Equal Gain Transmission in Multiple-Input Multiple-Output Wireless Systems

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***Abstract*—Multiple-input multiple-output (MIMO) wireless systems are of interest due to their ability to provide substantial gains in capacity and quality. This paper proposes equal gain transmission (EGT) to provide diversity advantage in MIMO systems experiencing Rayleigh fading. The applications of EGT with selection diversity combining, equal gain combining, and maximum ratio combining are addressed. It is proven that systems using EGT with any of these combining schemes achieve full diversity order when transmitting over a memoryless, flat-fading Rayleigh matrix channel with independent entries. Since, in practice, full channel knowledge at the transmitter is difficult to realize, a quantized version of EGT is proposed. An algorithm to construct a beamforming vector codebook that guarantees full diversity order is presented. Monte–Carlo simulation comparisons with various beamforming and combining systems illustrate the performance as a function of quantization.**

***Index Terms*—Diversity methods, equal gain transmission (EGT), multiple-input multiple-output (MIMO) systems, Rayleigh channels.**

I. Introduction

ANTENNA diversity has been shown to improve mean signal strength and reduce signal-level fluctuations in fading channels [1]. These benefits are a direct result of the fact that sufficiently spaced antennas encounter approximately independent fading channels. Antenna diversity can be utilized at the transmitter and/or receiver. Receive antenna diversity systems intelligently combine the multiple received copies to provide a higher average receive signal-to-noise ratio (SNR) (see [2]–[4], and the references therein). Transmit antenna diversity is more difficult to obtain, since it requires either channel-dependent beamforming or channel-independent space–time coding [5], [6].

Classical wireless research focused on the case where antenna diversity was employed exclusively at either the transmitter or receiver. When multiple antennas are only available at the transmitter, beamforming techniques such as selection diversity transmission (SDT), equal gain transmission (EGT), and maximum ratio transmission (MRT) have been used to exploit the diversity available from the multiple-input single-output (MISO) wireless channel. On the other hand, for systems with multiple antennas only at the receiver, combining schemes such as selection diversity combining (SDC), equal gain combining (EGC), and maximum ratio combining (MRC) have been used to obtain diversity advantage from the corresponding single-input multiple-output (SIMO) wireless channel.

When antenna diversity is employed at both the transmitter and receiver, the multiple-input multiple-output (MIMO) channel encountered in the memoryless case is a matrix. Beamforming and combining can be used in MIMO communication channels, however, the beamforming vector and receive combining vector must now be jointly designed to maximize the receive SNR. MIMO MRT and MRC were addressed in [7] and shown to provide full diversity order. Systems using SDT and MRC were studied in [8], and also shown to provide full diversity order. Designing MIMO beamforming and combining vectors is nontrivial, and in many cases, involves an optimization problem that can not be easily solved in real-time systems.

EGT has more modest transmit amplifier requirements than MRT, since it does not require the antenna amplifiers to modify the amplitudes of the transmitted signals. This property allows inexpensive amplifiers to be used at each antenna as long as the gains are carefully matched. For example, SIMO EGC and MISO EGT have already been considered as low-complexity alternatives to MRC and MRT, respectively (see [2], [9]–[12] and the references therein). Despite the importance of MIMO communication systems, the application of EGT to these systems has not yet been addressed.

In this paper we propose EGT, combined with either SDC, EGC, or MRC at the receiver, to provide full diversity order in MIMO wireless systems transmitting over memoryless, independent and identically distributed (i.i.d.) Rayleigh fading channels. We jointly solve for the optimal beamforming and combining vectors by maximizing the received SNR. For the cases considered, it is possible to find the optimum combining vector as a function of the beamforming vector; finding the optimum beamforming vector usually requires a nonlinear optimization. We prove that any beamforming and combining system whose set of possible beamforming vectors contains a subset of M_t orthogonal vectors and whose set of possible receive combining vectors contains a subset of M_r orthogonal vectors, where M_t and M_r are the number of transmit and receive antennas, respectively, provides full diversity order. We use this proof to show that MIMO systems using EGT combined with SDC, EGC, or MRC obtain full diversity order.

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One problem encountered during implementation of MISO and MIMO beamforming systems is that full channel knowledge is required at the transmitter to design optimal beamforming vectors. In many systems, such as those using frequency-division duplexing, it is impossible to obtain complete channel information at the transmitter. One solution to this problem is to let the receiver design the beamforming vector and then send the vector to the transmitter [13], [14]. Since infinite resolution is impossible, it is preferable to quantize the set of possible beamforming vectors into a codebook and then send only the codebook entry of the desired beamforming vector. Quantized MRT for MISO systems was addressed in [13], while MISO quantized equal gain transmission (QEGT) was discussed in [14]. QEGT has also been chosen as one of the closed-loop beamforming techniques in wideband code-division multiple access (W-CDMA) [15]. Due to the difficulty of finding the optimal beamforming vector in beamforming and combining systems, MIMO quantized beamforming represents a much more difficult problem than in MISO systems [23].

Since full channel knowledge is often not available at the transmitter, we propose and study MIMO QEGT. We develop an algorithm for QEGT codebook construction that guarantees full diversity order for memoryless, i.i.d. Rayleigh fading channels, given that a sufficient number of bits are allocated for feedback. This minimum number of bits depends directly on the number of transmit antennas. We show that when the number of bits allocated for feedback is equal to $\log_2 M_t$, the beamforming scheme performs, on average, identically to SDT. An important side benefit of QEGT is that the optimal beamforming vector can be found through a low-complexity brute-force search, as opposed to a nonlinear optimization.

This paper is organized as follows. Section II reviews MIMO communication with beamforming and combining. Basic performance properties of MIMO beamforming and combining systems are presented in Section III. We discuss EGT systems with SDC, EGC, and MRC in Section IV. We propose MIMO QEGT and provide a full-diversity codebook design method in Section V. In Section VI, we show simulation results that verify the performance analysis of EGT and QEGT systems. We provide some conclusions in Section VII.

