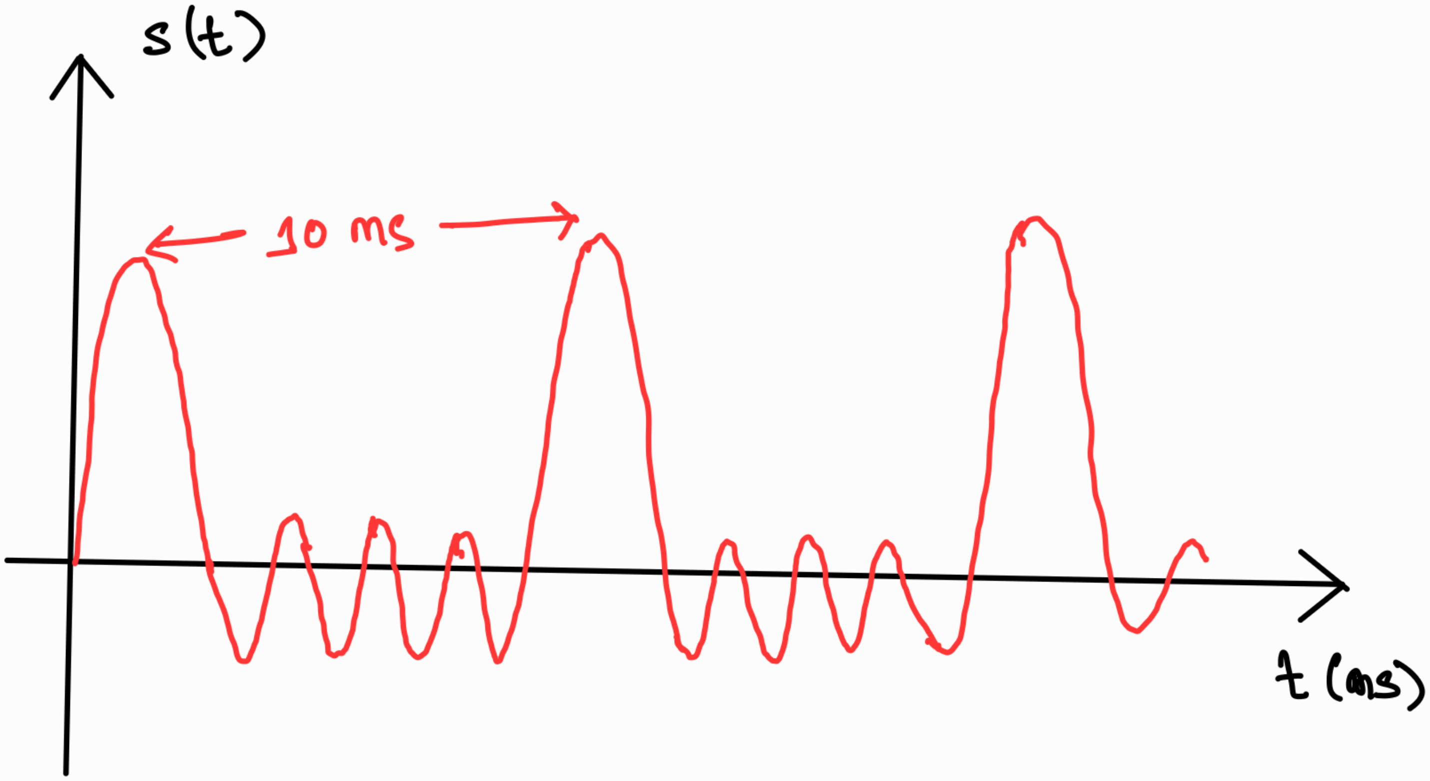
Formant frequencies: f_1 , f_2

$$f_1 = \frac{\omega_1 f_s}{2\pi} = \frac{\pi/6 \cdot 12}{2\pi} = \frac{5}{6} \text{ kHz}$$

$$f_2 = \frac{\omega_2 f_s}{2\pi} = \frac{\pi/2 \cdot 12}{2\pi} = \frac{5}{2} \text{ kHz}$$

Pole 1 is closer to the unit circle than pole 2 and hence f_1 has a higher peak. The zero is also closer to pole 2 which further reduces the peak at f_2.

