ECE 438 **Assignment No. 6** **Spring 2023**

1. Consider the signal

$$x[n] = \cos(\omega_1 n) + a\cos(\omega_2 n) + bd[n],$$

where a and b are constants and $d[n]$ is a sequence of independent Gaussian random variables with zero mean and unit variance.

a. Write a MATLAB or Python program that will:

i. plot x[n],

ii. compute the N point DFT $X[k]$ (using FFT routines available within MATLAB or Python),

iii. plot $|X[k]|$.

Turn in a printout of your M-file or Python program with your homework.

b. Run your program and generate output for the cases shown in the table below. Turn in the plots generated for each case.

c. Discuss the significance of each case. For each case, show only the sample values of the DFT in your plot, using stem plots. For Cases 5-8, the question that is of interest is whether or not one can determine that there are two separate frequency components present.

Case	N	ω_1	a	ω_2	b
1	20	0.62831853	0.0	-	0.0
2	200	0.62831853	0.0	-	0.0
3	20	0.64402649	0.0	-	0.0
4	200	0.64402649	0.0	-	0.0
5	200	0.64402649	0.4	1.27234502	0.0
6	200	0.64402649	0.4	0.79168135	0.0
7	200	0.64402649	0.4	0.79168135	0.2
8	200	0.64402649	0.4	0.79168135	1.0

2. Consider the following length 12 sequences:

n	0	1	2	3	4	5	6	7	8	9
$x_1[n]$	0	1	2	3	3	3	2	1	0	0
$x_2[n]$	1	1	1	1	-1	-1	-1	-1	0	0

a. Calculate the aperiodic convolution of $x_1[n]$ and $x_2[n]$.

b. Calculate the periodic (period 10) convolution of $x_1[n]$ and $x_2[n]$.

c. To what length would the sequences need to be padded with zeros so that a portion of their periodic convolution would match the nonzero part of their aperiodic convolution?

3. A random variable X has density

$$f_X(x) = \begin{cases} \kappa x^2, & 0 \le x \le 1 \\ 0, & \text{else} \end{cases}$$

where κ is a constant.

a. Determine the value for κ that will make $f_X(x)$ a proper density function.

b. Sketch $f_X(x)$.

c. Compute the mean and variance of X.

d. What is the probability that $X \ge 0.5$?

4. A signal with Laplacian density function

$$f_X(x) = \tfrac{1}{4} e^{-|x/2|}$$

is **uniformly** quantized to 9 bits. Assume that the range of the quantizer is set to limit the probability of overload to 0.01, and that the quantization error can be approximated as being uniformly distributed within each quantization bin.

a. Compute the signal-to-noise ratio in units of dB, assuming that the quantization error when the signal is in overloaded can be neglected.

b. Compare with the approximate result obtained in class when it is assumed that the signal X is uniformly distributed over the range of the quantizer.