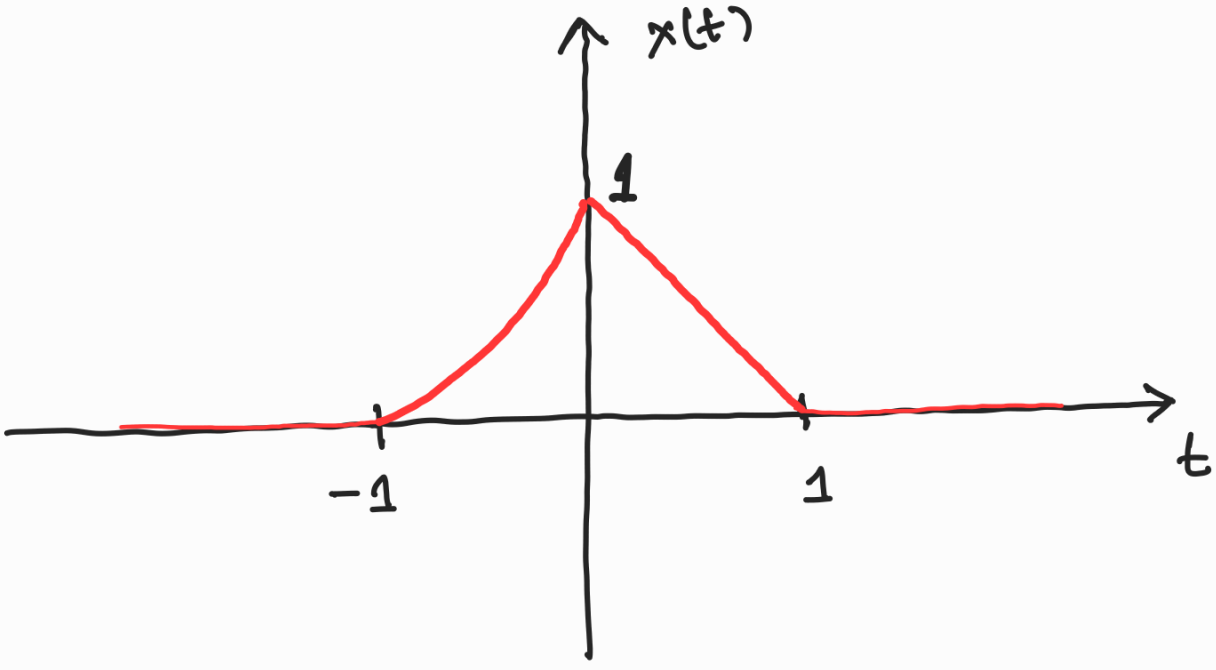
ECE-438 Assignment-2 Spring-2023

Problem-1

a)

$$x(t) = \begin{cases} 1-t, & 0 \le t \le 1 \\ (1+t)^2, & -1 \le t \le 0 \\ 0, & \text{else} \end{cases}$$

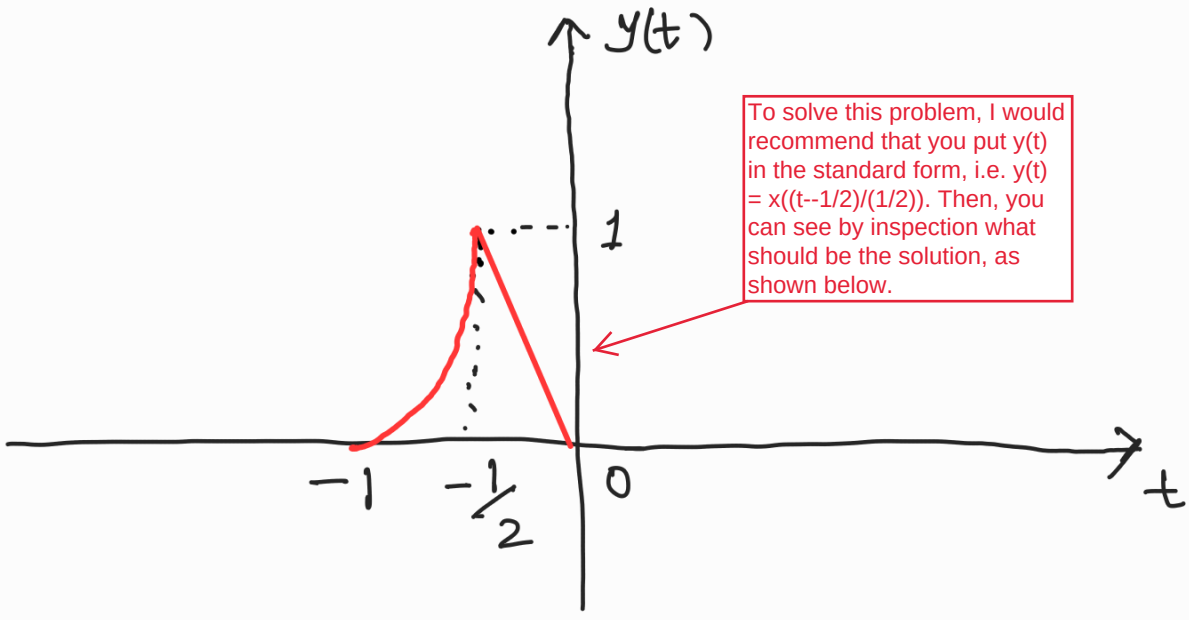


b)