II. System Overview

A MIMO system using beamforming and combining is illustrated in Fig. 1 with M_t transmit antennas and M_r receive antennas. A symbol s ($s \in \mathbb{C}$, the field of complex numbers) is multiplied by weight w_l $(w_l \in \mathbb{C})$ at the lth $(1 \le l \le M_t)$ transmit antenna. The signal y_k received by the kth $(1 \le k \le M_r)$ receive antenna is given by

$$y_k = \left(\sum_{l=1}^{M_t} h_{k,l} w_l\right) s + n_k \tag{1}$$

where $h_{k,l}$ is a memoryless fading channel that is constant over several channel uses and distributed according to $\mathcal{CN}(0,1)$, and n_k is a noise term distributed according to $\mathcal{CN}(0,N_0)$. We assume that $h_{k,l}$ is independent of $h_{i,j}$ if $k \neq i$ or $l \neq j$, and n_k is independent of n_i if $k \neq i$. Note that time dependence has been

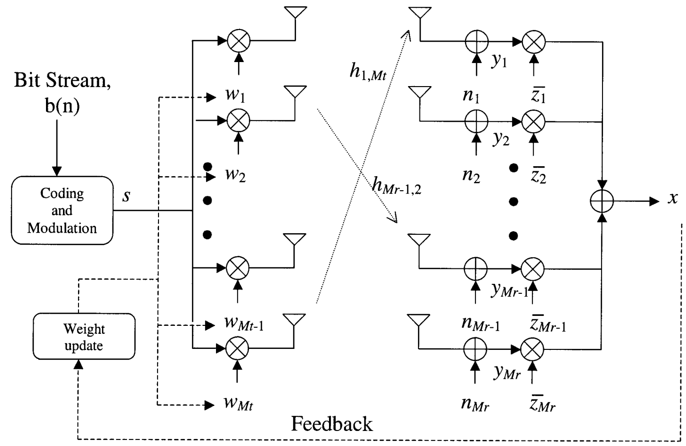

Fig. 1. Block diagram of a MIMO beamforming and combining system.

abstracted from the discussions by assuming that the channel is constant over several transmissions. The data received by the kth receive antenna, y_k, is multiplied by $\overline{z_k}$ ($z_k \in \mathbb{C}$ with $\bar{\cdot}$ denoting conjugation). The weighted output of each of the M_r receive antennas is then combined to produce x. This formulation allows the equivalent system to be written in matrix form as

$$x = (\mathbf{z}^H \mathbf{H} \mathbf{w})s + \mathbf{z}^H \mathbf{n} \tag{2}$$

with $\mathbf{w} = [w_1 \quad \ldots \quad w_{M_t}]^T$, $\mathbf{z} = [z_1 \quad \ldots \quad z_{M_r}]^T$, $\mathbf{n} = [n_1 \quad \ldots \quad n_{M_r}]^T$, and $\mathbf{H}$ denoting the $M_r \times M_t$ matrix with coordinate (k,l) equal to $h_{k,l}$ where ^{T} denotes transposition and ^{H} denotes conjugate transposition. We call $\mathbf{z}^H\mathbf{H}\mathbf{w}$ the effective channel. For optimum performance, $\mathbf{w}$ and $\mathbf{z}$ should be chosen as a function of the channel to minimize the probability of error.

The nearest neighbor union bound on the symbol-detection error probability can be stated [16] as

$$P_e \le N_e Q\left(\sqrt{\frac{d_{\min}^2 \gamma_r}{2}}\right) \tag{3}$$

where N_e is a real constant that is the average number of nearest neighbors per symbol, $d_{\min}$ is the minimum distance of the transmit constellation normalized to unit energy, γ_r is the receive SNR, and Q is the Gaussian Q-function. Note that N_e can be adjusted in order to provide a close approximation to the actual probability of error [16]. Since the Q-function is a monotonically decreasing function and $d_{\min}$ is assumed fixed, minimizing the bound requires that we maximize the SNR. It follows from (2) that

$$\gamma_r = \frac{\mathcal{E}_t|\mathbf{z}^H\mathbf{H}\mathbf{w}|^2}{\|\mathbf{z}\|_2^2 N_0} = \frac{\left(\mathcal{E}_t\|\mathbf{w}\|_2^2\right)\left|\frac{\mathbf{z}^H}{\|\mathbf{z}\|_2}\mathbf{H}\frac{\mathbf{w}}{\|\mathbf{w}\|_2}\right|^2}{N_0} \tag{4}$$

where $\|\cdot\|_2$ is the matrix two-norm, $|\cdot|$ is the absolute value, and $\mathcal{E}_t$ is the transmitted symbol's energy. Notice that (4) does not vary with $\|\mathbf{z}\|_2$, therefore, without loss of generality we can fix $\|\mathbf{z}\|_2 = 1$. We also can see that the transmitter transmits with total energy $\mathcal{E}_t\|\mathbf{w}\|_2^2$. Therefore, due to power constraints at the transmitter, we can take $\|\mathbf{w}\|_2 = 1$. With these assumptions, the instantaneous receive SNR, γ_r, can be expressed as

$$\gamma_r = \frac{\mathcal{E}_t \mathbf{\Gamma}_r}{N_0} \tag{5}$$

where $\mathbf{\Gamma}_r = |\mathbf{z}^H \mathbf{H} \mathbf{w}|^2$ is the effective channel gain.

Maximizing $\mathbf{\Gamma}_r$ is a multidimensional optimization problem. We will, therefore, employ standard linear programming terminology in dealing with the maximization. Recall that the set over which a cost function is optimized is called the *feasible set* [17]. We will denote the set of all possible beamforming vectors as the beamforming feasible set and the set of all possible combining vectors as the combining feasible set.

The beamforming feasible set defines the set over which the beamforming vector is chosen. When $\mathbf{w}$ can be any unit vector, the beamforming scheme is called MRT. A beamforming scheme where each transmit antenna l has weight w_l with $|w_l| = 1/\sqrt{M_t}$ is denoted by EGT. If $\mathbf{w}$ is constrained to be a column of $\mathbf{I}_{M_t}$, the $M_t \times M_t$ identity matrix, the beamforming scheme is called SDT.

In MIMO systems, the combining vectors need to be chosen in addition to the beamforming vectors, perhaps under different constraints. A receiver where $\mathbf{z}$ can be any unit vector is using MRC. An EGC receiver constrains each receive antenna weight z_k to have $|z_k| = 1/\sqrt{M_r}$. A receiver where $\mathbf{z}$ is a column of $\mathbf{I}_{M_r}$ is using SDC.

In this paper, EGT and EGC are considered. The definition of EGT allows us to express $\mathbf{w}$ as $\mathbf{w} = (1/\sqrt{M_t})e^{j\boldsymbol{\theta}} = (1/\sqrt{M_t}) \left[e^{j\theta_1} \quad e^{j\theta_2} \quad \ldots \quad e^{j\theta_{M_t}} \right]^T$, where $\boldsymbol{\theta} = [\theta_1 \quad \theta_2 \quad \ldots \quad \theta_{M_t}]^T$ and $\theta_k \in [0, 2\pi)$. As well, EGC vectors can be expressed as $\mathbf{z} = (1/\sqrt{M_r})e^{j\boldsymbol{\phi}}$, where $\boldsymbol{\phi} = [\phi_1 \quad \phi_2 \quad \ldots \quad \phi_{M_r}]^T$ and $\phi_i \in [0, 2\pi)$.

