Necessary and Sufficient Conditions for Full Diversity Order in Correlated Rayleigh Fading Beamforming and Combining Systems

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***Abstract*— Transmit beamforming and receive combining are low complexity, linear techniques that make use of the spatial diversity advantage provided by transmitters and/or receivers employing multiple antennas. There has been a growing interest in designing beamforming schemes for frequency division duplexing systems that use a limited amount of feedback from the receiver to the transmitter. This limited feedback conveys a beamforming vector chosen from a finite set known to both the transmitter and receiver. These techniques often use a set of beamforming vectors where the probability of error expression can not be easily formulated or bounded. It is of utmost importance to guarantee that the sets of beamforming and combining vectors are chosen such that full diversity order is achieved. For this reason, necessary and sufficient conditions on the sets of possible beamformers and combiners are derived that guarantee full diversity order in correlated Rayleigh fading.**

***Index Terms*-Beamforming, combining, correlated fading, diversity methods, MIMO systems, Rayleigh channels, transmit-receive diversity.**

I. Introduction

Transmit beamforming and/or receive combining are linear techniques that have been shown to improve mean signal strength and reduce signal level fluctuations in fading channels [1]. Wireless researchers studying beamforming and combining over the past 40 years have concentrated primarily on their application to single-input multiple-output (SIMO) or multiple-input single-output (MISO) wireless systems where multiple antennas are utilized exclusively at the transmitter or receiver. While probability of error expression analysis is difficult for these systems, there are many cases where exact results can be derived (see [2] and the references therein). These expressions are invaluable to system designers because they allow easy comparisons between beamforming, combining, and other vector channel signaling schemes.

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Over the past several years, it has become interesting to consider wireless systems using multiple-input multiple-output (MIMO) wireless communication links that employ multiple antennas at the transmitter and receiver. In these systems beamforming and combining are employed simultaneously, and thus the beamforming and combining vectors must be jointly designed, see e.g. [3]–[7]. Obtaining exact probability of error expressions that can be easily analyzed for MIMO beamforming and combining systems is often difficult, if not impossible. For this reason, the asymptotic measures of array gain and diversity order are often used to allow different diversity schemes to be compared.

Diversity order has been derived for many specific cases of MIMO beamforming and combining systems transmitting over memoryless and uncorrelated Rayleigh fading MIMO channels such as maximum ratio transmission (MRT) and maximum ratio combining (MRC) [3], [4], selection diversity transmission (SDT) and MRC [5], equal gain transmission (EGT) and MRC [6], equal gain transmission (EGT) and selection diversity combining (SDC) [6], EGT and equal gain combining (EGC) [6], and for various quantized beamforming schemes [6], [7]. In each of these systems the beamforming vector is chosen from some set of possible beamforming vectors and the combining vector is chosen from some set of possible combining vectors. These sets might be uncountable (MRT, MRC, EGT, EGC, etc.) or finite (see for example the quantized techniques in [6], [7]). Despite this work, the exact properties that the set of possible beamforming vectors and the set of possible combining vectors must possess to guarantee full diversity order in transmit and receive correlated Rayleigh fading has not yet been derived.

For this reason, we derive the necessary and sufficient conditions on the sets of beamforming and combining vectors that yield full diversity order when transmitting over memoryless, correlated Rayleigh fading channels. The uncorrelated result follows as a special case of this analysis. The derivation holds for any modulation scheme where the diversity orders of systems using MRT/MRC (systems using MRT and MRC) and SD (systems using selection diversity at both transmitter and receiver) over memoryless and uncorrelated Rayleigh fading channels are known to be the product of the number of transmit and receive antennas. We assume linear combining and the transmit/receive correlated Rayleigh channel model discussed in [8]–[10]. Simulations showing the probability of symbol error for a MIMO system using randomly generated beamforming and combining vectors verify our claim.