$$y(t) = x(2t+1)$$

$$x(t) \xrightarrow{t:t+1} x(t+1)$$

$$x(t+1) \xrightarrow{t:2t} x(2t+1)$$



To solve this problem, I would recommend that you put y(t) in the standard form, i.e. y(t) = x((t--1/2)/(1/2)). Then, you can see by inspection what should be the solution, as shown below.

Problem-2

a) $y_1[n] = \frac{1}{4}\left(x[n] - 2x[n-1] + x[n-2]\right)$

$$Y_1(w) = \frac{1}{4}\left[x(w) - 2e^{-jw}X(w) + e^{-2jw}X(w)\right]$$

$$\Rightarrow \frac{Y_1(w)}{X(w)} = \frac{1}{4}\left[1 - 2e^{-jw} + e^{-j2w}\right]$$

$$\Rightarrow H_1(w) = \frac{1}{4}\left[1 - 2e^{-jw} + e^{j2w}\right]$$

$$= \frac{1}{4}e^{-2jw}\left[e^{2jw} - 2e^{jw} + 1\right]$$

$$= \frac{1}{4}e^{-2jw}\left(e^{jw} - 1\right)^2$$

$$= \frac{1}{4}\left(1 - e^{-jw}\right)^2$$

This follows directly from the first line above.

$$= \frac{1}{4}\left[e^{-jw/2}\left(e^{jw/2} - e^{-jw/2}\right)\right]^2$$

$$= \frac{1}{4}e^{-jw}\left(2j\sin(w/2)\right)^2$$

$$= -e^{-jw}\sin^2(w/2)$$

$\therefore$ For $x[n] = e^{j\omega n}$

$$y_1[n] = H_1(\omega)\Big|_{\omega=\omega} e^{j\omega n}$$

$$= -e^{-j\omega}\sin^2(\omega/2)\, e^{j\omega n}$$

$$= -e^{j\omega(n-1)}\sin^2(\omega/2)$$

b) $y_2[n] = x[n] + x[n-1] + y_2[n-1]$

$$Y_2(\omega) = X(\omega) + e^{-j\omega}X(\omega) + e^{-j\omega}Y_2(\omega)$$

$$\Rightarrow (1-e^{-j\omega})\,Y_2(\omega) = (1+e^{-j\omega})\,X(\omega)$$

$$\Rightarrow \frac{Y_2(\omega)}{X(\omega)} = \frac{1+e^{-j\omega}}{1-e^{-j\omega}}$$

$$\Rightarrow H_2(\omega) = \frac{1+e^{-j\omega}}{1-e^{-j\omega}} = \frac{e^{j\omega/2} + e^{-j\omega/2}}{e^{j\omega/2} - e^{-j\omega/2}}$$

$$\Rightarrow H_2(\omega) = \frac{2\cos(\omega/2)}{2j\sin(\omega/2)}$$

$$= -j\tan(\omega/2)$$

This should be cot(omega/2) or 1/tan(omega/2)

$$y_2[n] = H_2(\omega)\Big|_{\omega=\omega} e^{j\omega n}$$

$$\therefore y_2[n] = -je^{j\omega n}\tan(\omega/2) \quad \{\text{steady state}\}$$

For transient response,

$$y_2[0] = x[0] + x[-1] + y_2[-1] = 1$$

$$y_2[n] = -je^{j\omega n}\tan(\omega/2) + \underbrace{c_1(-1)^n}_{\text{transient response}}$$

This is not the right form for the transient response. I was not asking for the transient response. To find the entire response, it is easiest to use Z transform techniques, which will be coming up soon.

$$\Rightarrow y_2[0] = 1 = -j\tan(\omega/2) + c_1$$

$$\Rightarrow c_1 = 1 + j\tan(\omega/2)$$

$\therefore$ For $x[n] = e^{j\omega n}$

$$y_2[n] = -je^{j\omega n}\tan(\omega/2) + 1 + j\tan(\omega/2)$$

$$= 1 + j\tan(\omega/2)\{1 - e^{j\omega n}\}$$

Problem - 3

(a) $x[n] = 5\cos\left(\pi(n+1)/3\right)$

$= 5\cos\left(\frac{\pi}{3}n + \frac{\pi}{3}\right)$

f : Digital freq.

F : Analog freq.

