ECE 438 Assignment No. 2 Spring 2023

1. Consider the CT signal

$$x(t)=\begin{cases} 1-t, & 0\le t\le 1 \\ (1+t)^2, & -1\le t\le 0 \\ 0, & \text{else} \end{cases}$$

   a. Carefully and accurately sketch $x(t)$. Be sure to dimension your axes.

   b. Let $y(t)=x(2t+1)$. Carefully and accurately sketch $y(t)$. Be sure to dimension your axes.

2. For each of the following signals, find the output $y[n]$ when the input is $x[n]=e^{j\omega n}$. Simplify your answer as much as possible.

   a. $y_1[n]=\frac{1}{4}\left(x[n]-2x[n-1]+x[n-2]\right)$.

   b. $y_2[n]=x[n]+x[n-1]+y_2[n-1],\quad y_2[n]=0, n<0$.

3 For each of the following signals, sketch the signal, and compute the DTFT $X(\omega)$. Simplify your answer as much as possible.

   a. $x[n]=5\cos\left(\pi(n+1)/3\right)$,

   c. $x[n]=\begin{cases} 1, & n=0 \\ 1, & n=5 \\ 0, & \text{else} \end{cases}$,

   d. $x[n]=\begin{cases} 1, & |n|\le 7 \\ 0, & \text{else} \end{cases}$.

4. Find an expression for the DTFT $Y(\omega)$ of the output in terms of the DTFT $X(\omega)$ of the input, when $y[n]$ and $x[n]$ are related by (Simplify your answer as much as possible, and assume that $x[n]$ is real-valued.):

   a. $y[n]=e^{j\pi n/2}x[n]$,

   b. $y[n]=\frac{1}{2}\left(x[n]-x[-n]\right)$.

5. Perform the convolution of the following pairs of signals by manually evaluating the summations, and carefully sketch your final answers:

   a. $\left(\frac{1}{2}\right)^{n-2} u[n-2]$ and $u[n+5]-u[n-1]$,

   b. $|n|^2\left(u[n+3]-u[n-3]\right)$ and $u[n]$.