II. System Overview and Preliminary Analysis

A memoryless, Rayleigh fading MIMO system with M_t transmit antennas and M_r receive antennas using beamforming and combining can be represented by the input/output relationship

$$y_k = (\mathbf{z}^*\mathbf{H}\mathbf{w})s_k + \mathbf{z}^*\mathbf{n} \tag{1}$$

where s_k is a single-dimensional real ($s_k \in \mathbb{R}$) or complex ($s_k \in \mathbb{C}$) symbol transmitted at the k^{th} channel use, $\mathbf{H}$ is an $M_r \times M_t$ channel matrix, $\mathbf{w}$ is a length M_t column vector representing the transmit beamforming vector, $\mathbf{z}$ is a length M_r column vector representing the receive combining vector, $\mathbf{n}$ is a length M_r column vector representing the system's additive noise, and * represents conjugation and transposition. The noise is modeled as having p^{th} entry n_p distributed according to $\mathcal{CN}(0, N_0)$ with n_p independent of n_q for $p \neq q$. The transmit constellation is assumed to be normalized such that it has expected value given by $E_{s_k}[|s_k|^2] = \mathcal{E}$ where $|\cdot|$ denotes absolute value. As well, we assume that $E_{s_k,s_l}[s_k{}^* s_l] = 0$ when $k \neq l$.

The sets that beamforming and combining vectors are chosen from play an important role in the beamforming and combining scheme. The set over which a cost function is optimized is called the *feasible set* [11] of the optimization. We will call the set of all possible beamforming vectors the *beamforming feasible set* and the set of all possible combining vectors the *combining feasible set*. For example, an SD system uses a beamforming feasible set consisting of the columns of the $M_t \times M_t$ identity matrix and a combining feasible set consisting of the columns of the $M_r \times M_r$ identity matrix.

The correlated Rayleigh fading assumption will be expressed by modeling the MIMO fading as

$$\mathbf{H} = \mathbf{R}_R^{1/2}\mathbf{G}\mathbf{R}_T^{1/2} \tag{2}$$

where $\mathbf{R}_R = \mathbf{R}_R^{1/2^*}\mathbf{R}_R^{1/2}$ is the receive covariance matrix and $\mathbf{R}_T = \mathbf{R}_T^{1/2}\mathbf{R}_T^{1/2^*}$ is the transmit covariance matrix. $\mathbf{G}$ is a random matrix with the (p,q) entry of $\mathbf{G}$, $g_{p,q}$, distributed according to $\mathcal{CN}(0,1)$ with g_{p_1,q_1} independent of g_{p_2,q_2} for $p_1 \neq p_2$ or $q_1 \neq q_2$. This model was first proposed in [8] and experimentally verified in [9]. This model has also been proposed by the IEEE 802.11 task group N [10]. In this paper, *we will assume that both $\mathbf{R}_R$ and $\mathbf{R}_T$ are full rank.* Note that when $\mathbf{R}_T = \mathbf{I}_{M_t}$, with $\mathbf{I}_{M_t}$ denoting the $M_t \times M_t$ identity matrix, and $\mathbf{R}_R = \mathbf{I}_{M_r}$ the system model represents the uncorrelated MIMO Rayleigh fading system. Thus the analysis applied to this correlated model obviously applies to the uncorrelated case.

The channel will be modeled as a quasi-static fading channel. The vectors $\mathbf{z}$ and $\mathbf{w}$ will be chosen for each channel realization in order to minimize the instantaneous probability of error. For a fixed channel realization $\mathbf{H}$, the received signal will have an SNR given by

$$\overline{\gamma}_r = \frac{|\mathbf{z}^*\mathbf{H}\mathbf{w}|^2\mathcal{E}}{\|\mathbf{z}\|_2^2 N_0} = \frac{\mathbf{\Gamma}_r\mathcal{E}}{N_0} \tag{3}$$

with $\|\cdot\|_2$ denoting the 2-norm. We will call the term $\mathbf{\Gamma}_r$ in (3) the *effective channel gain* of the effective single-input single-output channel. Note that this SNR expression is instantaneous only in the sense of $\mathbf{H}$, averaging is still done over the noise and transmitted symbol. For any given symbol, the average transmitted energy conditioned on the beamformer is given by $\|\mathbf{w}\|_2^2\mathcal{E}$, so in order to make proper transmit power comparisons we will fix $\|\mathbf{w}\|_2 = 1$ throughout the paper. The expression in (3) is also unchanged if $\mathbf{z}$ is multiplied by any nonzero complex number. Therefore without loss of generality let $\|\mathbf{z}\|_2 = 1$. After beamforming and combining, this system can be analyzed as a single antenna system with fading channel $\mathbf{z}^*\mathbf{H}\mathbf{w}$, noise $\mathbf{z}^*\mathbf{n}$, and effective channel gain $\mathbf{\Gamma}_r = |\mathbf{z}^*\mathbf{H}\mathbf{w}|^2$.

