1. (a)

$$\begin{aligned} x[n] &= \delta[n+1] - \delta[n] + \delta[n-1] \\ y[n] &= \frac{1}{2}\{x[n] + x[n-1]\} \\ &= \frac{1}{2}\{\delta[n+1] - \delta[n] + \delta[n-1] + \delta[n] - \delta[n-1] + \delta[n-2]\} \\ &= \frac{1}{2}\delta[n+1] + \frac{1}{2}\delta[n-2] \end{aligned}$$

(b)

$$h[n] = \frac{1}{2}\{\delta[n] + \delta[n-1]\}$$

Convolution of $h[n]$ and $x[n]$:

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \frac{1}{2}\{\delta[n] + \delta[n-1]\} * \{\delta[n+1] - \delta[n] + \delta[n-1]\} \\ &= \frac{1}{2}\{\delta[n+1] + \delta[n-2]\} \\ &= \frac{1}{2}\delta[n+1] + \frac{1}{2}\delta[n-2] \end{aligned}$$

(c) i.

$$\begin{aligned} y[n] &= \frac{1}{2}[e^{j\omega n} + e^{j\omega(n-1)}] = \frac{1}{2}[1 + e^{-j\omega}]e^{j\omega n} \\ H(\omega) &= \frac{1}{2}[1 + e^{-j\omega}] \end{aligned}$$

ii.

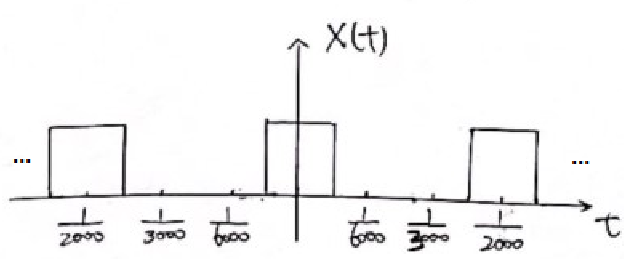
$$\begin{aligned} H(\omega) &= \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \\ &= \sum_{n=-\infty}^{\infty} \frac{1}{2}\{\delta[n] + \delta[n-1]\}e^{-j\omega n} \\ &= \frac{1}{2}(1 + e^{-j\omega}) \end{aligned}$$

(d)

$$
\begin{aligned}
X(\omega) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\
&= \sum_{n=-\infty}^{\infty} \delta[n+1] - \delta[n] + \delta[n-1]e^{-j\omega n} \\
&= e^{-j\omega} + e^{j\omega} - 1 \\
Y(\omega) &= H(\omega)X(\omega) \\
&= \frac{1}{2}(1+e^{-j\omega})(e^{-j\omega} + e^{j\omega} - 1) \\
&= \frac{1}{2}(e^{-j\omega} + e^{j\omega} - 1 + e^{-2j\omega} + 1 - e^{-j\omega}) \\
&= \frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-2j\omega} \\
y[n] &= \frac{1}{2}\delta[n+1] + \frac{1}{2}\delta[n-2]
\end{aligned}
$$

2. (a) $x(t) = rep_{\frac{1}{2000}}[rect(6000t)]$

   (b) The sketch is shown below.



   (c) Using CTFT Tranform Pairs:

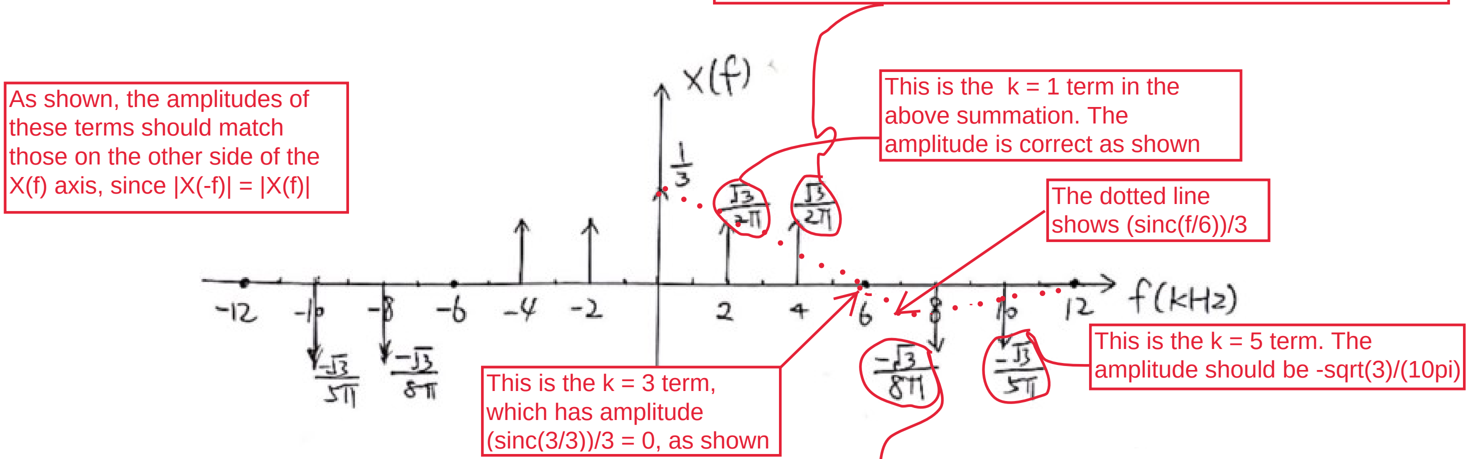
$$rect(t) \leftrightarrow sinc(f)$$

$$rep_T[x(t)] \leftrightarrow \frac{1}{T}comb_{\frac{1}{T}}[X(f)]$$

The Fourier Transform is:

$$
\begin{aligned}
X(f) &= 2000comb_{2000}[\frac{1}{6000}sinc(\frac{f}{6000})] \\
&= \frac{1}{3}\sum_{k=-\infty}^{\infty} sinc(\frac{k}{3})\delta(f - 2000k)
\end{aligned}
$$

(d) The sketch is shown below.