(c)



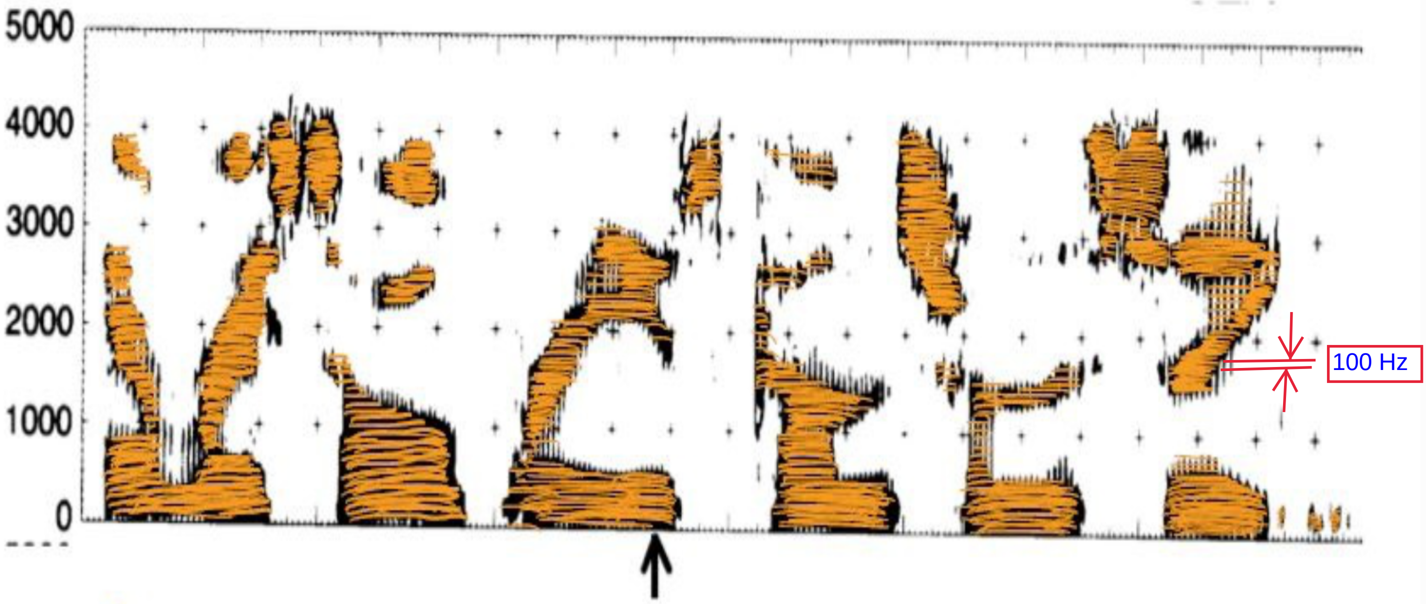
Problem-2

(a) The entire utterance includes 20 small grids, so each grid section corresponds to $\frac{2}{20} = 0.1$ sec. In each section there are 10 vertical strips on average.

$\therefore$ Estimated pitch period $= \frac{0.1}{10} = 0.01$ sec.
$= 10$ ms

(b) This is a wideband spectrum as the strips are vertical.

(c)



Convert vertical stripes to horizontal stripes

The horizontal stripes are 100 Hz apart.

Narrowband spectrum

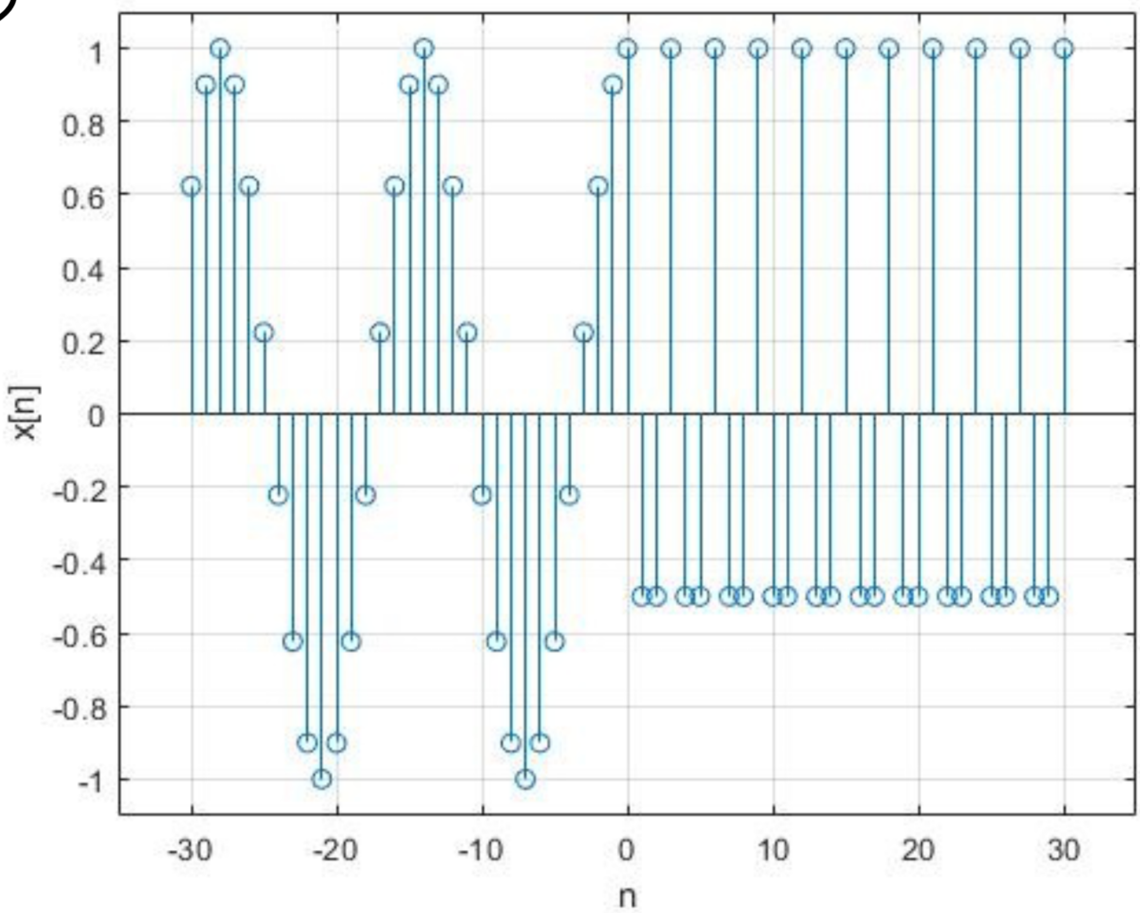
(d) Formant frequency $\simeq$ 250 Hz, 2200 Hz, 2800 Hz