It is important to note that uniqueness is not guaranteed for any beamforming and combining scheme. Multiplication of the beamforming vector $\mathbf{w}$ by $e^{j\xi}$ and the combining vector $\mathbf{z}$ by $e^{j\varphi}$ with $\xi, \varphi \in [0, 2\pi)$ does not change $\mathbf{\Gamma}_r$. For this reason, when optimizing a cost function, we will define $\arg\max$ to return the set of global maximizers. We later exploit this nonuniqueness to reduce the size of the solution set and thus, the amount of feedback in the QEGT system.

These transmission and combining methods can be intermixed together to suit different system requirements. If beamforming method A is used at the transmitter and combining method B is used at the receiver, we will call this an A/B system.

III. System Performance

Given no design constraints on the form of $\mathbf{w}$ or $\mathbf{z}$, the nearest-neighbor union bound tells us that the optimal solutions are the beamforming vector and combining vector that maximize γ_r. Since we assume that $\mathcal{E}_t$ and N_0 are fixed, this simplifies to maximizing the effective channel gain $\mathbf{\Gamma}_r$. *Lemma 1* gives a clear upper bound on $\mathbf{\Gamma}_r$.

Lemma 1: The SNR γ_r is maximized when $\mathbf{z}$ and $\mathbf{w}$ are the left and right singular vectors of $\mathbf{H}$, respectively, corresponding to the largest singular value of $\mathbf{H}$ with $\mathbf{\Gamma}_r = \|\mathbf{H}\|_2^2$.

The proof of *Lemma 1* is given in [18]. Since MRT and MRC pose no restrictions other than unit two-norm on the vectors $\mathbf{w}$ and $\mathbf{z}$, respectively, we can, therefore, conclude that for any channel matrix $\mathbf{H}$, the effective channel gain of a useful MRT/MRC system is $\|\mathbf{H}\|_2^2$. *Lemma 1* gives us an upper bound on $\mathbf{\Gamma}_r$ for EGT systems.

It is often difficult to compute meaningful, closed-form expressions for the average probability of symbol error (average taken with respect to the channel [4]) even for much simpler SIMO EGC systems [4], [10]–[12], [19]. We will, therefore, instead use the metrics of diversity order and array gain [1]. A system is said to have array gain A and diversity order D if the average probability of symbol error is inversely proportional to $A(\mathcal{E}_t/N_0)^D$ for $\mathcal{E}_t/N_0 \gg 0$.

Lemma 2: Let $\mathbf{\Gamma}_{r_1}, \mathbf{\Gamma}_{r_2}$ be the effective channel gains and D_1, D_2 be the diversity orders for two different MIMO beamforming and combining systems. If $\mathbf{\Gamma}_{r_1} \geq \mathbf{\Gamma}_{r_2}$ for all $\mathbf{H}$, then $D_1 \geq D_2$.

Proof: The nearest-neighbor upper bound tells us that for large $\mathcal{E}_t/N_0$, the probability of symbol error is a decreasing function of the effective channel gain. Therefore, if $\mathbf{\Gamma}_{r_1} \geq \mathbf{\Gamma}_{r_2}$ for any channel $\mathbf{H}$, then the average probability of symbol error for Scheme 1 will always be less than the average probability of symbol error for Scheme 2. Thus, there does not exist a $\mathcal{E}_t/N_0 \gg 0$ such that the average probabilities of symbol error are equivalent. We can, therefore, conclude that $D_1 \geq D_2$. ■

An important corollary that we will use later in upper bounding the diversity order of MIMO EGT systems follows from this lemma.

Corollary 1: For any $M_t \times M_r$ wireless systems using beamforming and combining, the diversity order is always less than or equal to $M_r M_t$ when transmitting over a memoryless, i.i.d. Rayleigh fading matrix channel.

Proof: By *Lemma 1*, the effective channel gain of MRT/MRC systems, which are known to have diversity order $M_r M_t$ (see [7] and [20]), will be greater than or equal to the effective channel gain of any other $M_t \times M_r$ beamforming and combining system. Therefore, by *Lemma 2*, for any $M_t \times M_r$ wireless system using beamforming and combining, the diversity order is always less than or equal to $M_r M_t$ when transmitting over a memoryless, i.i.d. Rayleigh fading matrix channel. ■

In our diversity advantage proofs we will also lower bound the diversity order. The following lemma provides an important result in the theory of beamforming and combining wireless systems transmitting over i.i.d. Rayleigh fading MIMO channels.

Lemma 3: If the beamforming feasible set and combining feasible set of an $M_t \times M_r$ beamforming and combining system contain M_t and M_r orthogonal vectors, respectively, then the system has a diversity order of $M_r M_t$ when transmitting over memoryless, i.i.d. MIMO Rayleigh fading channels.

Proof: Let A/B denote a beamforming and combining method satisfying the orthogonality conditions. *Corollary 1* tells us that the diversity order is upper bounded by $M_r M_t$. Let $\mathbf{U}_t$ be an $M_t \times M_t$ matrix whose columns are the M_t orthogonal beamforming vectors, and $\mathbf{U}_r$ be an $M_r \times M_r$ matrix whose columns are the M_r orthogonal combining vectors. Let $\mathbf{\Gamma}_{r_{\text{orth}}}$ be the effective channel gain for a beamforming and combining system that uses only the columns of $\mathbf{U}_t$ as beamforming vectors and the columns of $\mathbf{U}_r$ as combining vectors. The orthogonality of the columns and the unit two-norm requirement allow us to write that $\mathbf{U}_t^H\mathbf{U}_t = \mathbf{I}_{M_t}$ and $\mathbf{U}_r^H\mathbf{U}_r = \mathbf{I}_{M_r}$, meaning that $\mathbf{U}_t$ and $\mathbf{U}_r$ are both unitary.

Let $\mathbf{\Gamma}_{r_{\mathrm{orig}}}$ denote the effective channel gain of the original system. Since the columns of $\mathbf{U}_t$ are contained in the beamforming feasible set of the original system, and the columns of $\mathbf{U}_r$ are contained in the combining feasible set of the original system, we can conclude that $\mathbf{\Gamma}_{r_{\mathrm{orig}}} \geq \mathbf{\Gamma}_{r_{\mathrm{orth}}}$ for any channel realization $\mathbf{H}$. Therefore, the diversity order of the original system is greater than or equal to the diversity order of the restricted orthogonal system.