F_s : sampling rate

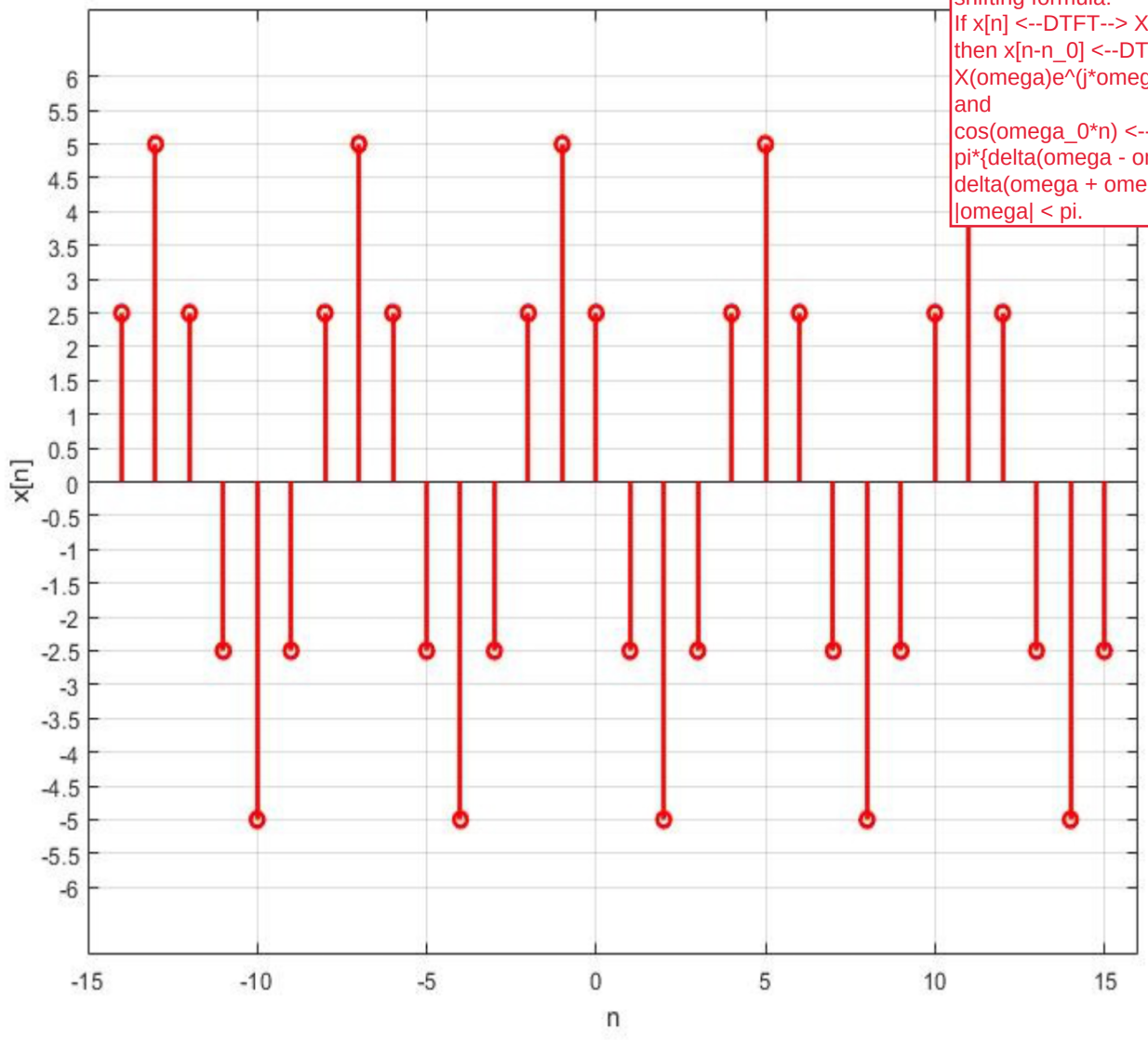
$\omega = 2\pi f = \frac{\pi}{3}$

$\Rightarrow f = \frac{1}{6}$

$\Rightarrow \frac{F}{F_s} = \frac{1}{6}$

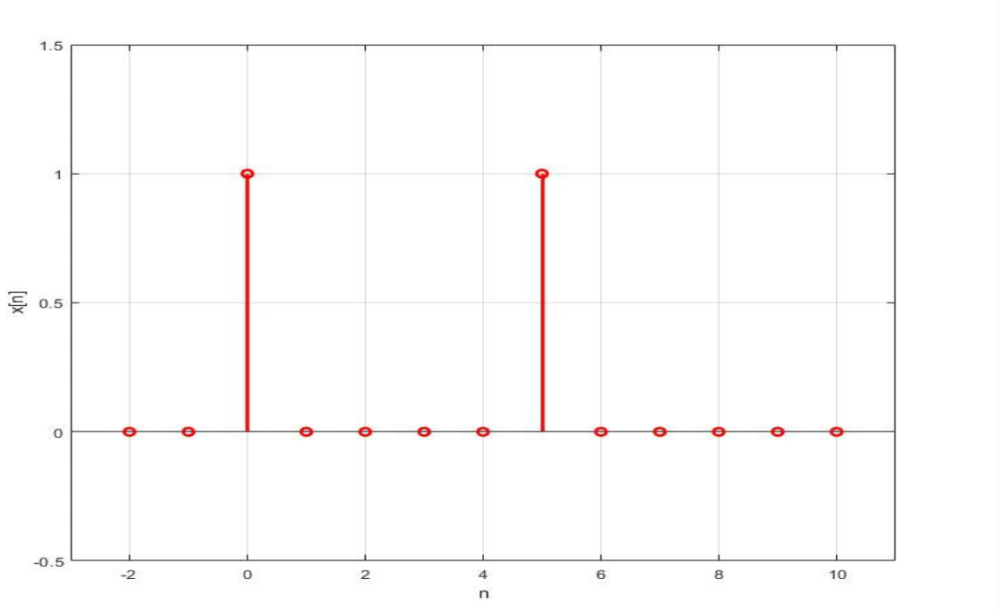
$\Rightarrow F_s = 6F$ (No aliasing)

This is correct. But it is a rather round-about way to get the answer. You should use the shifting formula:
If x[n] <--DTFT--> X(omega)