(e) IY

Problem - 3

$$x[n] = \begin{cases} \cos\left(\frac{\pi}{7}n\right), & n<0 \\ \cos\left(\frac{2\pi}{3}n\right), & n \geq 0 \end{cases}$$

(a)



plot of $x[n]$ in the range $n = [-30, 30]$

(b)

$$w[n] = \begin{cases} 1, & |n| \leq 12 \\ 0, & \text{else} \end{cases}$$

$$X(\omega, n) = \sum_k x[k]\, w[n-k]\, e^{-j\omega k}$$

$$\cos(\omega_0 k) \xleftrightarrow{DTFT} \pi\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right], \quad -\pi \le \omega \le \pi$$

$$w[k] \xleftrightarrow{DTFT} \text{psinc}_{25}(\omega) e^{-j12\omega} e^{j12\omega} = \text{psinc}_{25}(\omega)$$

$$w[k-n] \xleftrightarrow{DTFT} \text{psinc}_{25}(\omega) e^{-j\omega n} \longrightarrow (1)$$

Due to symmetry, $w[n-k] = w[k-n]$

$$w[n-k] = \begin{cases} 1, & n-12 \le k \le n+12 \\ 0, & \text{else} \end{cases}$$

i. $n \le -13$

$k < 0 \quad \forall\, n \le -13$

For $k<0$, $x[k] = \cos\left(\frac{\pi}{7}k\right)$

$$X(\omega, n) = \sum_k \cos\left(\frac{\pi}{7}k\right) w[n-k] e^{-j\omega k}$$

$$= DTFT\left\{\cos\left(\frac{\pi}{7}k\right) w[n-k]\right\}$$

Using (1) and modulation property of DTFT:

$$X(\omega,n) = \frac{1}{2}\left[\text{psinc}_{25}\left(\omega-\frac{\pi}{7}\right) e^{-jn\left(\omega-\frac{\pi}{7}\right)} + \text{psinc}_{25}\left(\omega+\frac{\pi}{7}\right) e^{-jn\left(\omega+\frac{\pi}{7}\right)}\right.$$

ii. $n \geq 13$

$k > 0 \quad \forall \, n \geq 13$

For $k > 0$, $x[k] = \cos\left(\frac{2\pi}{3} k\right)$

$$X(\omega, n) = \text{DTFT}\left\{ \cos\left(\frac{2\pi}{3} k\right) w[n-k] \right\}$$

Using ① and modulation property of DTFT:

$$X(\omega, n) = \frac{1}{2}\left[\text{psinc}_{25}\left(\omega - \frac{2\pi}{3}\right) e^{-jn\left(\omega - \frac{2\pi}{3}\right)} + \text{psinc}_{25}\left(\omega + \frac{2\pi}{3}\right) e^{-jn\left(\omega + \frac{2\pi}{3}\right)} \right]$$

iii. $n = 0$

For $n = 0$, $\quad -12 \leq k \leq 12$

$$X(\omega, 0) = \sum_{k} \cos\left(\frac{\pi}{7} k\right) w_1[k] e^{-j\omega k} + \cos\left(\frac{2\pi}{3} k\right) w_2[k] e^{-j\omega k} - 1$$