We will assume that $\mathbf{z}$, $\mathbf{H}$, and $\mathbf{w}$ are all known perfectly at the receiver. The transmitter is only required to have knowledge of the chosen $\mathbf{w}$. Thus this analysis applies to systems where the transmitter has no channel information and $\mathbf{w}$ is designed at the receiver then conveyed to the transmitter over a limited feedback channel.

The instantaneous probability of symbol error for any modulation scheme can be written as the conditional probability of symbol error for an additive white Gaussian noise (AWGN) channel given the receive SNR, denoted as $P_s(Error \mid \overline{\gamma}_r)$. The AWGN probability of error function $P_s(Error \mid \overline{\gamma}_r)$ will be a decreasing function of $\overline{\gamma}_r$. The average probability of symbol error is thus given by taking the expectation with respect to $\mathbf{H}$ as

$$P_s(Error) = E_{\mathbf{H}}[P_s(Error \mid \overline{\gamma}_r)]. \tag{4}$$

To minimize the conditional probability of error, we assume that the receiver chooses the vectors $\mathbf{w}$ and $\mathbf{z}$ that maximize (3). The receiver then sends the optimal beamforming vector $\mathbf{w}$ to the transmitter. Note that in the case where the beamforming feasible set is finite, this vector can be conveyed using only a limited number of feedback bits.

Obtaining exact expressions for $P_s(Error)$ with arbitrary combining schemes and constellations is quite difficult. Despite this fact, it is necessary to verify the system is making use of the full M_tM_r independently fading channels that are available over the MIMO link. For this reason, it is of interest to bound the diversity order of the beamforming and combining scheme. A MIMO wireless system is said to have *diversity order D* and *array gain A* if the average probability of symbol error decreases inversely proportional to $A(\frac{\mathcal{E}}{N_0})^D$ as $\frac{\mathcal{E}}{N_0} \to \infty$ [12].

MRT/MRC and SD wireless systems have been proven for many cases to maintain full diversity order. MRT/MRC was addressed in [3], [4] for M-ary phase sift keying (M-PSK), M-ary quadrature amplitude modulation (M-QAM), and M-ary amplitude modulation (M-AM). SD systems were addressed for M-PSK [13] and M-QAM [14].

III. Necessary and Sufficient Conditions

We will now present necessary and sufficient conditions on the beamforming and combining feasible sets. For convenience, the effective channel gain $\mathbf{\Gamma}_r = |\mathbf{z}^*\mathbf{H}\mathbf{w}|^2$ will be used instead of $\overline{\gamma}_r$ to simplify notation.

Theorem 1: A wireless system employing beamforming and combining over memoryless, correlated Rayleigh fading channels provides full diversity order if and only if the vectors in the beamforming feasible set span $\mathbb{C}^{M_t}$ and the vectors in the combining feasible set span $\mathbb{C}^{M_r}$.

Proof: We will first prove the sufficiency of the conditions. Suppose that the beamforming and combining vectors span $\mathbb{C}^{M_t}$ and $\mathbb{C}^{M_r}$, respectively. Note that the diversity order will always be less than or equal to M_tM_r because there are only M_tM_r independently fading parameters. Let $\mathcal{W}$ denote the beamforming feasible set and $\mathcal{Z}$ denote the combining feasible set. We can construct an invertible matrix $\mathbf{B} = \mathbf{R}_T^{1/2}\mathbf{W} = \mathbf{R}_T^{1/2}[\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_{M_t}]$ where $\mathbf{w}_i \in \mathcal{W}$ for all i. We can similarly construct an invertible matrix $\mathbf{C} = \mathbf{R}_R^{1/2^*}\mathbf{Z} = \mathbf{R}_R^{1/2^*}[\mathbf{z}_1\ \mathbf{z}_2\ \cdots\ \mathbf{z}_{M_r}]$ where $\mathbf{z}_i \in \mathcal{Z}$ for all i. Since the matrices are invertible, we can define a singular value decomposition of each matrix

$$\mathbf{B} = \mathbf{V}_L\mathbf{\Lambda}\mathbf{V}_R^* \quad \text{and} \quad \mathbf{C} = \mathbf{U}_L\mathbf{\Phi}\mathbf{U}_R^* \tag{5}$$

where $\mathbf{V}_L$ and $\mathbf{V}_R$ are $M_t \times M_t$ unitary matrices, $\mathbf{\Lambda}$ is a diagonal matrix with diagonal entries $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{M_t} > 0$, $\mathbf{U}_L$ and $\mathbf{U}_R$ are $M_r \times M_r$ unitary matrices, and $\mathbf{\Phi}$ is a diagonal matrix with diagonal entries $\phi_1 \geq \phi_2 \geq \ldots \geq \phi_{M_r} > 0$.