For any channel realization $\mathbf{H}$, we have that

$$\begin{aligned}\mathbf{\Gamma}_{r_{\mathrm{orth}}} &= \max_{1\leq m\leq M_r}\max_{1\leq n\leq M_t}\left|(\mathbf{U}_r)_m^H\mathbf{H}(\mathbf{U}_t)_n\right|^2\\ &= \max_{1\leq m\leq M_r}\max_{1\leq n\leq M_t}\left|\left(\mathbf{U}_r^H\mathbf{H}\mathbf{U}_t\right)_{m,n}\right|^2 \end{aligned}\tag{6}$$

with $(\mathbf{U}_t)_n$ denoting the nth column of $\mathbf{U}_t$, and $\left(\mathbf{U}_r^H\mathbf{H}\mathbf{U}_t\right)_{m,n}$ denoting the (m,n) entry of $\mathbf{U}_r^H\mathbf{H}\mathbf{U}_t$. We assumed that $\mathbf{H}$ was a complex normally distributed random matrix whose entries were all independent. By the invariance of complex normal random matrices to unitary transformation [21], $\mathbf{H}$ is equivalent in distribution to $\mathbf{U}_r^H\mathbf{H}\mathbf{U}_t$. Therefore

$$\mathbf{\Gamma}_{r_{\mathrm{orth}}} \stackrel{d}{=} \max_{1\leq m\leq M_r}\max_{1\leq n\leq M_t}|h_{m,n}|^2 \tag{7}$$

with $\stackrel{d}{=}$ denoting equivalence in distribution.

The distribution equivalent system defined in (7) is the one that chooses the pair of antennas with the largest gain channel. This is a selection diversity transmission and combining system. These systems are known to provide a diversity order of M_rM_t [3], [22].

We have now upper and lower bounded the diversity order of the A/B system by M_rM_t. We can conclude that any system using a beamforming feasible set and combining feasible set with M_t and M_r orthogonal vectors, respectively, has a diversity of order M_rM_t. ∎

IV. Equal Gain Transmission (EGT)

In this section, we will consider EGT in conjunction with SDC, EGC, and MRC. We will address the design of the beamforming vectors and the diversity performance for each of the combining schemes.

A. EGT/SDC

It is often convenient to employ SDC at the receiver because of its low-complexity implementation. A multiantenna receiver using SDC requires only a switch that can choose between M_r different antenna outputs and a single radio chain. SDC is also the only combining scheme where a general expression for the optimal EGT vector can be derived.

As discussed in Section II, we wish to choose $\mathbf{w}$ and $\mathbf{z}$ in order to maximize $\mathbf{\Gamma}_r = |\mathbf{z}^H\mathbf{H}\mathbf{w}|^2$. When SDC is employed at the receiver, $\mathbf{z}$ is one of the columns of $\mathbf{I}_{M_r}$. Therefore

$$\mathbf{\Gamma}_r = \max_{1\leq m\leq M_r}|(\mathbf{H}\mathbf{w})_m|^2 = \max_{1\leq m\leq M_r}\left|\sum_{n=1}^{M_t}h_{m,n}w_n\right|^2 \tag{8}$$

where $(\mathbf{H}\mathbf{w})_m$ is the mth entry of the vector $\mathbf{H}\mathbf{w}$.

Substituting the expression $\mathbf{w} = (1/\sqrt{M_t})e^{j\boldsymbol{\theta}}$ into (8), we find that

$$\mathbf{\Gamma}_r = \max_{1\leq m\leq M_r}\frac{1}{M_t}\left|\sum_{n=1}^{M_t}h_{m,n}e^{j\theta_n}\right|^2. \tag{9}$$

Notice that $\mathbf{\Gamma}_r$ is bounded by

$$\begin{aligned}\mathbf{\Gamma}_r &\leq \max_{1\leq m\leq M_r}\frac{1}{M_t}\left(\sum_{n=1}^{M_t}\left|h_{m,n}e^{j\theta_n}\right|\right)^2\\ &= \max_{1\leq m\leq M_r}\frac{1}{M_t}\left(\sum_{n=1}^{M_t}|h_{m,n}|\right)^2\\ &= \frac{1}{M_t}\|\mathbf{H}\|_\infty^2 \end{aligned}\tag{10}$$

where $\|\cdot\|_\infty$ is the matrix sup-norm.

The sup-norm can be rewritten in terms of the rows as $\|\mathbf{H}\|_\infty = \max_{1\leq m\leq M_r}\|\left(\mathbf{H}^T\right)_m\|_1$, where $\|\cdot\|_1$ is the one-norm and $\left(\mathbf{H}^T\right)_m$ is the transpose of the mth row of $\mathbf{H}$. Therefore, the bound in (10) is achievable by letting $\boldsymbol{\theta} = \xi - phase\left(\left(\mathbf{H}^T\right)_K\right)$, where $K \in \arg\max_{1\leq m\leq M_r}\left\|\left(\mathbf{H}^T\right)_m\right\|_1$, $\xi \in [0,2\pi)$, and the function $phase\colon \mathbb{C}^{M_t} \rightarrow [0,2\pi)^{M_t}$ returns the phase of each entry of a vector.

We now have an expression for the optimal EGT vector when SDC is employed. In this case, with an arbitrary ξ

$$\begin{aligned}&\mathbf{w} = \frac{1}{\sqrt{M_t}}e^{j\left(\xi - phase\left(\left(\mathbf{H}^T\right)_K\right)\right)} \text{ with}\\ &K \in \arg\max_{1\leq m\leq M_r}\left\|\left(\mathbf{H}^T\right)_m\right\|_1. \end{aligned}\tag{11}$$

With this beamformer, the receive SNR γ_r is

$$\gamma_r = \frac{\mathcal{E}_t\mathbf{\Gamma}_r}{N_0} = \frac{\mathcal{E}_t\|\mathbf{H}\|_\infty^2}{M_tN_0}. \tag{12}$$

Using *Lemma 3*, we can also comment on diversity order.

Theorem 1: The diversity order of a MIMO system using EGT and SDC is M_rM_t when transmitting over memoryless, i.i.d. MIMO Rayleigh fading channels.

Proof: Let $\mathbf{U}$ be the $M_t \times M_t$ point discrete Fourier transform (DFT) matrix where entry (k,l) of $\mathbf{U}$ is given by $(1/\sqrt{M_t})e^{j2\pi kl/M_t}$. By our definition, $\mathbf{U}\mathbf{U}^H = \mathbf{I}_{M_t}$ so $\mathbf{U}$ is unitary. The columns of $\mathbf{U}$ are all acceptable EGT vectors, so the beamforming feasible set contains M_t orthogonal vectors. The receive combiner uses the columns of $\mathbf{I}_{M_r}$ as a feasible set, thus, it contains M_r orthogonal vectors by definition. By *Lemma 3*, the EGT/SDC system has a diversity order of M_rM_t. ∎

B. EGT/EGC

While SDC is easily implemented, EGC receivers have been shown to improve the average probability of symbol-error performance [4]. EGCs require only moderate hardware complexity, because each of the receive antennas weights is restricted to have magnitude $1/\sqrt{M_r}$.