$$w_2[k] = \begin{cases} 1, & k = 0 \dots 12 \\ 0, & \text{else} \end{cases}$$

$$w_1[k] = w_2[-k]$$

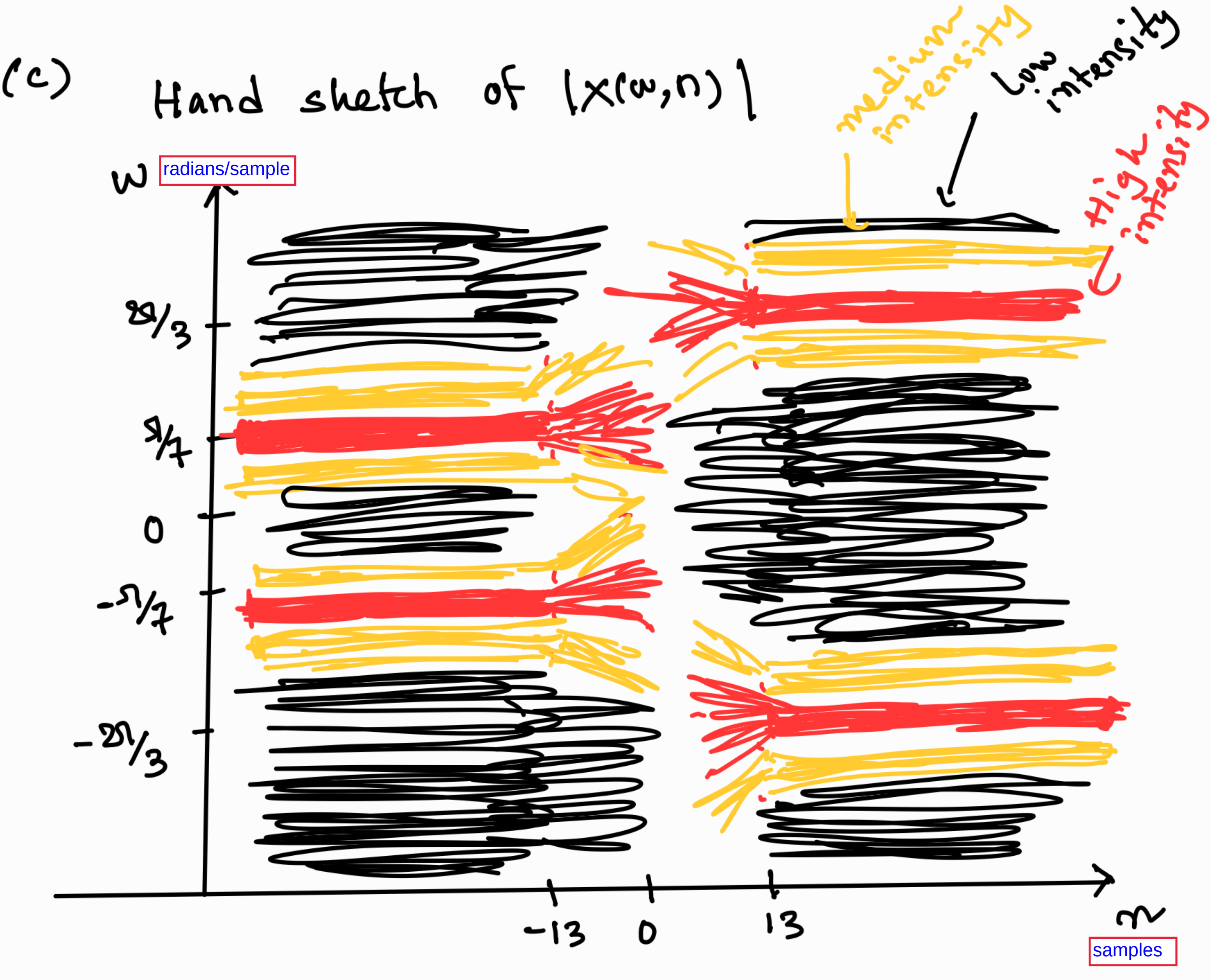
Because both w_1 and w_2 account for $k = 0$

$$\therefore\ w_2[k] \xleftrightarrow{\text{DTFT}} \text{psinc}_{13}(w)\, e^{-jw(12/2)}$$