then x[n-n_0] <--DTFT--> X(omega)e^(j*omega*n_0)
and
cos(omega_0*n) <--DTFT--> pi*{delta(omega - omega_0) + delta(omega + omega_0)}, where |omega| < pi.



(b) $$x[n] = \begin{cases} 1, & n = 0 \\ 1, & n = 5 \\ 0, & \text{else} \end{cases}$$

$$x[n] = \delta[n] + \delta[n-5]$$



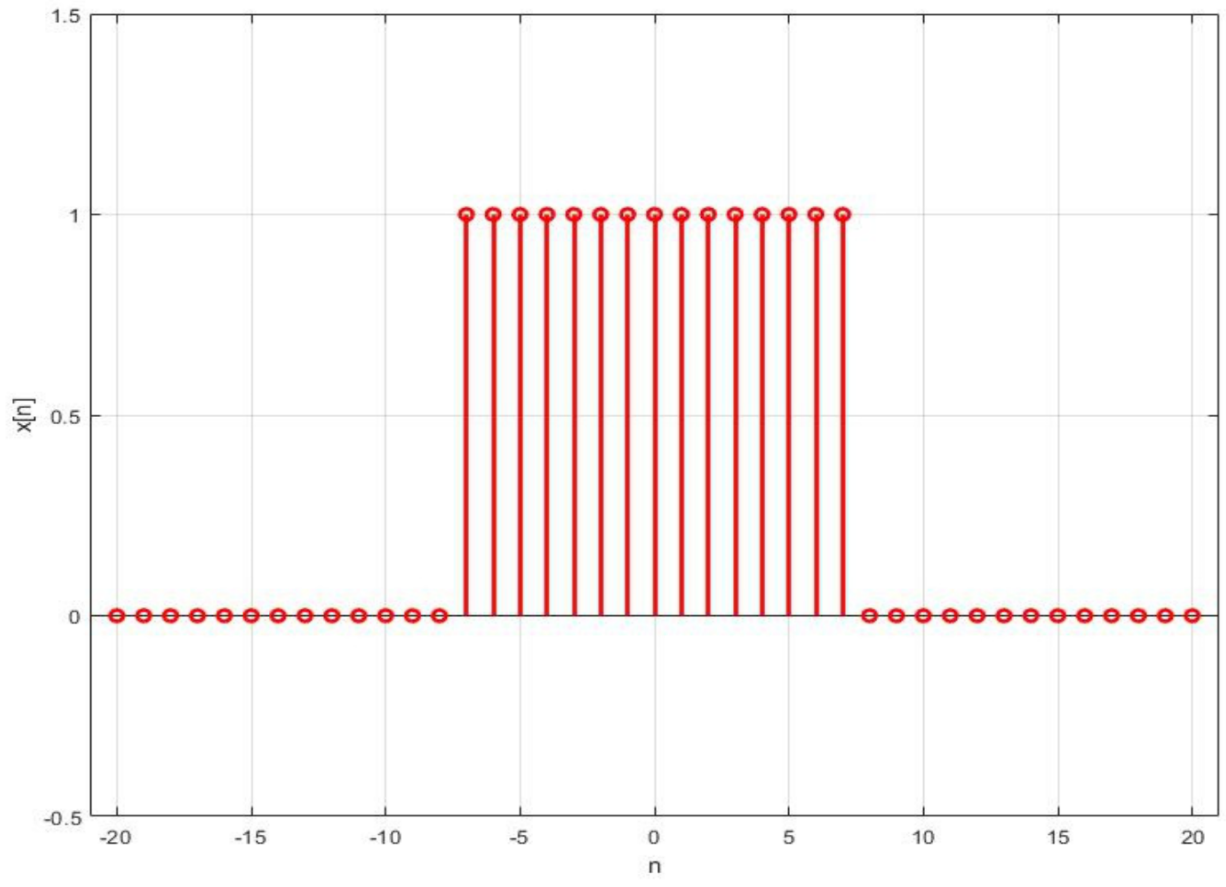
$$X(w) = F.T\{\delta[n]\} + e^{-5jw} F.T\{\delta[n]\}$$