The gain of the effective channel for an EGT/EGC system can be bounded by

$$\begin{aligned} \left|\mathbf{z}^H\mathbf{H}\mathbf{w}\right|^2 =& \frac{1}{M_r}\left|\sum_{m=1}^{M_r} e^{-j\phi_m}(\mathbf{H}\mathbf{w})_m\right|^2 \\ \leq& \frac{1}{M_r}\left(\sum_{m=1}^{M_r}|(\mathbf{H}\mathbf{w})_m|\right)^2 \\ =& \frac{1}{M_r}\|\mathbf{H}\mathbf{w}\|_1^2 \end{aligned} \tag{13}$$

where the inequality follows from the equal-gain properties of $\mathbf{z}$. The bound in (13) is achievable when $\mathbf{z} = (1/\sqrt{M_r})e^{j(\varphi+\text{phase}(\mathbf{H}\mathbf{w}))}$, where φ is an arbitrary phase angle.

Using the optimal EGC vector, $\mathbf{\Gamma}_r = (1/M_r)\|\mathbf{H}\mathbf{w}\|_1^2$. This can be rewritten for EGT as $\mathbf{\Gamma}_r = (1/(M_rM_t))\left\|\mathbf{H}e^{j\boldsymbol{\theta}}\right\|_1^2$. Therefore, the optimal phase vector $\boldsymbol{\theta}$ is given by

$$\boldsymbol{\theta} \in \arg\max_{\boldsymbol{\vartheta}\in[0,2\pi)^{M_t}} \left\|\mathbf{H}e^{j\boldsymbol{\vartheta}}\right\|_1. \tag{14}$$

The optimization problem defined by (14) has no known simple, closed-form solution. Again, note that the solution defined by (14) also does not have a unique solution. In fact, if $(1/\sqrt{M_t})e^{j\boldsymbol{\theta}}$ is an optimal EGT vector, then $(1/\sqrt{M_t})e^{j\xi}e^{j\boldsymbol{\theta}}$ is also optimal for any $\xi \in [0, 2\pi)$ because $\left\|\mathbf{H}e^{j\boldsymbol{\theta}}\right\|_1^2 = \left\|\mathbf{H}e^{j\xi}e^{j\boldsymbol{\theta}}\right\|_1^2$.

Theorem 2: A MIMO system using EGT and EGC achieves a diversity order of M_rM_t when transmitting over memoryless, i.i.d. MIMO Rayleigh fading channels.

Proof: We have shown in the proof of *Theorem 1* that the EGT feasible set contains a set of M_t orthogonal vectors. Similarly, let $\mathbf{V}$ be the $M_r \times M_r$ unitary DFT matrix. Each column of $\mathbf{V}$ is a possible EGC vector. Thus, the combining feasible set contains a set of M_r orthogonal vectors. *Lemma 3* tells us that an EGT/EGC system has a diversity order of M_rM_t. ■

C. EGT/MRC

MRC provides the best performance among all combining schemes, thanks to the absence of constraints placed on the set of possible combining vectors. The combining vector is designed specifically to maximize the effective channel gain $|\mathbf{z}^H\mathbf{H}\mathbf{w}|^2$.

For EGT/MRC systems, the effective channel gain can be upper bounded by

$$|\mathbf{z}^H\mathbf{H}\mathbf{w}|^2 \leq \|\mathbf{z}\|_2^2\|\mathbf{H}\mathbf{w}\|_2^2 = \|\mathbf{H}\mathbf{w}\|_2^2. \tag{15}$$

The upper bound in (15) is achievable if $\mathbf{z} = \mathbf{H}\mathbf{w}/\|\mathbf{H}\mathbf{w}\|_2$. Thus, the optimum phase vector $\boldsymbol{\theta}$ solves

$$\boldsymbol{\theta} \in \arg\max_{\boldsymbol{\vartheta}\in[0,2\pi)^{M_t}} \left\|\mathbf{H}e^{j\boldsymbol{\vartheta}}\right\|_2. \tag{16}$$

Once again, the phase vector $\boldsymbol{\theta}$ is not unique, because $\mathbf{w}$ can be arbitrarily multiplied by any unit gain of the form $e^{j\xi}$ with $\xi \in [0, 2\pi)$.

Theorem 3: A MIMO system using EGT and MRC has a diversity on the order of M_rM_t when transmitting over memoryless, i.i.d. MIMO Rayleigh fading channels.

Proof: We have already shown in the proof of *Theorem 1* that the EGT feasible set contains M_t orthogonal vectors. Note that each column of $\mathbf{I}_{M_r}$ is a possible MRC vector. Therefore, the MRC feasible set contains M_r orthogonal vectors. By *Lemma 3*, an EGT/MRC system has a diversity of order M_rM_t. ■

V. Quantized Equal Gain Transmission (QEGT)

In real-world systems, EGT is not an implementable solution for two main reasons, complexity and overhead. First, note that the optimization problems in (14) and (16) do not have closed-form solutions for arbitrary M_r and M_t. Implementation requires an iterative method, costing precious clock cycles. Convergence of such an iterative method to the global maximum is not guaranteed. Second, due to a limited feedback channel in most systems, it is impossible to send back high-precision phase angles. Wireless systems must always limit control data overhead in order to achieve large user data rates. If high-resolution phase angles were sent to the transmitter, this control overhead would overwhelm the limited feedback capacity.

One solution is to quantize the set of possible $\boldsymbol{\theta}$, creating a system called QEGT. This quantizes the space of beamforming vectors and eliminates the problem of finding the global maximum by using a brute-force search. As we show, the quantization can be quite low, reducing feedback requirements, without much performance sacrifice.

Suppose that B bits of quantization are used for each phase. Complete phase vector quantization would require BM_t bits of overhead. However, from (11), (14), and (16), a term of $e^{j\theta_1}$ can always be factored out without loss of generality. Thus, $\boldsymbol{\theta}$ can be written in the form $\boldsymbol{\theta} = [0\ (\theta_2 - \theta_1)\ \ldots\ (\theta_{M_t} - \theta_1)]^T$. Therefore, if B bits are used for each phase angle within $\boldsymbol{\theta}$, then only $B(M_t - 1)$ bits are necessary to define $M_t - 1$ antenna gains. Thus, by taking into account the nonuniqueness of the beamforming vector, we have reduced the amount of feedback.

Let $\mathcal{W}$ be the codebook, or set, of all possible QEGT vectors. For B bits of quantization, $\text{card}(\mathcal{W}) = 2^{B(M_t-1)}$, with $\text{card}(\cdot)$ denoting cardinality. A brute-force search through the possible vectors can be used to solve either (11), (14), or (16). We must now turn our attention to the design of the vectors within $\mathcal{W}$.