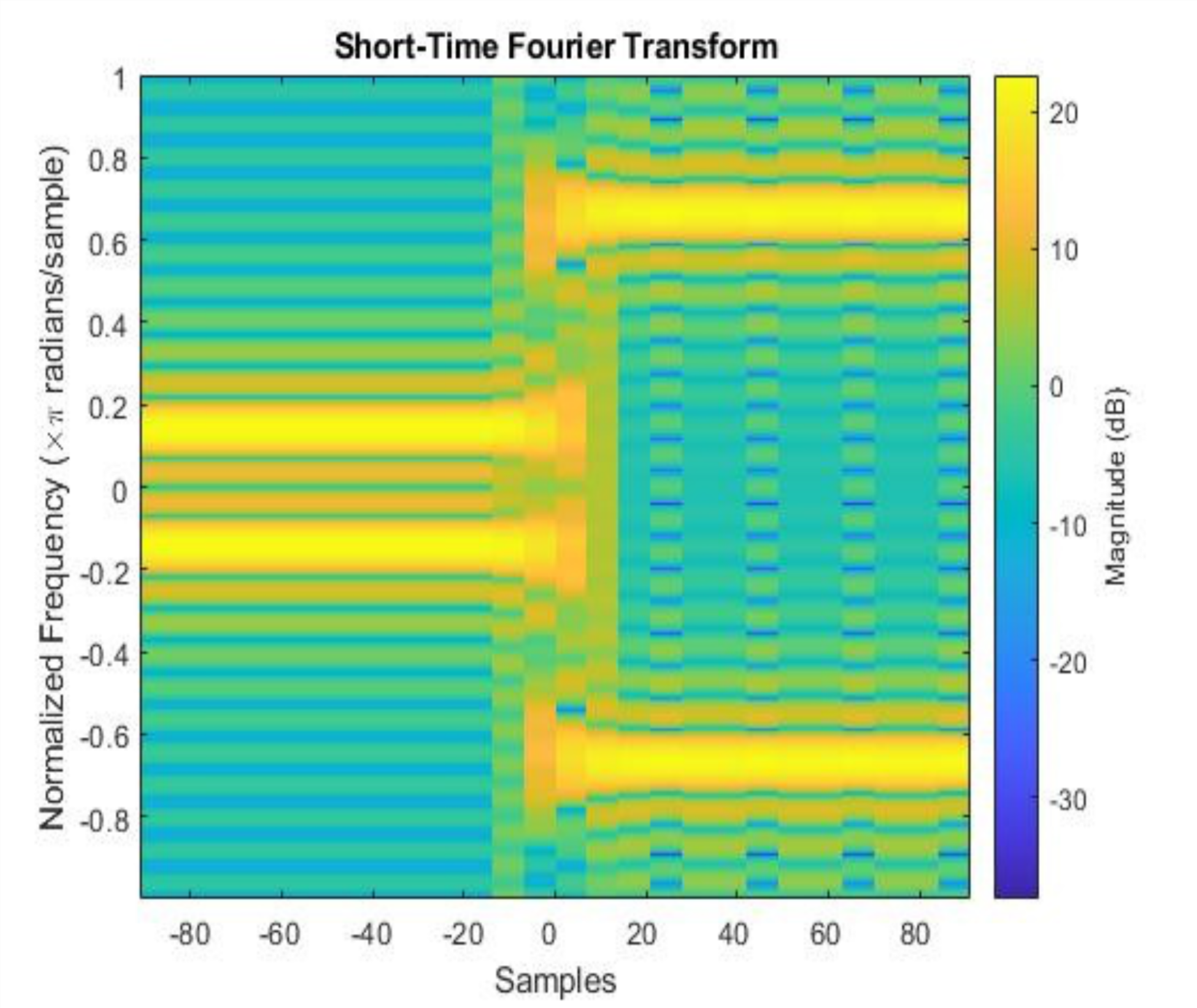
$$w_1[k] \xleftrightarrow{\text{DTFT}} \text{psinc}_{13}(w)\, e^{jw(12/2)}$$

$$X(w,0) = \frac{1}{2}\Big[\text{psinc}_{13}\left(w-\frac{\pi}{7}\right)e^{j6(w-\frac{\pi}{7})} + \text{psinc}_{13}\left(w+\frac{\pi}{7}\right)e^{j6(w+\frac{\pi}{7})} + \text{psinc}_{13}\left(w-\frac{2\pi}{3}\right)e^{-j6(w-2\pi/3)} + \text{psinc}_{13}\left(w+\frac{2\pi}{3}\right)e^{-j6(w+2\pi/3)}\Big] - 1$$

