Multimode Antenna Selection for Spatial Multiplexing Systems With Linear Receivers

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***Abstract*—Spatial multiplexing is a simple transmission technique for multiple-input multiple-output (MIMO) wireless communication links in which data is multiplexed across the transmit antennas. In Rayleigh fading matrix channels, however, spatial multiplexing with low-complexity linear receivers suffers due to a lack of diversity advantage. This paper proposes multimode antenna selection, which uses a low-rate feedback channel to improve the error rate performance of spatial multiplexing systems with linear receivers. In the proposed technique, both the number of substreams and the mapping of substreams to antennas are dynamically adjusted, for a fixed total data rate, to the channel based on limited feedback from the receiver. Dual-mode selection, where spatial multiplexing or selection diversity is adaptively chosen, dramatically improves the diversity gain achieved. Multimode selection (i.e., allowing any number of substreams to be dynamically selected) provides additional array gain. Various criteria for selecting the number of substreams and the optimal mapping of substreams to transmit antennas are derived. Relationships are made between the selection criteria and the eigenmodes of the channel. A probabilistic analysis of the selection criteria are provided for Rayleigh fading channels. Applications to nonlinear receivers are mentioned. Monte Carlo simulations demonstrate significant performance improvements in independent and identically distributed (i.i.d.) flat-fading Rayleigh matrix channels with minimal feedback.**

***Index Terms*—Antenna selection, diversity methods, MIMO systems.**

I. INTRODUCTION

MULTIPLE-INPUT multiple-output (MIMO) wireless communication channels, which are created through the use of arrays of transmit and receive antennas, can be exploited to improve capacity and reduce sensitivity to fading. One simple approach for taking advantage of the capacity of the MIMO wireless channel is spatial multiplexing, a component of Bell Labs Space Time (BLAST) [1], where the incoming data is divided into multiple substreams, and each substream is transmitted on a different transmit antenna [1], [2]. Given perfect channel knowledge at the receiver, a variety of methods including linear, successive, and maximum likelihood (ML) decoding can be used to remove the effect of the channel and reassemble the transmitted substreams [3]–[5].

Spatial multiplexing systems that use linear receivers, either zero-forcing (ZF) or minimum mean squared error (MMSE), are practically important due to their minimal complexity requirements. Unfortunately, in independent and identically distributed (i.i.d.) MIMO Rayleigh fading channels, the use of a linear receiver incurs a significant penalty: loss of diversity advantage relative to the ML receiver. In a system with M_t transmit antennas and M_r receive antennas, ZF receivers obtain a diversity gain on the order of $M_r - M_t + 1$ [5], whereas the diversity gain with ML receivers is M_r [3]. Although ML receivers do improve performance, they do not achieve the full $M_r M_t$ possible through more sophisticated space-time coding (see, e.g., [6]).

Contributions: In this paper, we propose to use a low-rate feedback channel to add a diversity mode to spatial multiplexing systems with linear receivers. The key innovation in our system is to allow both the number of substreams transmitted and the mapping of substreams to a subset of transmit antennas to be dynamically controlled based on feedback from the receiver. We *assume that the overall data rate remains the same, regardless of the number of substreams*; thus, the sole purpose of the feedback is to indicate the substream and antenna combination that provides the minimum error rate for the given data rate. Our work generalizes prior work on transmit antenna selection for MIMO systems with linear receivers [7], [8] by offering substantially more diversity advantage at the expense of additional feedback. In prior work, the number of transmitted substreams was fixed, and only the optimum subset among all possible transmit antennas was chosen. The diversity improvement was thus limited by the difference between the number of transmit antennas and the number of substreams. In contrast, by allowing both the number of substreams transmitted and the mapping of substreams to a subset of antennas, with optimal selection, we argue that it is possible to obtain a *diversity advantage proportional to the product of the number of transmit and receive antennas*. Further, the feedback requirements are surprisingly minimal—only M_t (the number of transmit antennas) bits per matrix channel.