(e) From c part, the Fourier Transform is:

$$X(f) = \frac{1}{3}\sum_{k=-\infty}^{\infty} sinc(\frac{k}{3})\delta(f - 2000k)$$

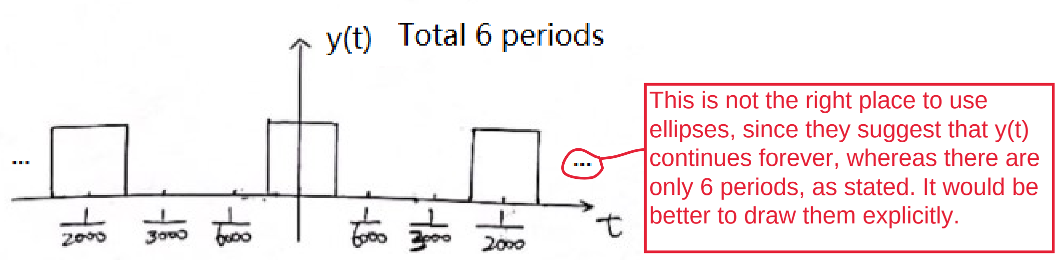
The direct inverse CTFT of $X(f)$:

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} \frac{1}{3}\sum_{k=-\infty}^{\infty} sinc(\frac{k}{3})\delta(f - 2000k)e^{j2\pi ft}df \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{3}sinc(\frac{k}{3})\int_{-\infty}^{\infty} \delta(f - 2000k)e^{j2\pi ft}df \\ &= \sum_{k=-\infty}^{\infty} \frac{1}{3}sinc(\frac{k}{3})e^{j2\pi 2000kt} \end{aligned}$$

So, the Fourier coefficient is $X_k = \frac{1}{3}sinc(\frac{k}{3})$.

(f) $y(t) = x(t) \times rect(\frac{t}{3\cdot 10^{-3}}) = x(t) \times rect(\frac{1000t}{3})$

(g) The sketch is shown below.



(h) Using the Fourier Transform Pair $rect(t) \leftrightarrow sinc(f)$ and the convolution theorem $x(t) * h(t) = X(f)H(f)$, the Fourier transform:

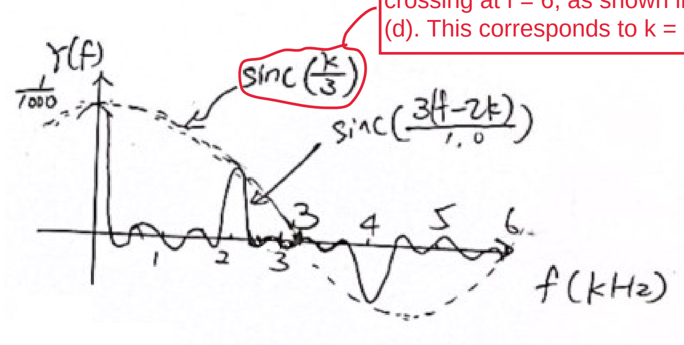
$$\begin{aligned} Y(f) &= X(f) * \frac{3}{1000} sinc(\frac{3f}{1000}) \\ &= \frac{1}{3} \sum_{k=-\infty}^{\infty} sinc(\frac{k}{3}) \delta(f - 2000k) * \frac{3}{1000} sinc(\frac{3f}{1000}) \\ &= \frac{1}{1000} \sum_{k=-\infty}^{\infty} sinc(\frac{k}{3}) sinc(\frac{3(f - 2000k)}{1000}) \end{aligned}$$

(i) The $sinc(3f/1000)$ function is replicated every 2 kHz. The zeros-crossings occur at interval 1/3 kHz. So there should be 6 zero-crossings between each replication at interval 2 kHz. The kth replication of $sinc(3f/1000)$ centered at 2 kHz, is weighted by $sinc(k/3)$.