A quantization scheme that does not maintain full diversity order is wasting valuable resources by not making use of the full M_rM_t independently fading channels arising from the multiantenna system. Therefore, when using QEGT, it is imperative to maintain full diversity order for small B. To proceed with the codebook design, note that the proofs of *Theorems 1, 2,* and *3* employ the $M_t \times M_t$ unitary DFT matrix. If our codebook always contains the columns of the $M_t \times M_t$ DFT matrix, then we are guaranteed by *Lemma 3* to have full diversity order for SDC, EGC, and MRC. Therefore, if $2^{B(M_t-1)} \geq M_t$ and $\mathbf{U}$ denotes the $M_t \times M_t$ unitary DFT matrix, we will require that for all $\mathbf{U}_i$, there exists $\mathbf{w} \in \mathcal{W}$ such that $\mathbf{w} = \mathbf{U}_i$.

By construction, the beamforming codebook *will always* contain the columns of the $M_t \times M_t$ unitary DFT matrix when

$B \geq \log_2(M_t)/(M_t - 1)$. We can conclude from *Lemma 3* that quantized systems such as QEGT/SDC, QEGT/EGC, and QEGT/MRC obtain a diversity advantage of order $M_r M_t$ if $B \geq (\log_2 M_t)/(M_t - 1)$.

The proposed algorithm uses a set $\mathcal{W}_1$ made up of the column vectors of the $RM_t \times RM_t$ DFT matrix truncated to M_t rows and scaled by $\sqrt{R}$, where R is an integer that satisfies $R \geq (2^{B(M_t-1)})/M_t$. Since M_t divides RM_t, the set $\mathcal{W}_1$ will contain the set of column vectors $\mathcal{W}_2$ of the $M_t \times M_t$ unitary DFT matrix. The codebook then chooses $\left(2^{B(M_t-1)} - M_t\right)$ vectors from $\mathcal{W}_1 \backslash \mathcal{W}_2$, with $\backslash$ denoting set difference, that minimize an absolute correlation requirement and adds them to $\mathcal{W}_2$. $\mathcal{W} = \mathcal{W}_2$ can then be used as a full-diversity transmitter codebook. The algorithm is given in its entirety below.

1. Fix a constant R such that $RM_t \geq 2^{B(M_t-1)}$.
2. Construct a matrix $\mathbf{A}$ where $\mathbf{A}$ consists of the first M_t rows of the $RM_t \times RM_t$ unitary DFT matrix. Scale this matrix by $\sqrt{R}$ to guarantee unit vector columns.
3. Construct a set of vectors $\mathcal{W}_1$ where the members of $\mathcal{W}_1$ are the columns of $\mathbf{A}$.
4. Let the set $\mathcal{W}_2$ be the columns of the $M_t \times M_t$ unitary DFT matrix.
5. Choose the vector $\mathbf{w} \in \mathcal{W}_1 \backslash \mathcal{W}_2$ such that $\forall\ \mathbf{v} \in \mathcal{W}_1 \backslash \mathcal{W}_2$, $f(\mathbf{v}) \geq f(\mathbf{w})$ where f is defined as

$$f(\mathbf{w}) = \max_{\mathbf{x} \in \mathcal{W}_2} |\mathbf{x}^H \mathbf{w}|. \tag{17}$$

Set $\mathcal{W}_2 = \mathcal{W}_2 \bigcup \{\mathbf{w}\}$.
6. Repeat 5 until $\operatorname{card}(\mathcal{W}_2) = 2^{B(M_t-1)}$.

The intuition behind this algorithm is to begin with a codebook of only M_t orthogonal vectors, and then add vectors one-by-one to this codebook, such that the vector added at each step is "distant" from the current codevectors. We have shown in Section III that for any $\xi \in [0, 2\pi)$, the beamforming vectors $\mathbf{w}$ and $e^{j\xi}\mathbf{w}$ provide the same receive SNR. We will, therefore, try to maximize the phase-invariant distance d between any two vectors defined by

$$d(\mathbf{w}_1, \mathbf{w}_2) = \min_{\xi \in [0, 2\pi)} \left\| \mathbf{w}_1 - e^{j\xi}\mathbf{w}_2 \right\|_2 = \sqrt{2 - 2\left|\mathbf{w}_1^H \mathbf{w}_2\right|} \tag{18}$$

where $\mathbf{w}_1$ and $\mathbf{w}_2$ are unit vectors. Thus, $f(\mathbf{w})$ returns the absolute correlation corresponding to the phase-invariant distance of the closest vector in $\mathcal{W}_2$ to $\mathbf{w}$ [21].

Two points are imperative to note about this algorithm. First, as R grows large, $\mathcal{W}$ approaches an optimal equal gain codebook in terms of the cost function. Second, as B increases, it is possible to approach a true EGT system, since for any phase vector entry θ_i, k can be chosen, given RM_t and B such that the error of $|e^{j\theta_i} - e^{j2\pi k/(RM_t)}|$ goes to zero as B (and thus, R) grows large. This shows that QEGT can perform arbitrarily close to EGT.

Selecting an optimal B requires making tradeoffs between the amount of tolerable feedback and array gain. As a rule of thumb, we have found that QEGT using a total feedback of at least M_t bits, or rather $B \geq M_t/(M_t - 1)$, provides performance almost identical to unquantized EGT. Notice that when $B = (\log_2 M_t)/(M_t - 1)$, $\operatorname{card}(\mathcal{W}_2) = M_t$. In this case, the beamforming feasible set will contain exactly M_t orthogonal vectors. The following observation gives an exact performance analysis for this case.

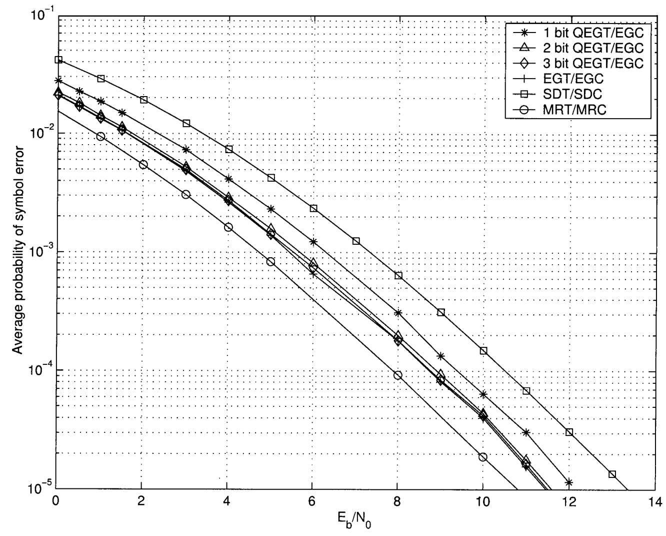

Fig. 2. Average probability of symbol error for 2×2 systems using QEGT/EGC with various weight quantizations, EGT/EGC, SDT/SDC, and MRT/MRC.

