Due 1 March 2023 at 11:59 PM EST via Gradescope

ECE 438 Assignment No. 5 Spring 2023

1. Consider a **causal** DT LTI system described by the following *non-recursive* difference equation (moving average filter)

$$y[n]=\frac{1}{6}\left\{x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]+x[n-5]\right\}.$$

   a. Find the impulse response $h[n]$ for this filter. Is it of finite or infinite duration?

   b. Find the transfer function $H(z)$ for this filter.

   c. Sketch the locations of poles and zeros in the complex z-plane.

   Hint: To factor $H(z)$, use the geometric series and the fact that the roots of the polynomial $z^N - r_0 = 0$ are given by

$$z_k = \left|r_0\right|^{1/N} e^{j\left[(\arg r_0)/N+2\pi k/N\right]}, k=0,\ldots,N-1,$$

   i.e. $z^N - r_0 = (z-z_0)(z-z_1)(z-z_2)\ldots(z-z_{N-1})$.

2. Consider a **causal** DT LTI system described by the following *recursive* difference equation

$$y[n]=\frac{1}{6}\left\{x[n]-x[n-6]\right\}+y[n-1].$$

   a. Find the transfer function $H(z)$ for this filter.

   b. Sketch the locations of poles and zeros in the complex z-plane.

   Hint: See Part c of Problem 4.

   4. Find the impulse response $h[n]$ for this filter by computing the inverse ZT of $H(z)$. Is it of finite or infinite duration?

3. The signal $x(t)=0.4\cos\left[2\pi(300)t\right]+1.2\cos\left[2\pi(3600)t\right]$ is sampled at 10 kHz to produce the digital signal $x[n]$. You compute a 2048-point DFT $X[k]$ of this signal. Find the approximate values of k and the amplitudes $|X[k]|$ corresponding to the spectral peaks in the analog signal.

4. This problem explores effect of signal length and zero-padding on the DFT. For each case below, do the following:

(i) Compute an exact expression for the N-point DFT $X[k], k = 0,...,N-1$ of the digital signal $x[n], n = 0,...,N-1$.

(ii) Carefully sketch the magnitude $|X[k]|, k = 0,...,N-1$ of your result (by hand), based on your answer to part (i) above.

a. $x[n] = \cos\left(2\pi(3/10)n\right), n = 0,...,9 \; (N = 10)$,

b. $x[n] = \cos\left(2\pi(3/10)n\right), n = 0,...,19 \; (N = 20)$,

c. $$x[n] = \begin{cases} \cos\left(2\pi(3/10)n\right), & n = 0,...,9 \\ 0, & n = 10,...,19 \end{cases} \quad (N = 20),$$

d. $x[n] = \cos\left(2\pi(3/20)n\right), n = 0,...,9 \; (N = 10)$,

e. $x[n] = \cos\left(2\pi(3/20)n\right), n = 0,...,19 \; (N = 20)$,

f. $$x[n] = \begin{cases} \cos\left(2\pi(3/20)n\right), & n = 0,...,9 \\ 0, & n = 10,...,19 \end{cases} \quad (N = 20).$$