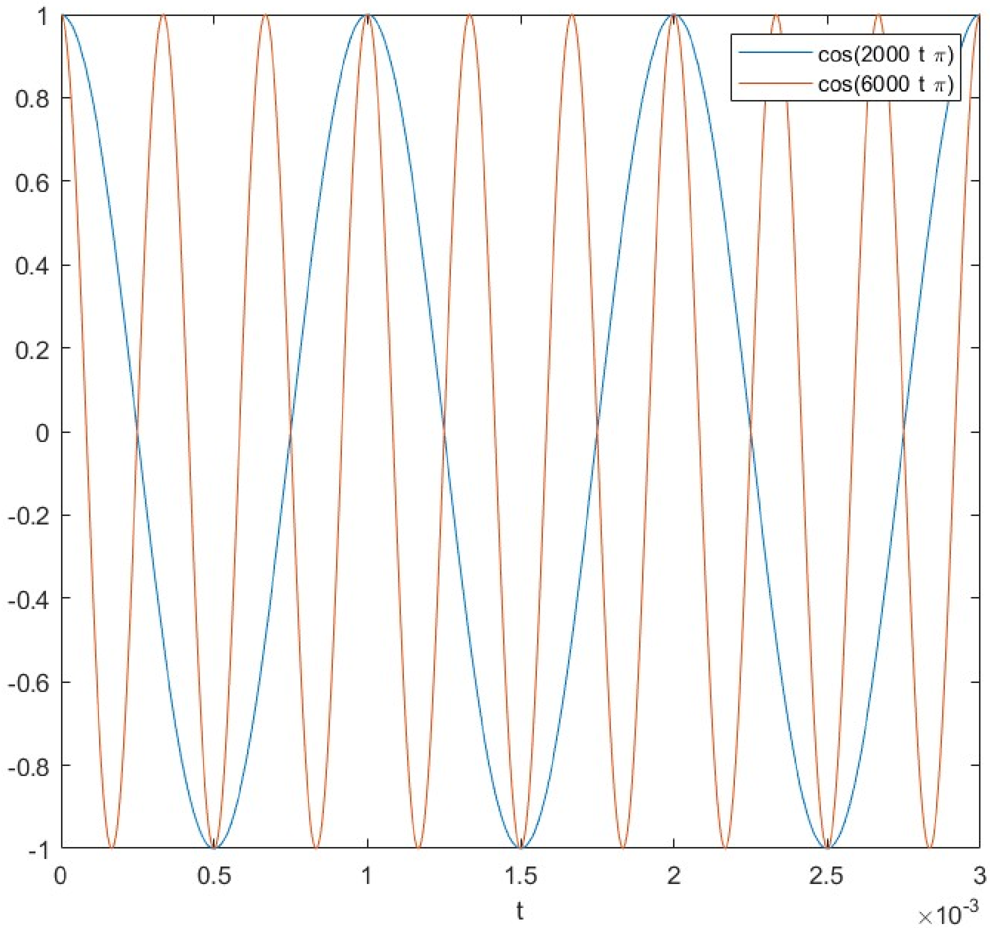
2k

The sketch is shown below.

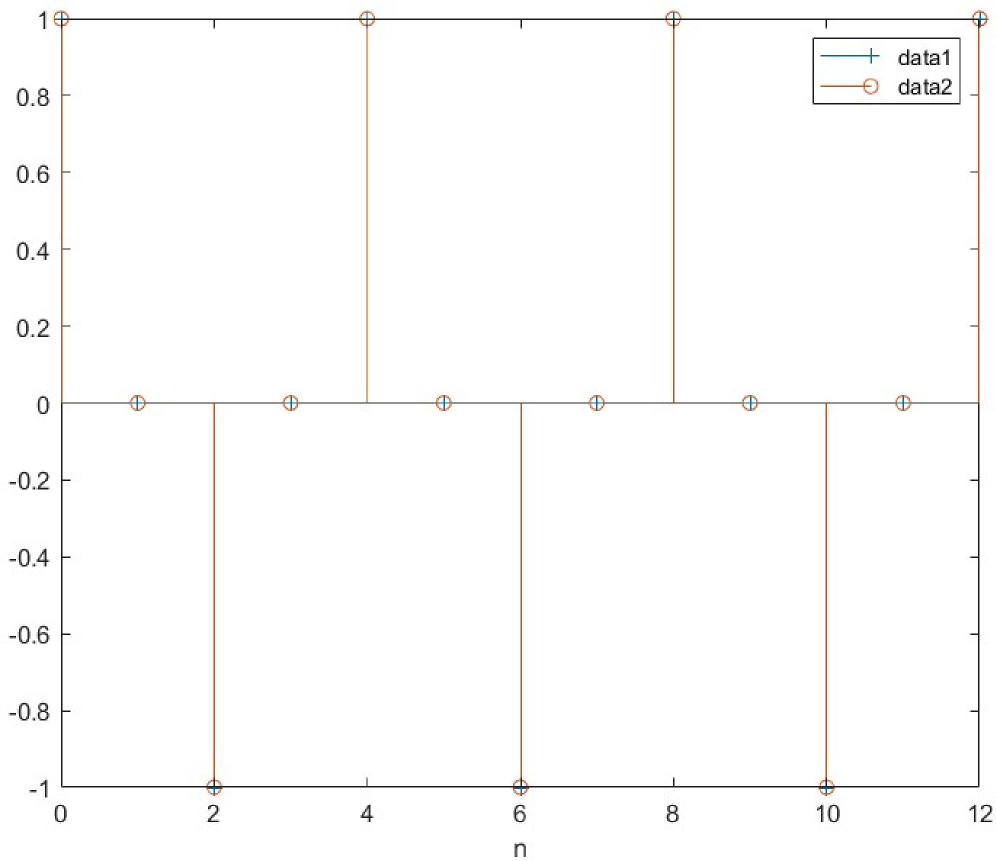
This should be sinc(f/6), which has its first zero-crossing at f = 6, as shown in the sketch for part (d). This corresponds to k = 3.



3. (a) The plot is shown below.



(b) The plot is shown below.