(c) Hand sketch of $|X(w,n)|$



Matlab generated spectogram

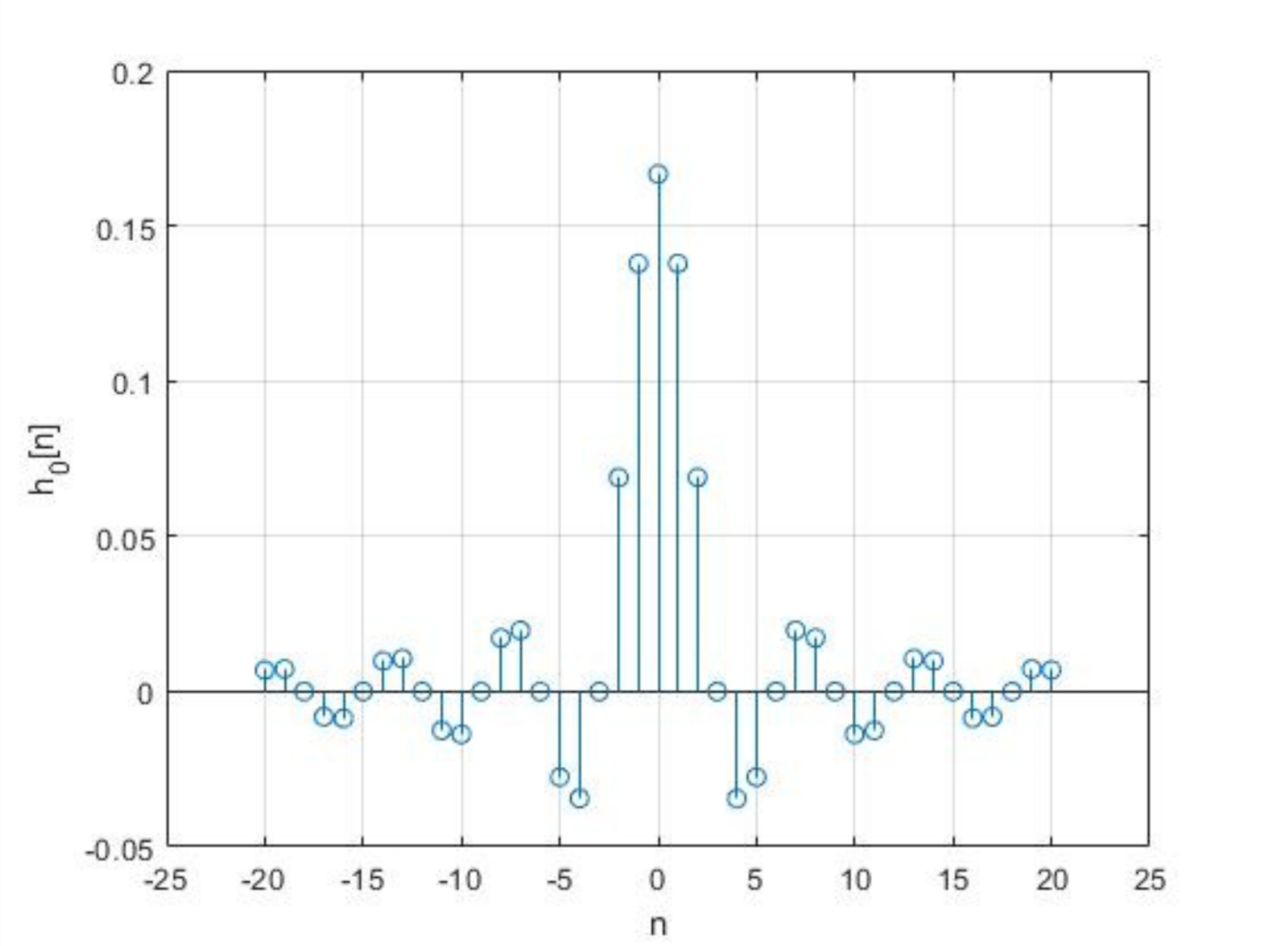


$|X(w,n)|$ in the range $n = [-100, 100]$

Problem - 4

$$h_0[n] = \frac{1}{L}\,\text{sinc}\left(\frac{n}{L}\right)\cos\left(\frac{\pi n}{L}\right)$$

(a) sketch $h_0[n]$ for $L=6$



(b) $$h_0[n] = \underbrace{\frac{1}{L}\,\text{sinc}\left(\frac{n}{L}\right)}_{x[n]}\ \underbrace{\cos\left(\frac{\pi n}{L}\right)}_{y[n]}$$

$$x[n] = \frac{1}{L}\,\text{sinc}\left(\frac{n}{L}\right) \xleftrightarrow{\text{DTFT}} X(\omega) = \begin{cases} 1, & |\omega| < \frac{\pi}{L} \\ 0, & \text{else} \end{cases}$$

$$y[n] = \cos\left(\frac{\pi n}{L}\right) \xleftrightarrow{\text{DTFT}} Y(\omega) = \pi\left[\delta\left(\omega - \frac{\pi}{L}\right) + \delta\left(\omega + \frac{\pi}{L}\right)\right]$$