We consider two important cases that we call dual-mode antenna selection and multimode antenna selection. In the dual-mode case, we allow either spatial multiplexing with all the transmit antennas or selection diversity with transmission from the single best transmit antenna. Because of the fixed rate assumption, in the later case, the constellation transmitted on a single antenna is much larger than when all the antennas

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are used. Essentially, this allows us to trade off between multiplexing and diversity based on instantaneous channel information, similar to the approach in [9] and [10]. We propose various criteria for selecting the optimum subset in flat-fading matrix channels, assuming a zero-delay and error-free feedback path. Based on Euclidean distance arguments, we show that the choice of diversity or multiplexing can be made as a function of the condition number of the channel and the minimum Euclidean distance of the constellations. Using this result, we study the probability of selecting multiplexing versus diversity, as a function of the number of antennas and the rate, for MIMO Rayleigh fading channels using the distribution of the condition number. We then generalize our results to the case of multimode selection, where all possible numbers of substreams and subsets of the transmit antennas are allowed. We derive various suboptimal selection criteria, and we show that the multimode approach provides a smoother way to trade off between diversity and multiplexing. Compared with dual-mode selection, this approach achieves the same diversity gain but more array gain due to improved coupling of the transmitted signal into the channel. The probability of selecting a given number of antennas is presented and evaluated numerically. We also show that dual-mode and multimode antenna selection can be used with nonlinear receivers such as the Vertical-Bell Labs Space Time (V-BLAST) and ML receiver. Monte Carlo simulations illustrate the performance improvement for dual-mode and multimode with various selection strategies.

Relations with Prior Work: The concept of adapting both the number of substreams and the substream rate has been proposed in different contexts in [11]–[13]. Dynamically selecting the number of substreams and the optimal subset based on knowledge of the channel correlation matrices was investigated in [11] for various receivers to improve the performance of spatial multiplexing in correlated fading channels. Both the antennas used for transmission, and the rate of the constellation on each antenna is varied. While the selection framework is similar to that in [11], our work is different because we adapt the constellation, the number of substreams, and the antenna subset based on the instantaneous channel conditions, as opposed to the correlation of the channel [14], [15]. Our approach is suitable when the channel is slowly fading, as in pedestrian environments, whereas the approach in [11] is preferable when the channel is fading too fast to adapt to the instantaneous channel. Partial knowledge of the dominant channel singular vectors is used in [12] to perform precoding. In other work, instantaneous channel knowledge is used to vary the number of substreams to control interference in a MIMO interference channel in [13]. In contrast, we adapt the number of substreams, substream rate, and optimal subset to improve throughput and diversity gain in noninterference fading channels. Extension to interference channels is an interesting topic for future research.

One way to view our work is in terms of the diversity-multiplexing tradeoff [16]. The work in [16] studies multiplexing and diversity from an information theoretic point-of-view based on achievable rates. In particular, the authors quantify the amount of diversity and multiplexing gain for any code by bounding the average probability of error using the outage probability. Our work is different in that it deals with a specific code (spatial multiplexing), includes feedback of channel state information, and deals with practical receivers. The work in [16] and in other information-theoretic treatments of the diversity-multiplexing tradeoff [17], [18] is useful for studying the fundamental diversity-multiplexing tradeoff, whereas our work is useful for practically achieving the benefits of both diversity and multiplexing with low complexity and limited feedback.

Compared with other probability-of-error-based work on the diversity-multiplexing tradeoff [9], [19], [20], we take a pragmatic view based on instantaneous channel state information. In particular, [19] studies the multiplexing and diversity tradeoff in terms of the average probability of error (specifically for polarized channels), which requires making certain assumptions about the distribution of the channel. This comparison is useful when either diversity or multiplexing is chosen for a succession of channel realizations but may vary as a function of the second-order statistics of the channel. Our approach is similar to that taken in our prior work [9], with the exception that we consider selection diversity instead of space-time block coding and linear receivers instead of ML for spatial multiplexing, and we allow for multimode transmission. Overall, an advantage of our formulation is that we obtain a constructive means for switching between diversity and multiplexing that is useful in actual wireless systems (see, e.g., [21] and [22] for details). A related form of spatial mode selection is proposed in [20] for transmit and receiver MMSE precoding. This work, however, requires full knowledge of the transmit precoder for the chosen mode and is not readily implementable with limited feedback.

Multimode selection can be viewed in the context of precoding and quantized precoding. Previous work on precoding (e.g., [23]) assumed perfect channel knowledge at the transmitter to construct an optimal precoder. Our approach uses only a finite number of possible precoders, thus enabling implementation in frequency-division duplex systems, where the channel is often not available at the transmitter. Related work on quantized precoding has focused on quantized beamforming [24]–[26] and quantized precoding for spatial multiplexing systems [27]. Our approach differs from prior work in that we use a very coarse quantization, and we allow for substream selection, in addition to subset selection. Incorporation and extension of the results of [27] is possible in the proposed framework but is treated elsewhere due to lack of space [28].