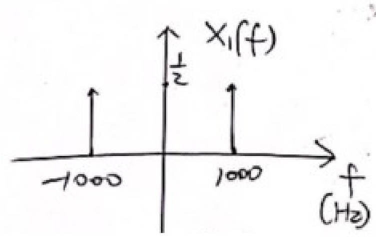


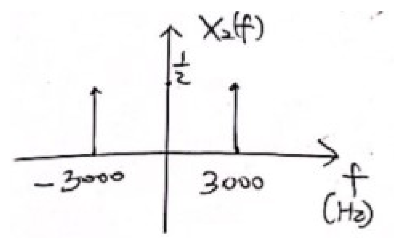
(c) Apply the Fourier Transform pairs

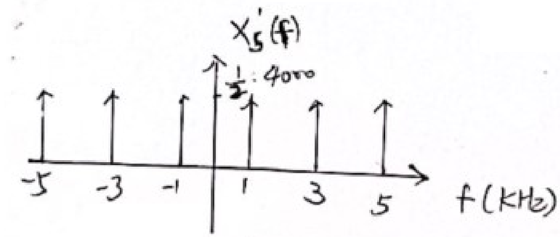
$$cos(2\pi f_0 t) \leftrightarrow \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$$

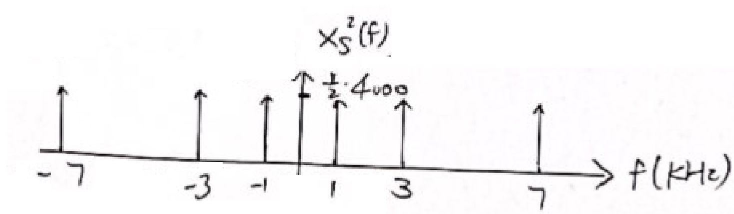
$$comb_T[x(t)] \leftrightarrow \frac{1}{T} rep_{\frac{1}{T}}[X(f)]$$

$$\begin{aligned}
X_1(f) &= \frac{1}{2}(\delta(f - 1000) + \delta(f + 1000)) \\
X_2(f) &= \frac{1}{2}(\delta(f - 3000) + \delta(f + 3000)) \\
X_s^1(f) &= 4000 rep_{4000}[\frac{1}{2}(\delta(f - 1000) + \delta(f + 1000))] \\
&= 4000 \sum_{k=-\infty}^{\infty} [\frac{1}{2}(\delta(f - 4000k - 1000) + \delta(f - 4000k + 1000))] \\
X_s^2(f) &= 4000 rep_{4000}[\frac{1}{2}(\delta(f - 3000) + \delta(f + 3000))] \\
&= 4000 \sum_{k=-\infty}^{\infty} [\frac{1}{2}(\delta(f - 4000k - 3000) + \delta(f - 4000k + 3000))]
\end{aligned}$$





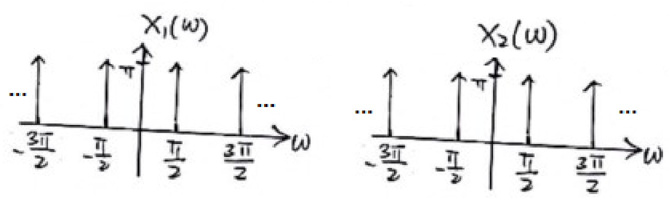




(d) Apply the Fourier Transform pairs

$$cos(\omega_0 n) = \pi \times rep_{2\pi}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

$$\begin{aligned} X_1(\omega) &= \pi \times rep_{2\pi}[\delta(\omega - \frac{2000\pi}{4000}) + \delta(\omega + \frac{2000\pi}{4000})] \\ &= \pi \sum_{k=-\infty}^{\infty} (\delta(\omega - 2\pi k - \frac{\pi}{2}) + \delta(\omega - 2\pi k + \frac{\pi}{2})) \\ X_2(\omega) &= \pi \times rep_{2\pi}[\delta(\omega - \frac{6000\pi}{4000}) + \delta(\omega + \frac{6000\pi}{4000})] \\ &= \pi \sum_{k=-\infty}^{\infty} (\delta(\omega - 2\pi k - \frac{3\pi}{2}) + \delta(\omega - 2\pi k + \frac{3\pi}{2})) \end{aligned}$$

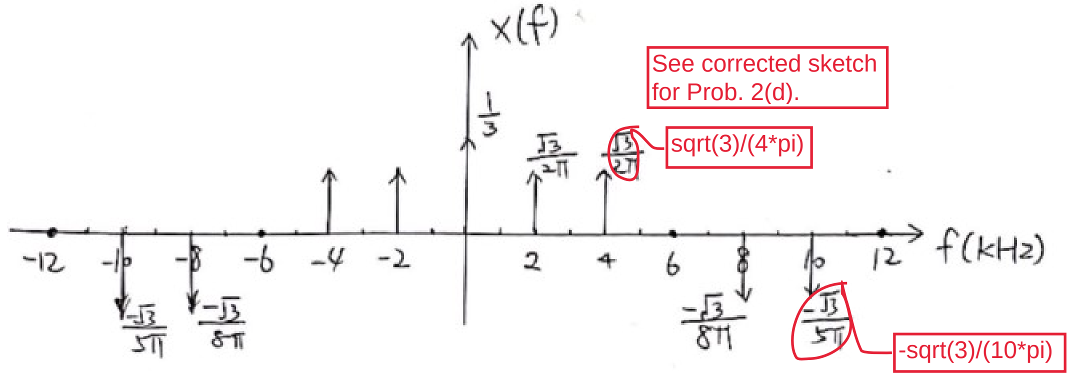


(e) In part a, signals $x_1(t)$ and $x_2(t)$ are different from each other. However, both signals would appear the same after sampling, as shown in part d. This is because the selection of the sampling frequency. Similarly, when looking at the CTFTs and DTFTs of the two signals, although both appear to be different, they are actually the same.

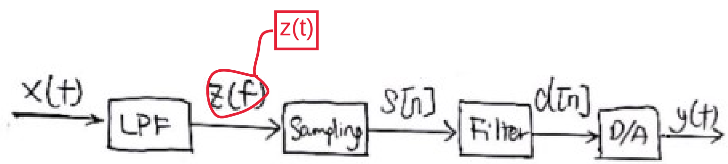
4. Same as problem 2,

$$X(f) = \frac{1}{3} \sum_{k=-\infty}^{\infty} sinc(\frac{k}{3})\delta(f - 2000k)$$

and the sketch of $X(f)$ is shown below.