Observation: If $B = (\log_2 M_t)/(M_t - 1)$, the system is equivalent in distribution to an SDT system with the same combining scheme.

The proof of this follows easily from the distribution invariance of memoryless, i.i.d. Rayleigh fading matrices to multiplication by unitary matrices. The implication of this observation is that when $B = (\log_2 M_t)/(M_t - 1)$, our algorithm becomes a modified selection diversity beamformer. The only difference is that the beamforming vectors have been "rotated" by the unitary DFT matrix.

VI. SIMULATIONS

For this section, we simulated the average probability of symbol error with various antenna configurations and beamforming schemes. All simulations used i.i.d. Rayleigh fading with $h_{k,l}$ distributed according to $\mathcal{CN}(0,1)$. Monte–Carlo simulations ran over 1.5 million iterations per SNR point.

Experiment 1: We considered an $M_t = M_r = 2$ MIMO QEGT/EGC system with various values of B and $R = 2^{B(M_t-1)}/M_t$, so $\mathcal{W}_1 \backslash \mathcal{W}_2$ was the empty set at the conclusion of the algorithm. Unquantized EGT/EGC, SDT/MRC, SDT/SDC, and MRT/MRC systems were also simulated. Each simulated system used binary phase-shift keying (BPSK) modulation. Fig. 2 shows the results from this experiment. Notice that all the quantized curves have a diversity order of four. The array gain between one and two bits of quantization is approximately 0.6 dB. However, the array gain between two and three bits of quantization is only about 0.08 dB. This is indicative that QEGT approaches EGT performance as B increases.

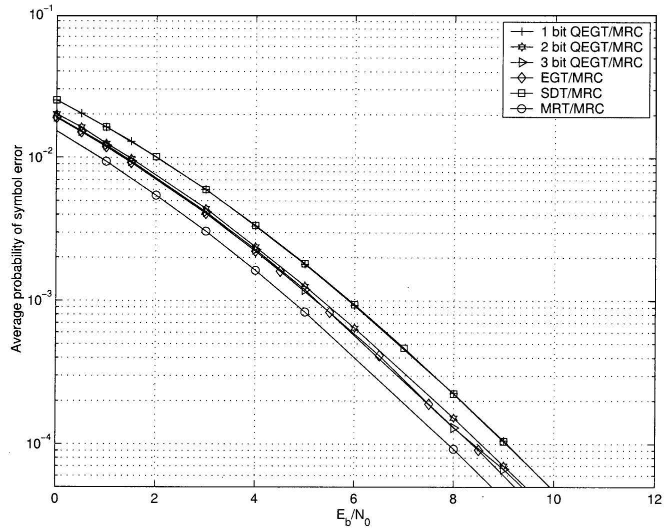

Fig. 3. Average probability of symbol error for 2×2 systems using QEGT/MRC with various weight quantizations, EGT/MRC, SDT/MRC, and MRT/MRC.

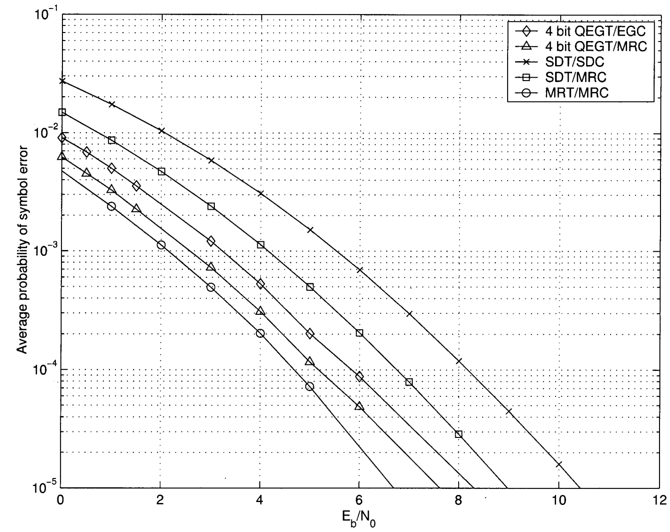

Fig. 4. Average probability of symbol error for 2×3 systems using QEGT/EGC with four bits of feedback per weight, QEGT/MRC with four bits of feedback per weight, SDT/SDC, SDT/MRC, and MRT/MRC.

Experiment 2: This experiment considered $M_t = M_r = 2$ beamforming and combining systems using QEGT/MRC with various values of B, unquantized EGT/MRC, SDT/MRC, and MRT/MRC. All simulations used BPSK modulation. Fig. 3 shows the performance. It is important to note that for $B = 1$, the average symbol-error rate (SER) curve for a QEGT/MRC system is on top of the average SER curve for an SDT/MRC system. This verifies the observation presented for this special case. We took $R = 2^{B(M_t - 1)}/M_t$ once again. The array gain between one and two bits of quantization is approximately 0.6 dB. Once again, the gain of around 0.05 dB between two and three bits quantization is much smaller. The diversity order is seen to be four, as expected.

Experiment 3: In the third experiment, we took $M_t = 2$, $M_r = 3$, and transmitted BPSK symbols. We considered QEGT/EGC and QEGT/MRC with $B = 4$, which provide a close approximation to EGT/EGC and EGT/MRC performance, respectively. Here, R was taken to be $2^{4-1} = 8$. SDT/SDC, SDT/MRC, and MRT/MRC were also simulated. The results are shown in Fig. 4. This plot shows that using MRC instead of EGC at the receiver with EGT gives around a 0.8 dB gain. The diversity order for all of the curves is seen to be six, as one would expect.

Experiment 4: This experiment used $M_t = M_r = 3$ with BPSK. The results are shown in Fig. 5. We considered QEGT/EGC and QEGT/MRC with $B = 4$, which again provide a close approximation to EGT/EGC and EGT/MRC performance, respectively. SDT/SDC, SDT/MRC, and MRT/MRC were simulated for comparison. Here, we took $R = 243$. This value of R led to a nonempty $\mathcal{W}_1 \backslash \mathcal{W}_2$ when the algorithm was completed. The array gain difference between receive MRC and EGC with EGT is around 0.6 dB. The diversity order for all of the plotted curves is nine.