For this system,

$$\begin{aligned}
\mathbf{\Gamma}_r &= \max_{\mathbf{w}\in\mathcal{W}} \max_{\mathbf{z}\in\mathcal{Z}} |\mathbf{z}^*\mathbf{H}\mathbf{w}|^2 \\
&\geq \max_{1\leq p\leq M_r} \max_{1\leq q\leq M_t} \left|\mathbf{z}_p^*\mathbf{H}\mathbf{w}_q\right|^2 \\
&= \max_{1\leq p\leq M_r} \max_{1\leq q\leq M_t} \left|(\mathbf{U}_R)_p^*\,\mathbf{\Phi}\mathbf{U}_L^*\mathbf{G}\mathbf{V}_L\mathbf{\Lambda}\,(\mathbf{V}_R)_q\right|^2 \\
&\stackrel{d}{=} \max_{1\leq p\leq M_r} \max_{1\leq q\leq M_t} \left|(\mathbf{U}_R^*\mathbf{\Phi}\mathbf{G}\mathbf{\Lambda}\mathbf{V}_R)_{(p,q)}\right|^2 \qquad (6)\\
&\geq \frac{1}{M_tM_r}\|\mathbf{\Phi}\mathbf{G}\mathbf{\Lambda}\|_2^2 \\
&\geq \frac{1}{M_tM_r}\phi_{M_r}^2\lambda_{M_t}^2 \max_{1\leq p\leq M_r} \max_{1\leq q\leq M_t} |g_{p,q}|^2 \qquad (7)
\end{aligned}$$

where $(\mathbf{A})_p$ represents the p^{th} column of a matrix $\mathbf{A}$, $(\mathbf{A})_{(p,q)}$ is the (p,q) entry of a matrix $\mathbf{A}$, and $\stackrel{d}{=}$ denotes equivalence in distribution. Eq. (6) used the invariance of complex normal matrices to unitary transformation [15], and (7) follows from the matrix norm bounds described in [16].

Noting that the maximum over the entries $|g_{p,q}|^2$ is the effective channel gain for SD systems, we thus have that

$$P_s(Error) \leq E_{\mathbf{G}}\left[P_s\left(Error \;\middle|\; \frac{1}{M_tM_r}\phi_{M_r}^2\lambda_{M_t}^2\overline{\gamma}_{r_{SD}}\right)\right] \tag{8}$$

with

$$\overline{\gamma}_{r_{SD}} = \max_{1\leq p\leq M_r} \max_{1\leq q\leq M_t} |g_{p,q}|^2 \frac{\mathcal{E}}{N_0}.$$

This is the probability of symbol error for an uncorrelated Rayleigh fading SD system with array gain shift of $\frac{1}{M_tM_r}\phi_{M_r}^2\lambda_{M_t}^2$ which provides a diversity of order M_tM_r. We have thus proven that the system achieves a diversity order of M_tM_r.

Now we prove necessity. Let the subspace spanned by the vectors in $\mathcal{W}$ after multiplication by $\mathbf{R}_T^{1/2}$, denoted by $\mathcal{S}_W$, be of dimension M_w and the subspace spanned by the vectors in $\mathcal{Z}$ after multiplication by $\mathbf{R}_R^{1/2^*}$, denoted by $\mathcal{S}_Z$, be of dimension M_z. Suppose that $M_wM_z < M_tM_r$. We can then find an $M_t \times M_w$ matrix $\mathbf{V}$ that spans $\mathcal{S}_W$ and an $M_r \times M_z$ matrix $\mathbf{U}$ that spans $\mathcal{S}_Z$ with $\mathbf{V}^*\mathbf{V} = \mathbf{I}_{M_w}$ and $\mathbf{U}^*\mathbf{U} = \mathbf{I}_{M_z}$. For both matrices, we can construct square unitary matrices $\overline{\mathbf{V}}$ and $\overline{\mathbf{U}}$ by concatenating $\mathrm{M}_t - M_w$ and $M_r - M_z$ orthonormal vectors to $\mathbf{V}$ and $\mathbf{U}$ respectively.