$|\omega| \leq \pi$

Multiplication in time leads to convolution in frequency domain

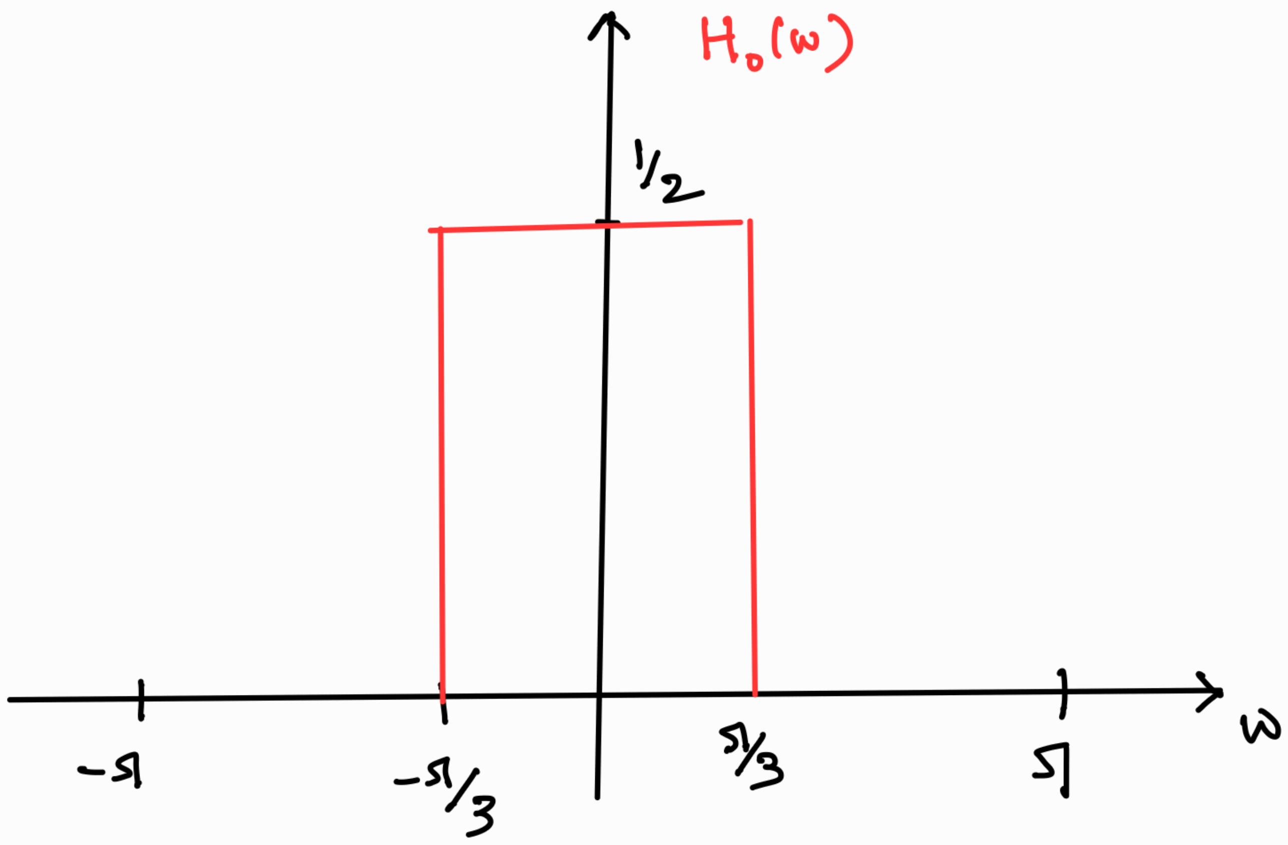
As $h_0[n] = x[n]y[n]$

$$H_0(\omega) = \frac{1}{2\pi} X(\omega) * Y(\omega)$$

$$= \frac{1}{2}\left[X\left(\omega - \frac{\pi}{L}\right) + X\left(\omega + \frac{\pi}{L}\right)\right]$$

$$= \begin{cases} \frac{1}{2}, & |\omega| \le 2\pi/L \\ 0, & \text{else} \end{cases}$$

(c) $L = 6$: $H_0(\omega) = \begin{cases} 1/2, & |\omega| \le \pi/3 \\ 0, & \text{else} \end{cases}$



(d) $h_0[n] = \frac{1}{L}, \quad n = 0$

$h_0[kL] = \frac{1}{L} \operatorname{sinc}(k) \cos(k\pi) = 0, \quad \forall k \neq 0$

$$\therefore \; h_0[n] = \begin{cases} \frac{1}{L}, & n = 0 \\ 0, & n = kL, \; \forall k \neq 0 \end{cases}$$