Multimode antenna selection is also related to the suboptimal per-antenna rate and power control algorithms proposed in [29]. The algorithm in [29] uses the notion of a "power unit" given by the total transmit power divided by the number of transmit antennas. The optimal placement of the power units is then allocated using a brute force search. Our algorithm differs from this because we only consider turning antennas "on" or "off" without per-antenna power control. We also address the problem from a probability-of-error approach instead of an ergodic capacity approach.

Finally, our work is a natural extension of the existing literature on antenna subset selection for spatial multiplexing systems [7], [8], [30]–[35]]. The work in [30] and [32] studies antenna selection from the capacity point of view, whereas the work in [7], [8] and [31] studies selection based from the perspective of error probability. In this paper, we develop generalized transmit selection algorithms with the goal of improving error rate along the lines of [7] and [31]. Our approach differs from previous work in that we fix the overall rate and allow the total number of substreams to vary as well as the optimal subset. We require

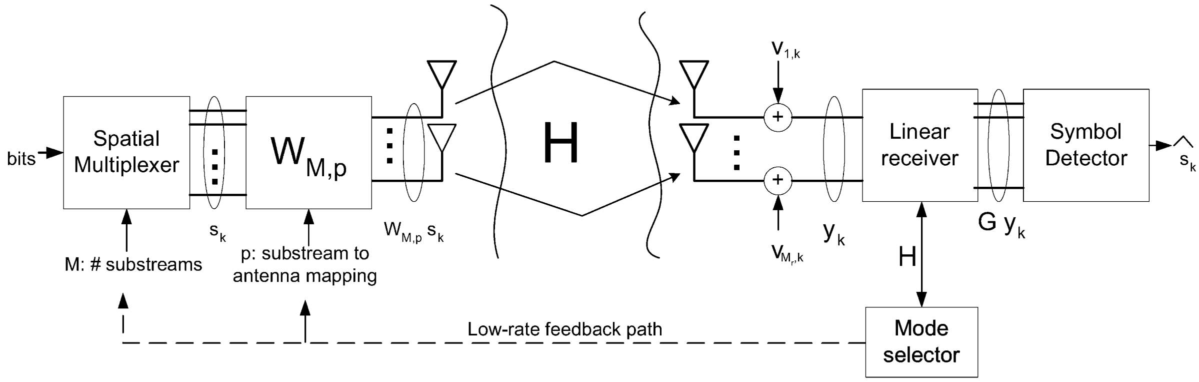

Fig. 1. Spatial multiplexing system with feedback as considered in this paper. The mode selector determines the optimal transmission mode for a given channel (from a finite set of transmission modes) and sends the index of this mode back to the transmitter.

slightly more feedback, but we can obtain full diversity, even with an equal number of transmit and receive antennas, unlike [7]. We consider a brute force search for the optimal subset; deriving suboptimal selection algorithms along the lines of [33] and [35] is an interesting extension.

Organization: This paper is organized as follows. In Section II, we introduce the system model, our assumptions, and the corresponding mathematical models. Then, in Section III, we present motivation for dual-mode and multimode selection. In Section IV, we study dual-mode selection, where either spatial multiplexing or a single transmit antenna is used for transmission. This provides a dynamic interpretation of the diversity-multiplexing tradeoff. In Section V, we generalize our results to the case of multimode selection where both the number of substreams as well as the best subset of transmit antennas is chosen. Section VI provides some probabilistic analysis of the switching criteria, whereas Section VII discusses how multimode antenna selection can be applied to nonlinear receivers. Section VIII illustrates error rate improvements via Monte Carlo simulations. In Section IX, we make some conclusions and provide suggestions for future work.

II. System Model

Consider the M_t-transmit antenna by M_r-receive antenna MIMO communication system illustrated in Fig. 1 that transmits R bits per channel use. The system consists of a spatial multiplexer that produces M substreams, a spatial mapper that maps these M streams to transmit antennas, a matrix propagation channel that is a function of the wireless environment, and a space-time receiver that uses an estimate of the total channel to decide on the transmitted bit stream. A low-rate feedback path is used to determine the number of substreams M and mapping from substreams to antennas.