Therefore,

$$\begin{aligned}
\mathbf{\Gamma}_r &= \max_{\mathbf{w}\in\mathcal{W}} \max_{\mathbf{z}\in\mathcal{Z}} |\mathbf{z}^*\mathbf{H}\mathbf{w}|^2 \\
&\leq \max_{\mathbf{a}:\mathbf{a}^*\mathbf{a}=1} \max_{\mathbf{d}:\mathbf{d}^*\mathbf{d}=1} \beta\,|\mathbf{d}^*\mathbf{U}^*\mathbf{G}\mathbf{V}\mathbf{a}|^2 \\
&= \max_{\mathbf{a}:\mathbf{a}^*\mathbf{a}=1} \max_{\mathbf{d}:\mathbf{d}^*\mathbf{d}=1} \beta\left|\begin{pmatrix}\mathbf{d}\\ \mathbf{0}\end{pmatrix}^*\overline{\mathbf{U}}^*\mathbf{G}\overline{\mathbf{V}}\begin{pmatrix}\mathbf{a}\\ \widetilde{\mathbf{0}}\end{pmatrix}\right|^2 \\
&\stackrel{d}{=} \max_{\mathbf{a}:\mathbf{a}^*\mathbf{a}=1} \max_{\mathbf{d}:\mathbf{d}^*\mathbf{d}=1} \beta\left|\begin{pmatrix}\mathbf{d}\\ \mathbf{0}\end{pmatrix}^*\mathbf{G}\begin{pmatrix}\mathbf{a}\\ \widetilde{\mathbf{0}}\end{pmatrix}\right|^2 \\
&= \beta\left\|\mathbf{G}_{(1:M_z,1:M_w)}\right\|_2^2 \qquad (9)
\end{aligned}$$

where $\beta = \|\mathbf{R}_T\|_2\|\mathbf{R}_R\|_2$, $\mathbf{0}$ and $\widetilde{\mathbf{0}}$ are zero vectors, and $\mathbf{G}_{(1:M_z,1:M_w)}$ represents the matrix formed from the first M_z rows and M_w columns of $\mathbf{G}$. The expression in (9) is the effective channel gain for an $M_w \times M_z$ MRT/MRC system with an array gain shift. Therefore,

$$P_s(Error) \geq E_{\mathbf{G}}\left[P_s\left(Error \;\middle|\; \|\mathbf{R}_T\|_2\|\mathbf{R}_R\|_2\overline{\gamma}_{r_{MRC}}\right)\right] \tag{10}$$

with

$$\overline{\gamma}_{r_{MRC}} = \|\mathbf{G}_{(1:M_z,1:M_w)}\|_2^2\frac{\mathcal{E}}{N_0}.$$

Note that an $M_w \times M_z$ MRT/MRC system provides a diversity order of M_wM_z. Thus, the bounded system provides a diversity order less than or equal to M_wM_z and thus less than M_tM_r. Since this bound is true for arbitrary $\frac{\mathcal{E}}{N_0}$, we can conclude that the system does not achieve full diversity. ■

IV. SIMULATIONS

We present Monte Carlo simulations to verify our diversity proofs. The beamforming and combining schemes in each case were generated randomly.

Experiment #1: This experiment shows the importance of spanning $\mathbb{C}^{M_t}$. We simulated a 2×1 system using BPSK with two different beamforming feasible sets. The probability of symbol error results are shown in Fig. 1. The first used a single vector feasible set $\{\mathbf{w}_1\}$, and this system has been shown to have diversity order one. We also simulated a second order diversity system by using the feasible set $\{\mathbf{w}_1, \sqrt{0.99}\mathbf{w}_1 + \sqrt{0.01}\mathbf{w}_2\}$ where $\mathbf{w}_1^*\mathbf{w}_2 = 0$. As Fig. 1 illustrates, even adding a highly correlated second vector to the feasible set guarantees full diversity order. The probability of error performance of a two vector beamforming feasible set consisting of $\{\mathbf{w}_1, \sqrt{0.95}\mathbf{w}_1 + \sqrt{0.05}\mathbf{w}_2\}$ is also shown to demonstrate the array gain from decreasing the correlation of the vectors in the feasible set. This agrees with the analysis performed in [7].