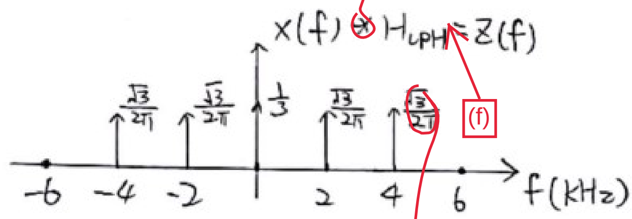


The block diagram of the described process is shown below.

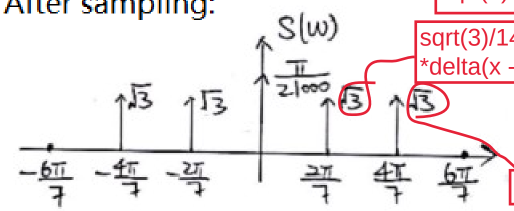


(a) $f_c = 7kHz$, $f_s = 14kHz$

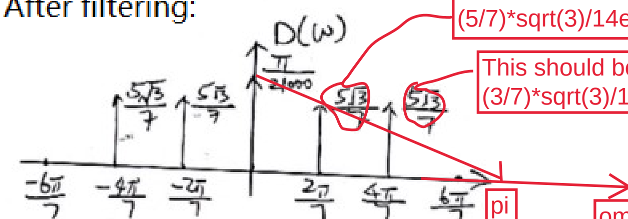
After low pass filter:



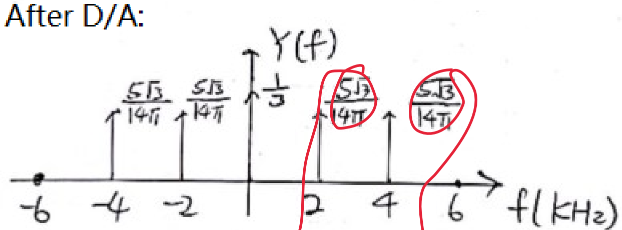
After sampling:



After filtering:



After D/A:



I think that we should assume that the filter gain of the D/A converter restores the frequency components in z(t) to their original amplitudes, taking into account the filter which passes the DC component with no change, and attenuates the 2kHz term by 5/7and the 4kHz term by 3/7.

The cutoff frequency is $\frac{f_s}{2} = 7$ kHz.

(5/7)*sqrt(3)/2*pi)

$$Y(f) = \frac{1}{3}\delta(f) + \frac{5\sqrt{3}}{14\pi}(\delta(f-2000) + \delta(f+2000)) + \frac{3\sqrt{3}}{14\pi}(\delta(f-4000) + \delta(f+4000))$$

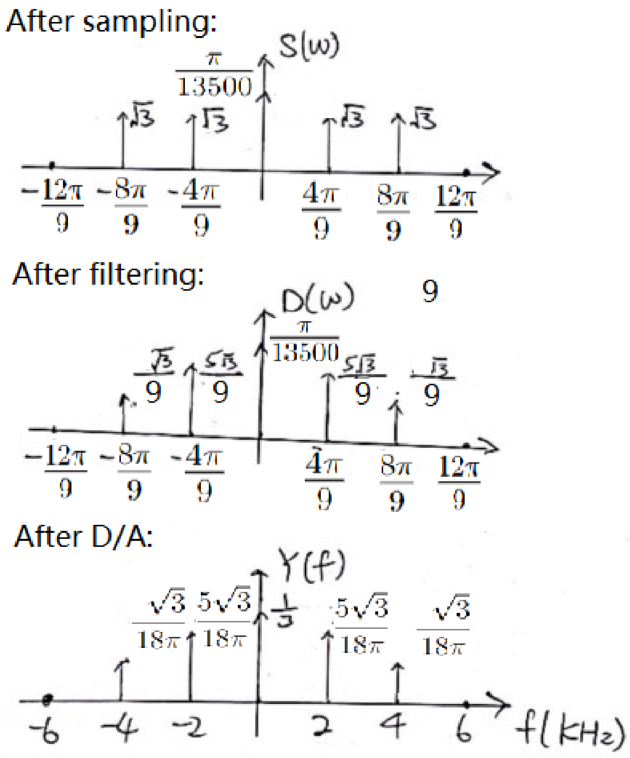
(3/7)*sqrt(3)/(4*pi)

$$y(t) = \frac{1}{3} + \frac{5\sqrt{3}}{14\pi}cos(2\pi 2000t) + \frac{3\sqrt{3}}{14\pi}cos(2\pi 4000t)$$

3/7)*sqrt(3)/pi

(5/7)*sqrt(3)/pi

(b) $f_c = 7kHz, f_s = 9kHz$ Please see pp. 13-15 of this document for a solution to Problem 4(b).