Experiment 5: This experiment shows that the performance of QEGT systems is independent of the modulation scheme. Fig. 6 shows the average SER for an MRT/MRC system using four-point quadrature amplitude modulation (4-QAM), a

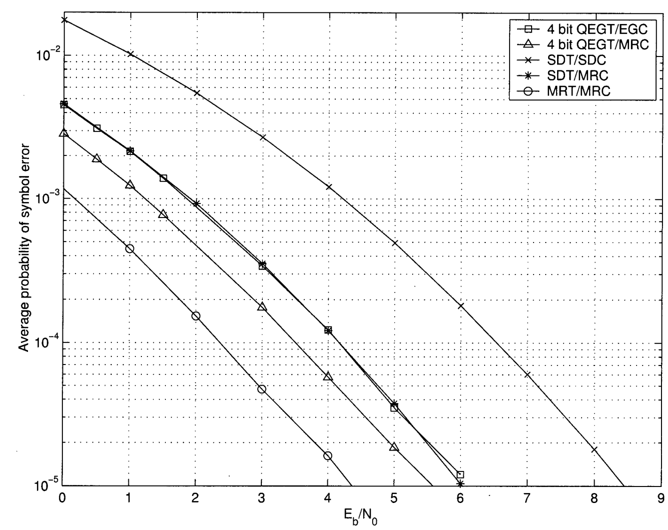

Fig. 5. Average probability of symbol error for 3×3 systems using QEGT/EGC with four bits of feedback per weight, QEGT/MRC with four bits of feedback per weight, SDT/SDC, SDT/MRC, and MRT/MRC.

QEGT/MRC system with $B = 3$ using 4-QAM, an MRT/MRC system using 16-QAM, and a QEGT/MRC system with $B = 3$ using 16-QAM. We used $R = 10^3$. All systems used $M_t = 2$ and $M_r = 4$. For both modulation schemes, the MRT/MRC system has an array gain of approximately 0.4 dB over the QEGT/MRC system.

Experiment 6: This experiment illustrates the benefits of employing transmit *and* receive antenna diversity over simply receive diversity. In Fig. 7, the average SER curves are shown for an $M_t = 1$ and $M_r = 4$ EGC system, an $M_t = 2$ and $M_r = 4$ QEGT/EGC system with $B = 3$ and $R = 10^3$, and an $M_t = 1$ and $M_r = 8$ EGC system. Each simulation used 4-QAM. The 2×4 QEGT/EGC system outperforms the 1×4 EGC system by approximately 3.4 dB at an error rate of 10^{-3}. The 2×4 QEGT/EGC system also provides eighth-order diversity compared with fourth-order diversity of the 1×4 EGC system. Thus, adding another transmit antenna provides

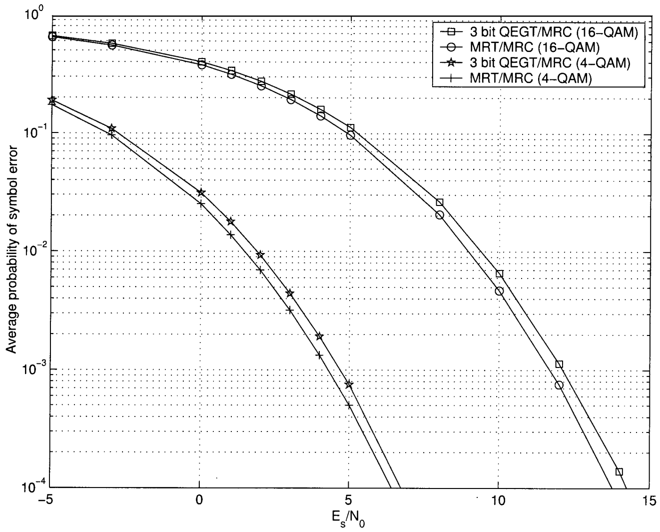

Fig. 6. Bound and average probability of symbol error for a 2×4 QEGT/MRC system with three bits of feedback per weight with various modulations.

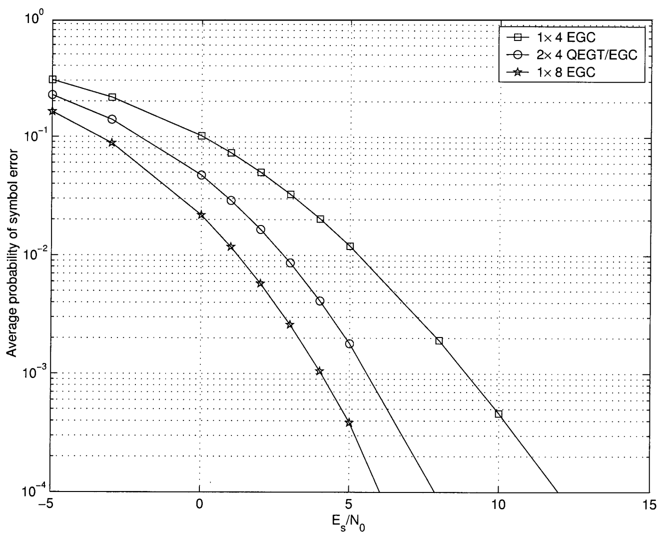

Fig. 7. Average probability of symbol error for a 2×4 QEGT/EGC system with three bits of feedback per weight, a 1×4 receive EGC system, and a 1×8 receive EGC system.

substantial performance gains. The 1×8 EGC system also provides eighth-order diversity and provides approximately a 1.5 dB array gain over the 2×4 QEGT/EGC system. This performance increase comes at greater cost, because the 1×8 system requires three more antennas than the 2×4 system.

VII. Conclusion

In this paper, we examined EGT for MIMO wireless systems operating in memoryless, MIMO Rayleigh fading channels. We specifically examined the design and performance of EGT when used with receive SDC, EGC, or MRC. We showed that in each of these cases, the beamforming and combining system obtains full diversity order. We proposed a quantized version of EGT for systems without transmitter channel knowledge. We presented a codebook design method for QEGT that guarantees full diversity order.

The primary performance limitation of QEGT derives from the equal gain assumption. In other work [21], [22] we show that quantized MRT provides further performance improvement at the expense of a signal peak-to-average ratio increase. A thorough probabilistic analysis of Rayleigh fading MIMO channels is needed in order to understand the performance of quantized beamforming systems [21], [22]. Another point of future interest is the derivation of exact expressions for the average probability of error for MIMO equal gain systems. Many papers have derived closed-form probability of error expressions for the SIMO equal gain case [4], [10]–[12], [19], but there has been little work on deriving exact probability of error expressions for MIMO EGT systems.

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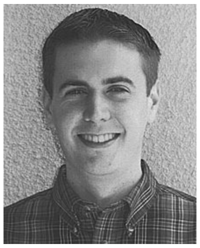

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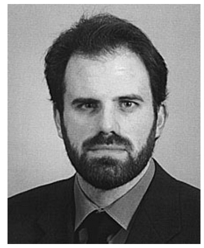

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