Experiment #2: Fig. 2 demonstrates the probability of symbol error performance for various randomly generated beamforming and combining feasible sets on a 4×2 wireless

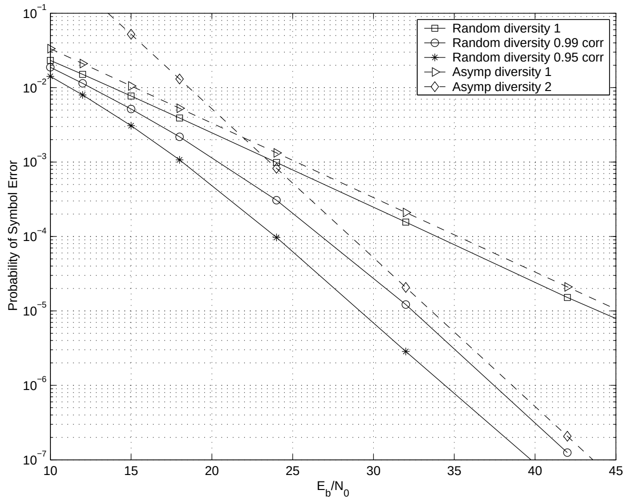

Fig. 1. Experiment showing the full diversity order of full rank beamformers on a 2×1 system.

system transmitting 16-QAM. The diversity order of a system using a beamforming feasible set and combining feasible set that spans $\mathbb{C}^4$ and $\mathbb{C}^2$, respectively, is shown to be eight. When the beamforming feasible set only spans a two-dimensional subspace the diversity order is seen to be four. Similarly when the beamformer feasible set spans a one-dimensional subspace and the combiner feasible set spans a one-dimensional subspace the diversity order is one. To demonstrate experimentally that the results work for correlated MIMO systems, randomly generated, full rank $\mathbf{R}_T$ and $\mathbf{R}_R$ were used with beamforming and combining sets that span $\mathbb{C}^4$ and $\mathbb{C}^2$, respectively. As evident from the plot, this system also provides eighth order diversity.

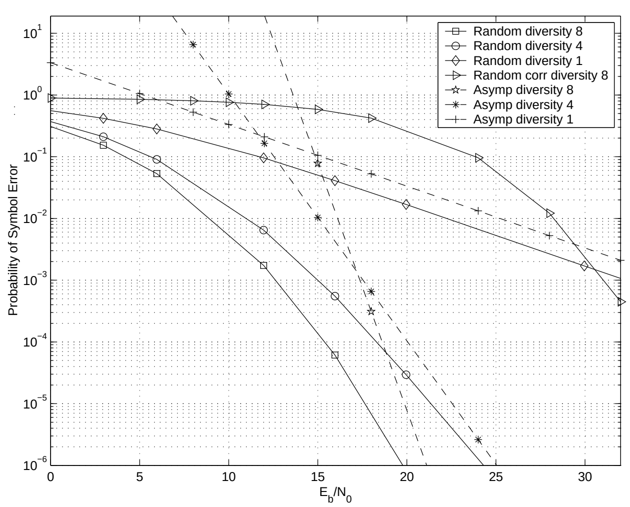

Fig. 2. Experiment showing various diversity orders obtained by generating random beamforming and combining sets of various ranks on a 4×2 system.

V. Conclusion

In this paper, we gave necessary and sufficient conditions on the feasible set of beamforming and combining systems in order to guarantee full diversity order when signaling over memoryless, correlated Rayleigh fading channels. These conditions allow the diversity order of beamforming and combining systems, which often are difficult to analyze analytically especially in the limited feedback case, to be easily verified without simulation. The proofs in this paper work for any modulation scheme with provable full diversity order in MRT/MRC and SD systems that transmit over memoryless and uncorrelated Rayleigh fading channels. Future work in this area could involve the analysis of the effect that the beamforming and combining feasible sets play on array gain. This has been looked at in detail for the uncorrelated case with MISO [17] and MIMO systems [7]. Another interesting area of future research is the diversity order of precoded space-time block codes. In [18], we analyze necessary and sufficient conditions for full diversity order in precoded orthogonal space-time block coded systems.

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