Due Wednesday 08 February 2023 at 11:59 PM via GradeScope

ECE 438 **Assignment No. 3** **Spring 2023**

1. Consider a DT LTI system described by the following equation

$$y[n] = \frac{1}{2}\left\{x[n] + x[n-1]\right\}.$$

Find the response of this system to the input

$$x[n] = \begin{cases} 0, & n < -1 \\ 1, & n = -1 \\ -1, & n = 0 \\ 1, & n = 1 \\ 0, & n > 1 \end{cases}$$

by the following approaches:

a. directly substitute $x[n]$ into the difference equation describing the system;

b. find the impulse response $h[n]$ and convolve it with $x[n]$;

c. find the frequency response $H(\omega)$ by the following two approaches:

   i. apply the input $e^{j\omega n}$ to the difference equation describing the system,

   ii. find the DTFT of the impulse response,

   verify that both methods lead to the same result, then find the DTFT $X(\omega)$ of the input, multiply it by $H(\omega)$ to yield the DTFT $Y(\omega)$ of the output, and finally calculate the inverse DTFT $y[n]$.

Hints:

i. There is no need to simplify the frequency response or the DTFT of the input.

ii. To evaluate the inverse DTFT of $Y(\omega)$, simply put it in the series form $Y(\omega) = \sum_n y[n] e^{-j\omega n}$, and identify the terms $y[n]$ in the series.

d. Verify that all three approaches for finding $y[n]$ lead to the same result.

2. Consider a non-negative unit amplitude square wave $x(t)$ with period $\frac{1}{2} \bullet 10^{-3}$ sec. and a 33.3% duty cycle. Assume that the waveform is an even function of time. So $x(t) = x(-t)$.

a. Write an expression for $x(t)$ in terms of standard signals and operators.

b. Sketch $x(t)$. Be sure to dimension all important quantities in your plot.

c. Using standard transform pairs and transform relations, find a simple expression for the CTFT $X(f)$ of $x(t)$.

d. Sketch $X(f)$. Be sure to dimension all important quantities in your plot.

e. By taking the direct inverse CTFT of the expression for the $X(f)$ that you obtained in part c), and using the relationship $e^{j2\pi f_0 t} \overset{CTFT}{\leftrightarrow} \delta(f - f_0)$, express the signal $x(t)$ in the form of a Fourier series: $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T}$, where T is the period of the signal. Be sure to show an exact expression for the Fourier coefficients X_k.

Consider a new signal $y(t) = \begin{cases} x(t), & |t| < 1.5 \times 10^{-3} \\ 0, & \text{else} \end{cases}$, where $x(t)$ is defined as above.

f. Write an expression for $y(t)$ in terms of $x(t)$ and standard signals and operators.

g. Sketch $y(t)$. Be sure to dimension all important quantities in your plot.

h. Using standard transform pairs and transform relations, find a simple expression for the CTFT $Y(f)$ of $y(t)$.

i. Sketch $Y(f)$. Be sure to dimension all important quantities in your plot.

3. Consider the two CT signals $x_1(t) = \cos\left(2\pi(1000)t\right)$ and $x_2(t) = \cos\left(2\pi(3000)t\right)$.

a. Using Matlab or Python, plot both $x_1(t)$ and $x_2(t)$ on the same axes for $0 \le t \le 3 \bullet 10^{-3}$.

Define two DT signals according to $x_1[n] = x_1(nT)$ and $x_2[n] = x_2(nT)$, where $T = \frac{1}{4000}$ sec.

b. Using Matlab or Python, plot both $x_1[n]$ and $x_2[n]$ on the same axes for $0 \le n \le 12$.

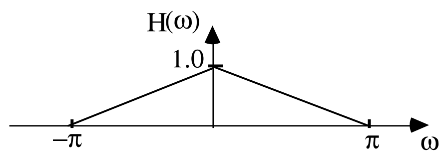
Let $x_s^1(t) = \text{comb}_T\left[x_1(t)\right]$ and $x_s^2(t) = \text{comb}_T\left[x_2(t)\right]$.

c. Derive and sketch the CTFTs $X_1(f)$, $X_2(f)$, $X_s^1(f)$, and $X_s^2(f)$ for the corresponding time-domain signals.

d. Derive and sketch the DTFTs $X_1(\omega)$, and $X_2(\omega)$ for the corresponding time-domain signals.

e. Discuss all your results for this problem.

4. The non-negative unit amplitude square wave with period $\frac{1}{2} \bullet 10^{-3}$ sec. and a 33.3% duty cycle from Problem 2 is filtered with an ideal low pass analog filter with cutoff at f_c kHz, and then sampled with an ideal sampler at a rate of f_s kHz, filtered with a digital filter having the frequency response $H(\omega)$ shown below, and then reconstructed as an analog signal $y(t)$ with an ideal D/A convertor with a cutoff frequency of $f_s / 2$ kHz.



Find the output $y(t)$ for the following values of f_c and f_s

a. $f_c = 7$ kHz, $f_s = 14$ kHz

b. $f_c = 7$ kHz, $f_s = 9$ kHz

c. $f_c = 3$ kHz, $f_s = 7$ kHz