ECE 438 **Assignment No. 4** **Spring 2023**

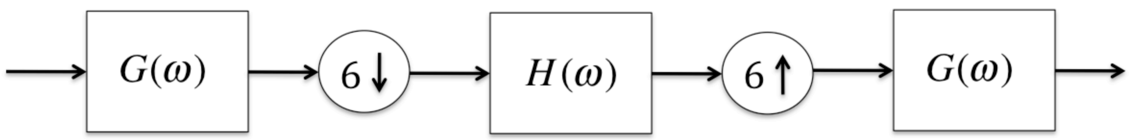
1. Consider the digital filter described by the following difference equation

$$y[n]=\frac{1}{3}\big(x[n]+x[n-1]+x[n-2]\big)$$

   a. Find a simple expression for the frequency response $H(\omega)$ of this filter.

   b. Sketch the magnitude of $H(\omega)$.

   Now consider the following digital system



   where $H(\omega)$ is the filter from parts a and b and $G(\omega)$ is an ideal low-pass filter with a cutoff frequency of $\pi/6$ rad/sample and unity gain in the passband.

   c. Find the overall frequency response $F(\omega)$ for this system.

   d. Sketch the magnitude of $F(\omega)$.

   e. Discuss the possible advantages of a system like that shown above compared to directly implementing a digital filter with frequency response $F(\omega)$ as a single stage.

2. For the signals below, do the following: (i) sketch the signal, (ii) find the ZT if it exists, (iii) sketch the ROC for the ZT, showing all poles and zeros.

   a. $x[n]=\left(\frac{1}{5}\right)^n \cos(\pi n/5)u[n]$

   b. $x[n]=\begin{cases}\left(\frac{1}{2}\right)^n, & n\geq 0\\ 3^n, & n\leq -1\end{cases}$

   c. $x[n]=\begin{cases}3^n, & n\geq 0\\ \left(\frac{1}{2}\right)^n, & n\leq -1\end{cases}$

Hint: You should, as much as possible, use known signal-ZT pairs and ZT properties, rather than evaluating the ZT sum directly.

3. Consider the ZT

$$X(z)=\frac{1}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+3z^{-1}\right)}$$

Sketch the 3 different ROC's that are possible for this ZT; and for each ROC, find the corresponding signal $x[n]$. Simplify your answers as much as possible. Also, for each of the three signals $x[n]$, state whether or not the DTFT $X(\omega)$ exists for that signal.

4. Consider a causal LTI system with transfer function

$$H(z)=\frac{\left(1-\frac{1}{4}z^{-2}\right)}{\left(1+\frac{1}{4}z^{-2}\right)}$$

a. Sketch the locations of the poles and zeros.

b. Use the graphical approach to determine the magnitude and phase of the frequency response $H(\omega)$, for $\omega=0,\pi/6,\ \pi/3,$and $\pi/2$. Based on these values, sketch the magnitude and phase of the frequency response for $-\pi\le\omega\le\pi$. (Be sure to show your work.)

c. Is the system stable, Explain why or why not?

d. Find the difference equation for $y[n]$ in terms of $x[n]$, corresponding to this transfer function $H(z)$.