Due Wednesday 29 March 2023 at 11:59p EDT via Gradescope

ECE 438 **Assignment No. 7** **Spring 2023**

1. Consider the digital stereo multiplexer discussed in class on Monday 20 March 2023.

   a. Download the M-file `stereo_mux.m` and the speech files erf1s1t0 and ysf1s1t0 from the "Multiplexing" section (see left margin of web-page) of the web site:

   b. http://www.ecn.purdue.edu/VISE/ee438/demos/. You can also obtain these files from the course website under Module 1.8, which is under Legacy Lecture Notes.

   b. Draw the block diagram of a stereo demultiplexer system that will recover the left and right channel signals $x_l[n]$ and $x_r[n]$ from the multiplexed signal $x_{mux}[n]$. Be sure to identify all parameters of your system, such as modulator frequencies, filter cutoff frequencies, etc.

   c. Write a Python or M-file that will implement your block diagram from part a. It should take the signal `xmux` produced by the M-file `stereo_mux.m`, and generate from it the original signals `xl` and `xr` (bandlimited to 3.125 kHz, of course).

   d. Turn in plots of the original and demultiplexed bandlimited left channel signals, and the spectra of these signals.

   e. Play back your reconstructed signals, and compare to the original signals. Comment on any differences that you observe between the original and demultiplexed signals.

2. Consider two random variables X and Y which are jointly distributed according to the following bivariate density function

$$f_{XY}(x,y)=\begin{cases} 4, & 0\le x\le 0.5, 0\le y\le x, \\ 4, & 0.5\le x\le 1, x\le y\le 1, \\ 0, & \text{else} \end{cases}$$

   a. Sketch $f_{XY}(x,y)$.

   b. Find the marginal densities $f_X(x)$ and $f_Y(y)$.

   c. Are X and Y independent?

   d. Find the mean and variance of X and Y and the correlation coefficient ρ_{XY} between them.

3. Suppose that X and Y are two random variables with means μ_X and μ_Y, variances σ_X^2 and σ_Y^2, and correlation coefficient ρ_{XY}. We cannot observe X;

but we can observe Y. We wish to form an estimate $\hat{X} = g(y)$ for X, given that Y is observed and has value y. We constrain the estimate to have the form $\hat{X} = aY + b$, where a and b are constants.

a. Show that the values for a and b in terms of μ_X, μ_Y, σ_X^2, σ_Y^2, and ρ_{XY} that minimize the mean-squared error of the estimate are:

$$a = \frac{\rho_{XY}\sigma_X}{\sigma_Y}, \qquad b = \frac{\mu_X\sigma_Y - \rho_{XY}\sigma_X\mu_Y}{\sigma_Y}.$$

i.e. choose a and b to minimize $\varepsilon = E\{|\hat{X} - X|^2\}$. You may assume that all quantities are real-valued. *Hint:* Find an expression for ε in terms of a and b, μ_X, μ_Y, σ_X^2, σ_Y^2, and ρ_{XY}. Then differentiate separately with respect to each of the unknowns a and b.

b. Show that the mean-squared error for this estimator in terms of μ_X, μ_Y, σ_X^2, σ_Y^2, and ρ_{XY} is given by:

$$\varepsilon = \sigma_X^2\left(1 - \rho_{XY}^2\right).$$

c. Discuss the significance of the values for a, b, and ε for the following three cases: i.) $\rho_{XY} = 0$, ii.) $\rho_{XY} = 1$, iii.) $\rho_{XY} = -1$.

4. Let $X[n]$ be a wide-sense stationary sequence of random variables with zero mean and autocorrelation function

$$r_{xx}[n] = \begin{cases} 1, & n = 0 \\ \frac{3}{4}, & |n| = 1 \\ \frac{1}{4}, & |n| = 2 \\ 0, & \text{else} \end{cases}$$

Suppose that this sequence is filtered to generate the output sequence

$$y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-1]$$

a. Find the mean of the sequence $Y[n]$.

b. Find the cross-correlation $r_{XY}[n]$ between X and Y.

c. Find the autocorrelation $r_{YY}[n]$ of the output Y .