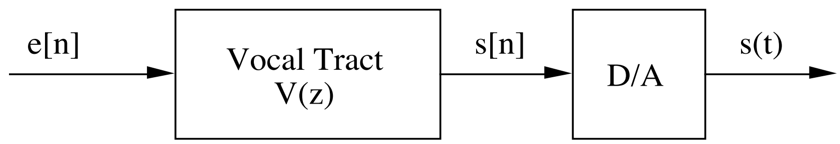
ECE 438 **Assignment No. 8** **Spring 2023**

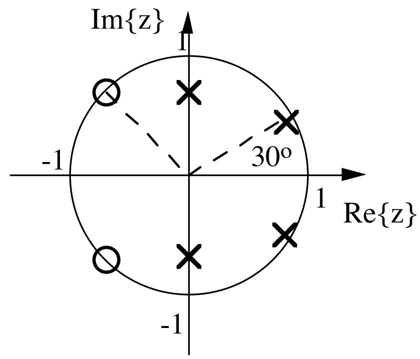
1. The digital synthesizer for voiced speech shown below operates at a 12 kHz sampling rate.



The excitation is given by

$$e[n] = \sum_{k=-\infty}^{\infty} \delta[n - 120k] .$$

The vocal tract transfer function $V(z)$ has poles and zeros at the locations shown below:

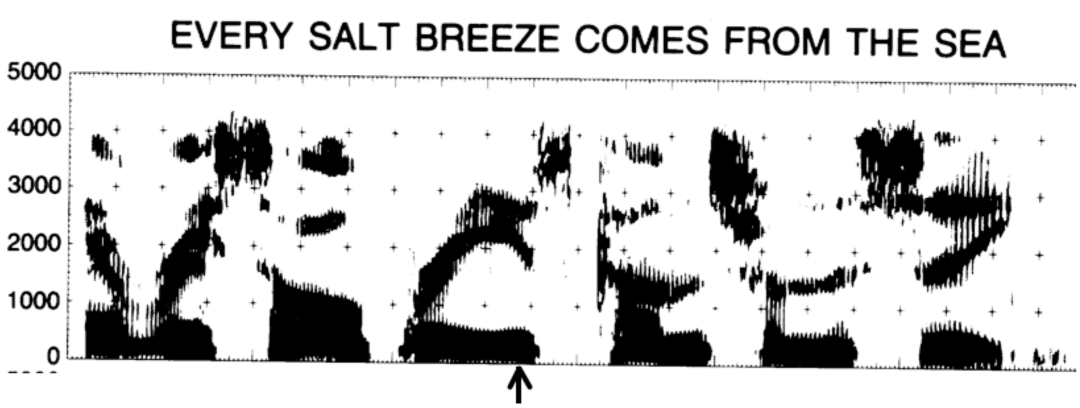


a. What is the pitch period in seconds and the pitch frequency in Hz?

b. Find the formant frequencies in Hz, and rank them according to their strength, *i.e.* how peaked the vocal tract response is at the corresponding frequency. Explain the reasoning underlying your ranking.

c. Sketch what the speech waveform $s(t)$ might look like.

2. Consider the spectrogram shown below for the utterance "Every salt breeze comes from the sea."



a. Assuming the entire utterance lasted 2 sec, *very* roughly estimate the pitch period.

b. Is this a wideband or narrowband spectrogram?

c. If your answer to part a, was "wideband", sketch what a narrowband spectrogram for this same signal would look like. On the other hand, if your answer to part a, was "narrowband", sketch what a wideband spectrogram for this same signal would look like.

d. Identify the formant frequencies at the time marked by the arrow.

e. What phoneme do you think is being uttered at this point? Support your answer by comparison with the formant frequencies in the table below:

TABLE 2.2. Formant frequencies for typical vowels.

ARPABET Symbol for Vowel	IPA Symbol	Typical Word	F_1	F_2	F_3
IY	/i/	beet	270	2290	3010
IH	/ɪ/	bit	390	1990	2550
EH	/ɛ/	bet	530	1840	2480
AE	/æ/	bat	660	1720	2410
AH	/ʌ/	but	520	1190	2390
AA	/ɑ/	hot	730	1090	2440
AO	/ɔ/	bought	570	840	2410
UH	/U/	foot	440	1020	2240
UW	/u/	boot	300	870	2240
ER	/ɝ/	bird	490	1350	1690

3. Consider the signal

$$x[n] = \begin{cases} \cos(\pi n/7), & n < 0, \\ \cos(2\pi n/3), & n \geq 0. \end{cases}$$

a. Carefully sketch $x[n]$.

Assume a rectangular window

$$w[n] = \begin{cases} 1, & |n| \leq 12, \\ 0, & \text{else.} \end{cases}$$

b. Compute the STDTFT as defined below

$$X(\omega, n) = \sum_k x[k]w[n-k]e^{-jwk}$$

for the following cases (Be sure to express your answer in terms of the function $\text{psinc}_N(\cdot)$ for appropriate values of N):

i. $n \leq -13$

ii. $n \geq 13$

iii. $n = 0$

c. Sketch $|X(\omega,n)|$ for all n. Be sure to label important dimensions. (You may ignore the contributions of the phase terms when preparing your sketch.)

4. In class, we derived conditions for perfect reconstruction using an L-channel modulated filter bank in which the ℓ-th channel has unit sample response $h_\ell[n] = h_0[n]e^{j2\pi\ell/L}$, where $h_0[n]$ is the unit sample response of the 0-th channel. Thus the frequency response of each channel is just a shifted version of the frequency response of the 0-th channel. Consider a system with unit sample response for the 0-th channel given by

$$h_0[n] = \tfrac{1}{L}\,\mathrm{sinc(n/L)cos(\pi n/L)}\ .$$

a. Sketch the unit sample response $h_0[n]$ for $L = 6$.

b. Derive the frequency response $\mathrm{H}_0(\omega)$ corresponding to the unit sample response $h_0[n]$. Note that it may be easiest to do this by considering what frequency response $\mathrm{H}_0(\omega)$ will yield the correct inverse DTFT $h_0[n]$ from $h_0[n] = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} H_0(\omega)e^{j\omega n}\,d\omega$, for the given $h_0[n] = \tfrac{1}{L}\,\mathrm{sinc(n/L)cos(\pi n/L)}$, while also keeping in mind Euler's identity.

c. Sketch $\mathrm{H}_0(\omega)$ for $L = 6$.

d. Show that $h_0[n]$ satisfies the time-domain condition derived in class for perfect reconstruction with a modulated filter bank.

e. Show that $\mathrm{H}_0(\omega)$ satisfies the frequency-domain condition derived in class for perfect reconstruction with a modulated filter bank.

f. Comment on the frequency selectivity of the filter bank with base unit sample response $h_0[n]$ given above, compared with a filter bank having base unit sample response $h_0[n] = \tfrac{1}{L}\,\mathrm{sinc(n/L)}$.