$$= 1 + e^{-5jw}$$

$$= e^{-jw5/2}\left(e^{jw5/2} + e^{-jw5/2}\right)$$

$$= 2e^{-jw5/2}\cos\left(\frac{5}{2}w\right)$$

(c) $x[n] = \begin{cases} 1, & |n| \le 7 \\ 0, & \text{else} \end{cases}$



$$x[n] = u[n+7] - u[n-8]$$

$$= [u[n] - u[n-15]] * \delta[n+7]$$

$$X(\omega) \quad FT\{u[n] - u[n-15]\} e^{j\omega 7}$$

$$= \frac{\sin(\omega 15/2)}{\sin(\omega/2)} e^{-j\omega 7} e^{j\omega 7}$$

$$= \frac{\sin\left(\frac{15}{2}\omega\right)}{\sin(\omega/2)} = \text{psinc}_{15}(\omega)$$

Problem - 4

(a) $y[n] = e^{j\pi n/2} x[n]$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{j\pi n/2} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega - \frac{\pi}{2})n} = X\left(\omega - \frac{\pi}{2}\right)$$

This follows directly from the modulation property, which is one of the formulas.

(b) $y[n] = \frac{1}{2}\left(x[n] - x[-n]\right)$

$$Y(\omega) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(x[n] - x[-n]\right) e^{-j\omega n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} - \frac{1}{2} \sum_{n=-\infty}^{\infty} x[-n] e^{-j\omega n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} - \frac{1}{2} \sum_{n=-\infty}^{\infty} x[n] e^{-j(-\omega) n}$$

$$= \frac{1}{2}\left[X(\omega) - X(-\omega)\right]$$

As $x[n]$ is real valued,

if $|X(\omega)| = r(\omega)$, $\angle X(\omega) = \theta(\omega)$ $\xrightarrow{\text{then}}$ $|X(-\omega)| = r(\omega)$, $\angle X(-\omega) = -\theta(\omega)$

$$Y(\omega) = \frac{1}{2}\left[X(\omega) - X(-\omega)\right]$$

$$= \frac{1}{2}|X(\omega)|\, e^{j\angle X(\omega)} - \frac{1}{2}|X(-\omega)|\, e^{j\angle X(-\omega)}$$

$$= \frac{1}{2} r(\omega) e^{j\theta(\omega)} - \frac{1}{2} r(\omega) e^{-j\theta(\omega)}$$

$$= \frac{1}{2} r(\omega)\left[e^{j\theta(\omega)} - e^{-j\theta(\omega)}\right]$$