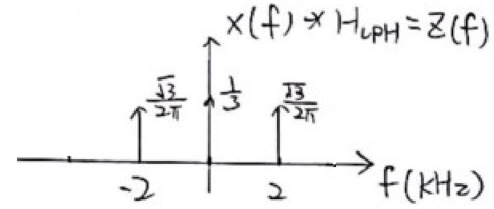
Since the input signal and the low pass filter are the same as part a, the low pass filtered signal for part b is the same as part a.
The cutoff frequency is $\frac{f_s}{2} = 4.5$ kHz.

$$Y(f) = \frac{1}{3}\delta(f) + \frac{5\sqrt{3}}{18\pi}(\delta(f-2000)+\delta(f+2000)) + \frac{\sqrt{3}}{18\pi}(\delta(f-4000)+\delta(f+4000))$$

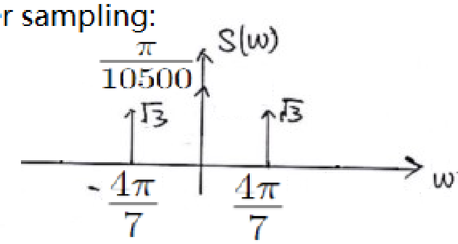
$$y(t) = \frac{1}{3} + \frac{5\sqrt{3}}{18\pi}cos(2\pi 2000t) + \frac{\sqrt{3}}{18\pi}cos(2\pi 4000t)$$

(c) $f_c = 3kHz$, $f_s = 7kHz$ Please see pp. 16 and 17 of this PDF document.

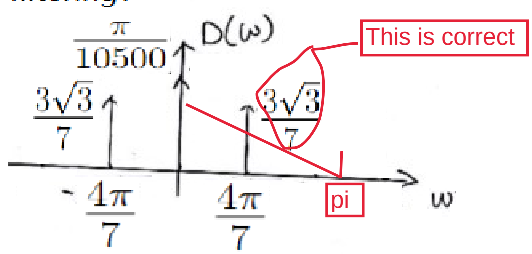
After low pass filter:



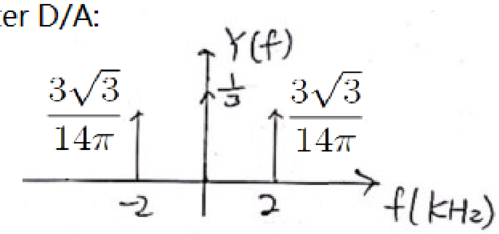
After sampling:



After filtering:



After D/A:

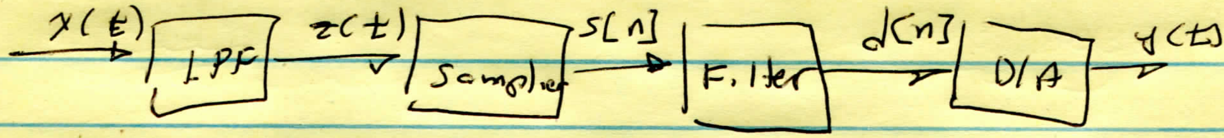


The curoff frequency is $\frac{f_s}{2} = 3.5$ kHz.

$$Y(f) = \frac{1}{3}\delta(f) + \frac{3\sqrt{3}}{14\pi}(\delta(f-2000) + \delta(f+2000))$$

$$y(t) = \frac{1}{3} + \frac{3\sqrt{3}}{14\pi}cos(2\pi 2000t)$$

4(b). from HW #3



$x(t)$ is a square wave with period $\frac{1}{2} \times 10^{-3}$ sec and 33.3 % duty cycle

$x(t) = rep_{\frac{1}{2}}\left[rect(t/\tfrac{1}{6})\right]$, t - msec

$X(f) = 2\, comb_{2}\left[\frac{1}{6} sinc(\tfrac{f}{6})\right]$, f - kHz

$$= \sum_{k=-\infty}^{\infty} \frac{1}{3} sinc\left(\frac{2k}{6}\right) \delta[f - 2k]$$

For this part, our filter has cutoff 7 kHz so

$$Z(f) = \frac{1}{3} sinc(0)\delta(f) + \frac{1}{3} sinc(\tfrac{1}{3})\left[\delta(f-2) + \delta(f+2)\right]$$

$$+ \frac{1}{3} sinc(\tfrac{2}{3})\left[\delta(f-4) + \delta(f+4)\right]$$

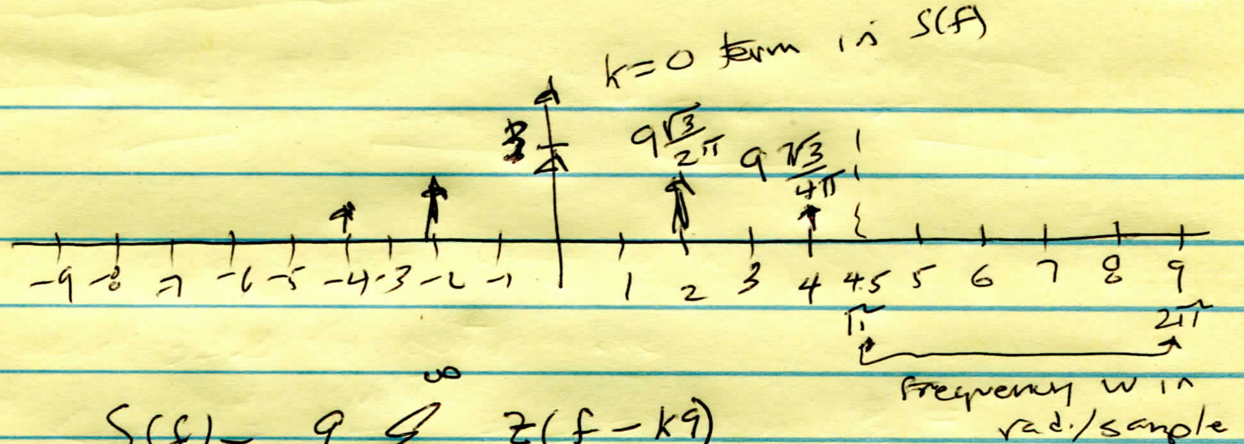
$k = 3$ term would be at $f = 6$ kHz; but it has zero amplitude because $sinc(\frac{3}{3}) = 0$.