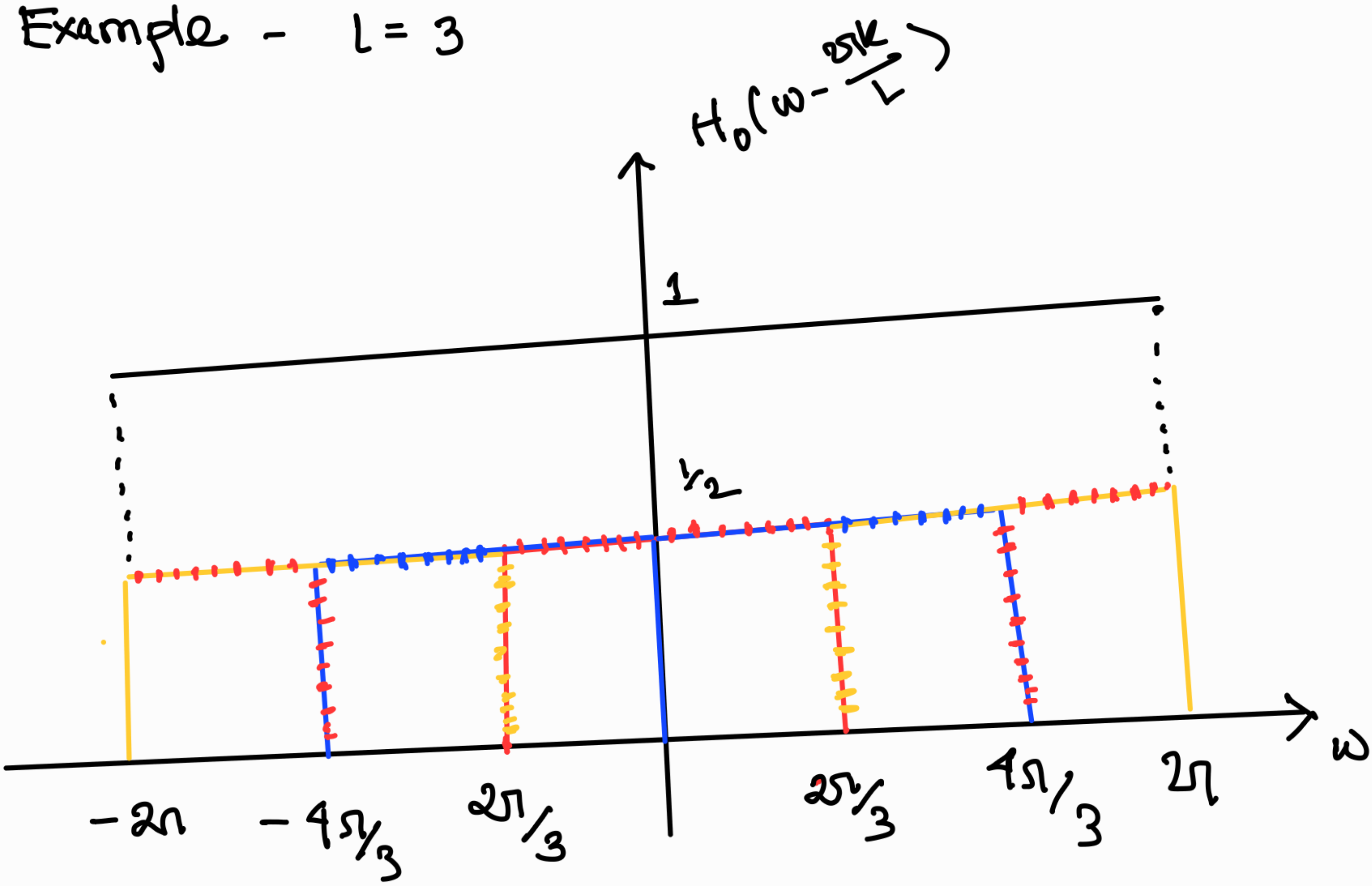
$\therefore$ $h_0[n]$ satisfies the time domain $cond^n$ for perfect reconstruction as the function only has one non-zero value in the desired sample points.

(e) $$\sum_{k=0}^{L-1} H_0\left(w - \frac{2\pi k}{L}\right) = 1$$

The summation takes two overlapping rectangles $\forall w, \; 0 \leq w \leq 2\pi$.

$\therefore$ $H_0(w)$ satisfies the frequency domain condition

Example - $L = 3$



(f) $$x[n] = \frac{1}{L}\,\mathrm{sinc}\left(\frac{n}{L}\right)$$

$$X(\omega) = \begin{cases} 1, & |\omega| < \frac{\pi}{L} \\ 0, & \text{else} \end{cases}$$

$$H_0(\omega) = \begin{cases} \frac{1}{2}, & |\omega| \le 2\pi/L \\ 0, & \text{else} \end{cases}$$

$h_0[n] = \frac{1}{L}\,\mathrm{sinc}\left(\frac{n}{L}\right)\cos\left(\frac{\pi}{L}n\right)$ has twice the bassband width compared with the filter $\frac{1}{L}\,\mathrm{sinc}\left(\frac{n}{L}\right)$. So frequency selectivity of $h_0[n]$ is worse.