$$= r(\omega)\, j\sin(\theta(\omega))$$

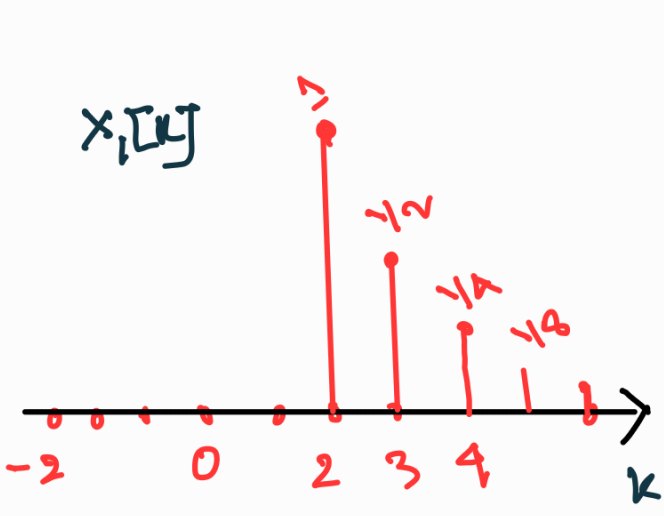
$$= j r(\omega)\sin(\theta(\omega))$$

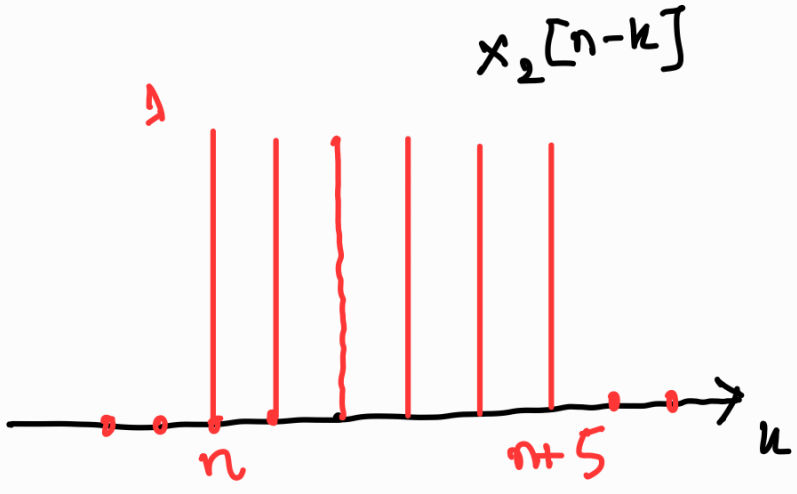
Problem-5

(a) $x_1[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2]$

$x_2[n] = u[n+5] - u[n-1]$

$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$





if $n < -4$, $y[n] = 0$

if $-4 \le n \le 1$,

$$y[n] = \sum_{k=2}^{n+6} x_1[k]\, x_2[n-k]$$

$$= \sum_{k=2}^{n+6} \left(\frac{1}{2}\right)^{k-2}$$

Let $l = k - 2$

$$\therefore y[n] = \sum_{l=0}^{n+4} \left(\frac{1}{2}\right)^{l}$$

$$\sum_{k=0}^{n} r^k = \frac{1 - r^{n+1}}{1 - r}$$

$$= \frac{1 - \left(\frac{1}{2}\right)^{n+5}}{1 - \frac{1}{2}}$$

$$= 2\left[1 - \left(\frac{1}{2}\right)^{n+5}\right], \quad -4 \le n \le 1$$

For $n \le 2$,

$$y[n] = \sum_{k=n}^{n+5} \left(\frac{1}{2}\right)^{k-2}$$

Let $m = k - n$

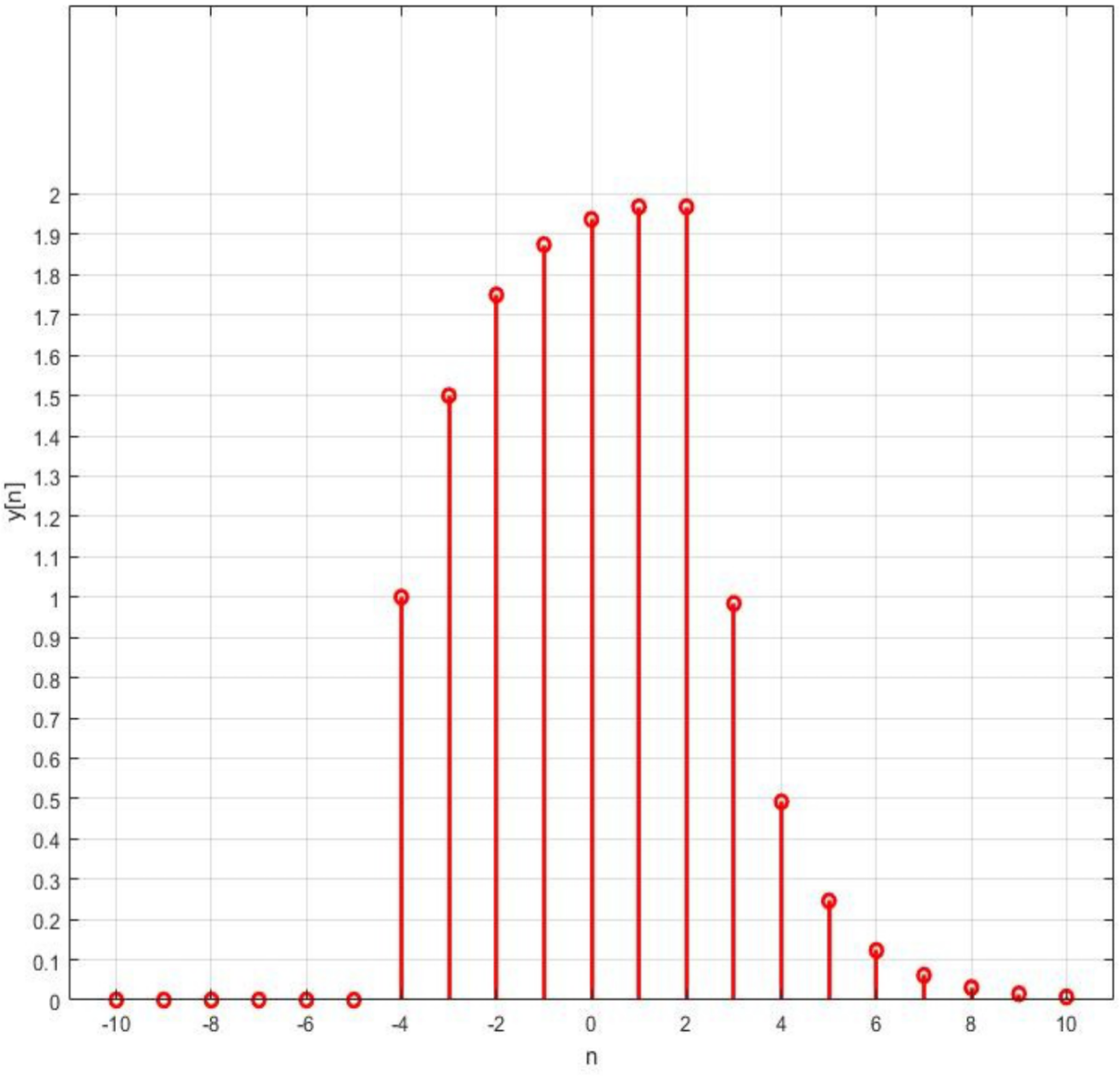
$$\therefore y[n] = \sum_{m=0}^{5} \left(\frac{1}{2}\right)^{m+n-2}$$