Next, we sample at $f_s = 9$ (kHz) or $T = \frac{1}{9}$ msec

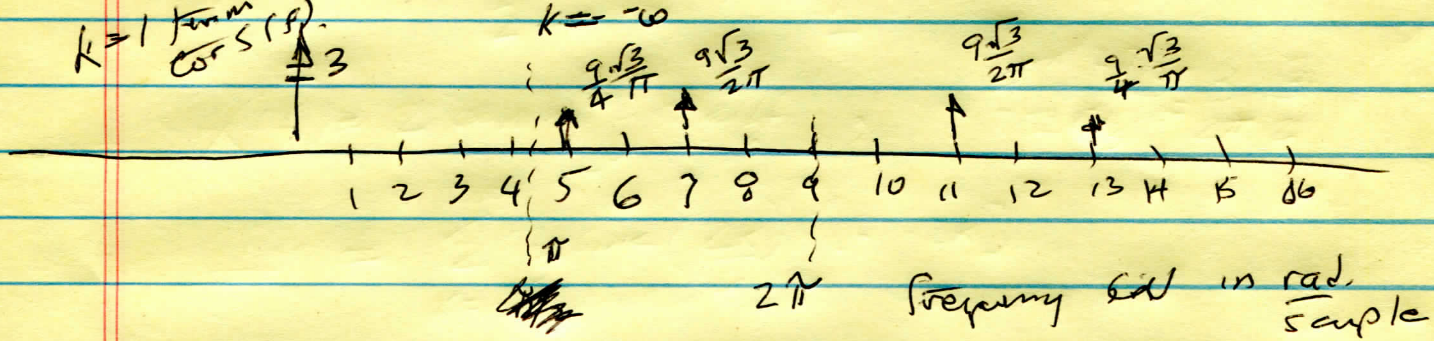
So $s(t) = comb_{\frac{1}{9}}[z(t)]$

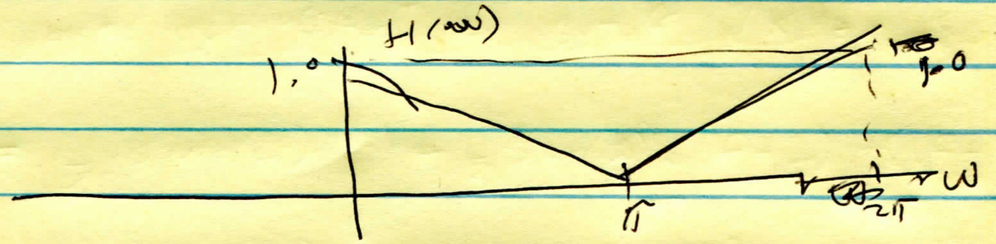
$S(f) = 9\, rep_{9}[Z(f)]$

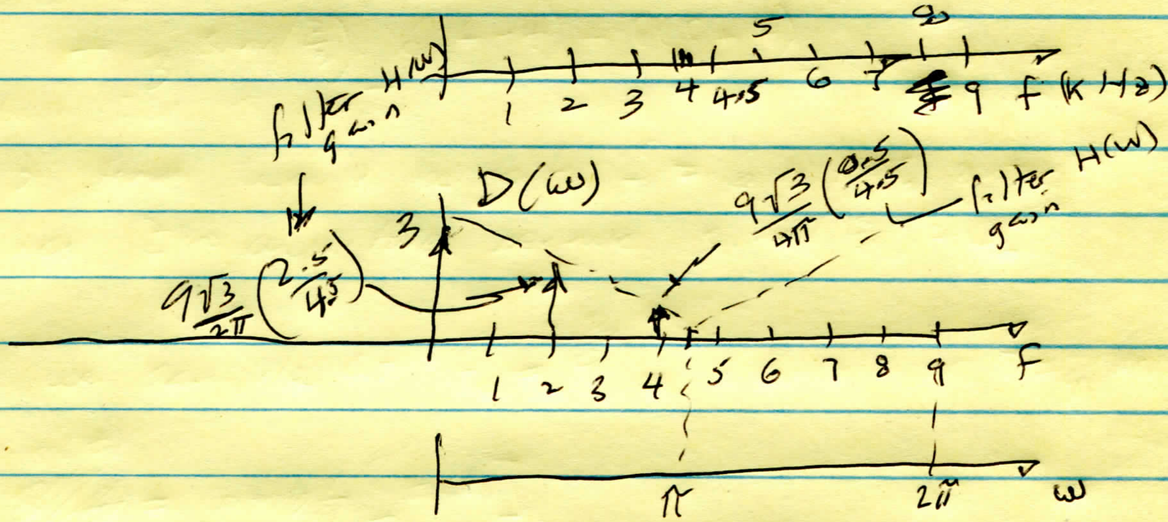
$sinc(\frac{1}{3}) = \frac{\sin(\pi/3)}{\pi/3} = \frac{3}{\pi}\frac{\sqrt{3}}{2}$ $\quad sinc(\frac{2}{3}) = \frac{\sin(2\pi/3)}{2\pi/3} = \frac{3}{2\pi}\frac{\sqrt{3}}{2}$



$$S(f) = 9 \sum_{k=-\infty}^{\infty} \delta(f - k9)$$







Since we are sampling at $f_s = 9\,kHz$, our ideal D/A effectively is an ideal LPF with cutoff at 4.5 kHz so only $k=0$ term in expansion for $S(f)$ survives. But it has been modified by the digital filter to yield $D(f)$

this is to get rid of the 9 from Eq. (**) near bottom of p. 1

Thus $Y(f) = \frac{1}{9}\left\{3\,\delta(f) + \frac{9\sqrt{3}}{2\pi}\left(\frac{2.5}{4.5}\right)\left\{\delta(f-2)+\delta(f+2)\right\}\right.$