$$= \left(\frac{1}{2}\right)^{n-2} \sum_{m=0}^{5} \left(\frac{1}{2}\right)^{m}$$

$$= \left(\frac{1}{2}\right)^{n-2} \frac{1 - \left(\frac{1}{2}\right)^{6}}{1 - \frac{1}{2}} = \left(\frac{1}{2}\right)^{n-3}\left[1 - \left(\frac{1}{2}\right)^{6}\right]$$

$$\therefore y[n] = \left(\frac{1}{2}\right)^{n-3} - \left(\frac{1}{2}\right)^{n+3}, \quad n \ge 2$$

$$\therefore \quad y[n] = \begin{cases} 0 & , \ n \leq -5 \\ 2\left[1-\left(\frac{1}{2}\right)^{n+5}\right] & , \ -4 \leq n \leq 1 \\ \left(\frac{1}{2}\right)^{n-3} - \left(\frac{1}{2}\right)^{n+3} & , \ n \geq 2 \end{cases}$$



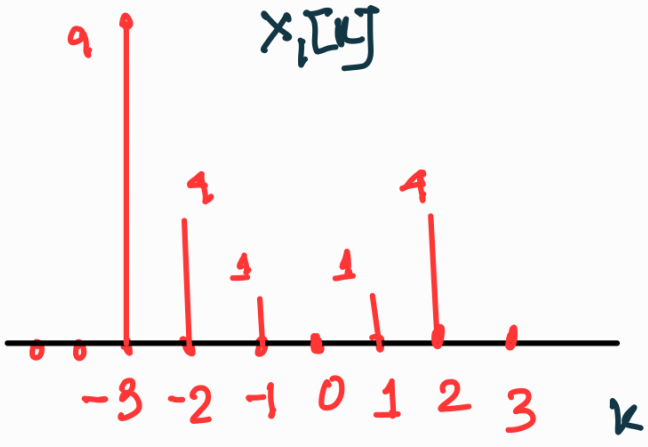
(b)

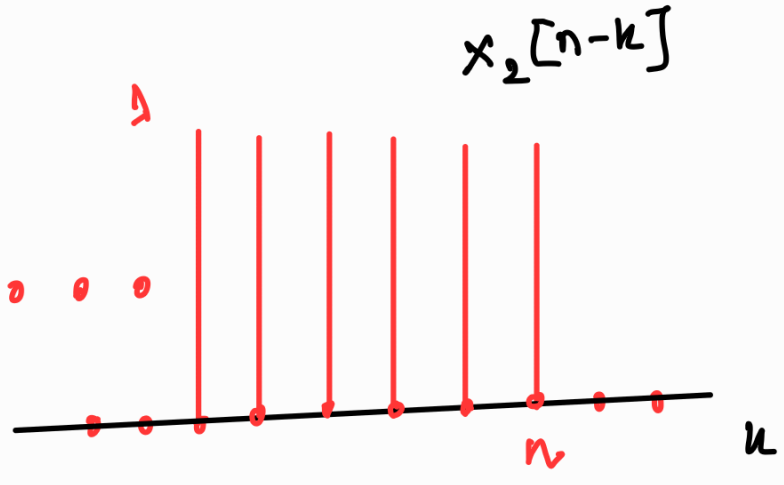
$$x_1[n] = |n|^2 \left(u[n+3] - u[n-3]\right)$$

$$x_2[n] = u[n]$$

Graphical method –

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$$





$$y[n] = \begin{cases} 0 & , \ n < -3 \\ 9 & , \ n = -3 \\ 9+4 & , \ n = -2 \\ 9+4+1 & , \ n = -1 \\ 9+4+1 & , \ n = 0 \\ 9+4+1+1 & , \ n = 1 \\ 9+4+1+1+4 & , \ n \geq 2 \end{cases}$$

$$\therefore y[n] = 9\delta[n+3] + 13\delta[n+2] + 14\delta[n+1] + 14\delta[n] + 15\delta[n-1] + 19u[n-2]$$

Alternate method -

$$y[n] = x_1[n] * x_2[n]$$

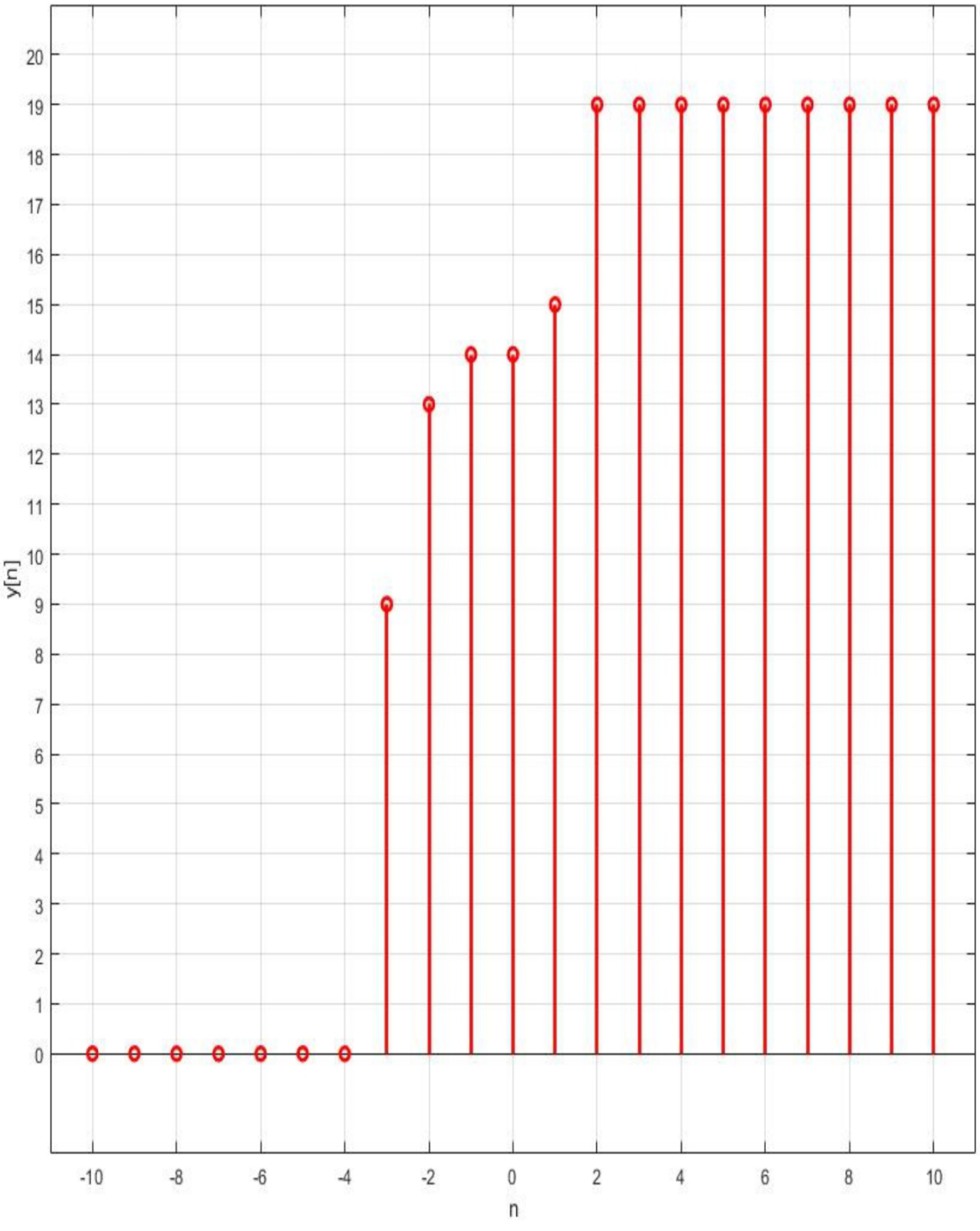
$$x_1[n] = 9\delta[n+3] + 4\delta[n+2] + \delta[n+1] + \delta[n-1] + 4\delta[n-2]$$

$$x_2[n] = u[n]$$

$$\therefore y[n] = 9u[n+3] + 4u[n+2] + u[n+1] + u[n-1] + 4u[n-2]$$

{ using the fact, $x[n] * \delta[n-k] = x[n-k]$ }

$$\therefore y[n] = 9\delta[n+3] + 13\delta[n+2] + 14\delta[n+1] + 14\delta[n] + 15\delta[n-1] + 19u[n-2]$$


y[n]
n
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10