$\left. + \frac{9\sqrt{3}}{4\pi}\left(\frac{0.5}{4.5}\right)\left\{\delta(f-4)+\delta(f+4)\right\}\right\}$

$\frac{2.5}{4.4}$

$= \frac{1}{3}\delta(f) + \frac{\sqrt{3}}{2\pi}(0.556)\left\{\delta(f-2)+\delta(f+2)\right\}$

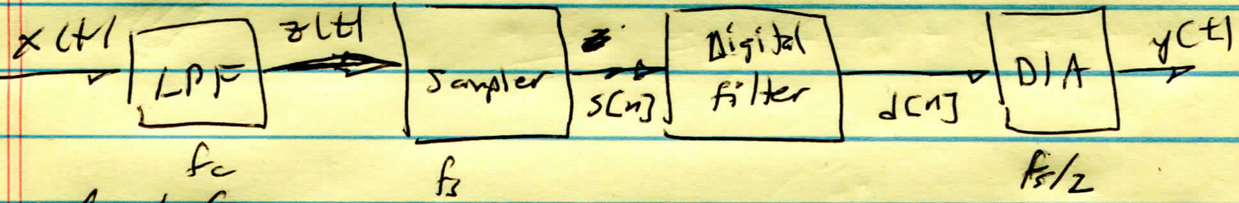
$+ \frac{\sqrt{3}}{12\pi}(0.111)\left\{\delta(f-4)+\delta(f+4)\right\}$

So $y(t) = \frac{1}{3} + \frac{\sqrt{3}}{\pi}(0.556)\cos(2\pi\, 2t) + \frac{\sqrt{3}}{2\pi}(0.111)\cos(2\pi(4)t)$

$f = 2$

t - msec

Problem 4(c) from HW #3



As before,

$$x(t) = rep_{\frac{1}{2}}\left[rect(t/\tfrac{1}{6})\right], \quad t - \text{msec.}$$

$$X(f) = \sum_{k=-\infty}^{\infty} \frac{1}{3}\, sinc\left(\frac{2k}{6}\right)\delta[f-2k]$$

For this part, $f_c = 3$ kHz, $f_s = 7$ kHz

So only the $k=0$, $k=-1$, & $k=+1$ terms survive the prefilter.

Thus $$Z(f) = \frac{1}{3}\delta(f) + \frac{1}{\pi}\left(\frac{\sqrt{3}}{2}\right)\{\delta[f-2] + \delta[f+2]\}$$

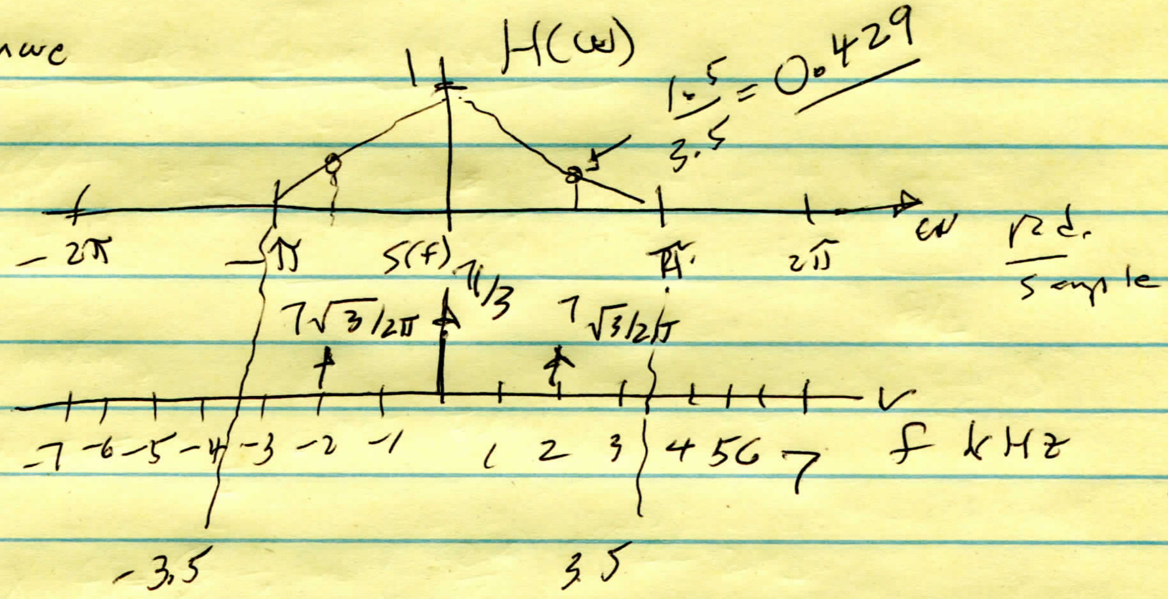
Next, we sample at $f_s = 7$ kHz

so $$S(f) = 7\, rep_7\left[Z(f)\right]$$

We have frequencies above 2 kHz; so there will be no aliasing.

Thus, we need to only consider the $k=0$ term in the replication for $S(f)$

We have



The ideal D/A will pass the k=0 term, i.e. Z(f), and will cancel the factor $f_s = 7$

So we have

$$y(t) = \frac{1}{3} + \frac{\sqrt{3}}{\pi}\cos(2\pi(2)t) \quad t - \text{msec.}$$