During each symbol period, R bits are demultiplexed into M different bit streams and modulated independently using the same constellation $\mathcal{S}$. Note that the constellation size per substream is $2^{R/M}$ so that a total of R bits are transmitted, regardless of the choice of M. The spatial multiplexer produces an M-dimensional symbol vector $\mathbf{s}_k$ at symbol period k, where $\mathbf{s}_k = [s_{k,1}, s_{k,2}, \ldots, s_{k,M}]^T$.[1] For convenience, we assume there is no error correction coding and that $\mathcal{E}_\mathbf{s}[\mathbf{s}_k\mathbf{s}_k^H] = (1/M)\mathbf{I}_M$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix. Notice that $\mathbf{s}_k$, and thus, the constellations that make up $\mathbf{s}_k$ are normalized so that the total transmit power is $\mathcal{E}_\mathbf{s}[\mathbf{s}^H\mathbf{s}] = 1$, irrespective of M. Throughout, it is assumed that $M \le M_r$.

Given M, the symbol vector $\mathbf{s}_k$ is transmitted over a subset of the available transmit antennas. Let $\mathcal{W}_M$ be the set of $\binom{M_t}{M}$ matrices taken by choosing M columns from $\mathbf{I}_{M_t}$. For example

$$\mathcal{W}_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

$$\mathcal{W}_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

and

$$\mathcal{W}_3 = \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}.$$

We will assume that for each M, $\mathcal{W}_M$ has been ordered so that it can be written as

$$\mathcal{W}_M = \left\{ \mathbf{W}_{M,1}, \mathbf{W}_{M,2}, \ldots, \mathbf{W}_{M,\binom{M_t}{M}} \right\}.$$

Each matrix $\mathbf{W}_{M,p}$ represents a *substream-to-antenna mapping*. The columns of the mapping matrices are simply selection diversity vectors that select the antenna to transmit the corresponding substream.

We will assume that the transmitter has no knowledge of the forward-link channel. Therefore, the optimal values of M and p (which are denoted M^* and p^*) are determined at the receiver and sent back to the transmitter over a limited rate feedback channel. Using the substream-to-antenna matrix framework, we will write the transmitted vector as $\sqrt{E_s}\mathbf{W}_{M^*,p^*}\mathbf{s}_k$, where E_s is the average transmit energy. Thus, as mentioned before, this framework includes antenna subset selection as a special case [7], [8], [30]–[32], [34] but is even more general.

After precoding, the qth transmit antenna transmits the qth entry of $\sqrt{E_s}\mathbf{W}_{M,p}\mathbf{s}_k$. We assume that the transmission bandwidth is much less than the coherence bandwidth of the channel and that the symbol period is much less than the coherence time. In this case, the channel between the qth transmit antenna and the rth receive antenna can be modeled by a complex gain $h_{r,q}$.

[1]We use * for conjugate, ^{T} for transpose, ^{H} for conjugate transpose, † for the matrix pseudo-inverse, and tr( ) for the trace operator that gives the sum of the diagonal elements of a matrix, $\|\cdot\|$ for the usual Euclidean vector norm, and $\mathcal{E}_s$ to denote expectation with respect to random variable s.

We assume that the antennas are sufficiently separated and that the environment is sufficiently rich in scatterers to allow $h_{r,q}$ to be modeled as a realization of a proper complex Gaussian random variable with distribution $\mathcal{CN}(0,1)$ that is uncorrelated in space.

Neglecting symbol timing errors and frequency offsets, the $M_r \times 1$ received signal vector after matched filtering and sampling is

$$\mathbf{y}_k = \sqrt{E_s}\mathbf{H}\mathbf{W}_{M,p}\mathbf{s}_k + \mathbf{v}_k \tag{1}$$

where $\mathbf{H}$ is a matrix with $h_{r,q}$ at entry (r,q), and $\mathbf{v}_k = [v_{k,1}, v_{k,2}, \ldots, v_{k,M_r}]^T$ is complex Gaussian noise with $v_{k,r}$ distributed according to $\mathcal{CN}(0, N_o)$ with $v_{k,r}$ independent of $v_{k,q}$ for $r \neq q$. We refer to $\mathbf{H}\mathbf{W}_{M,p}$ as the equivalent channel. In this paper, we assume that the channel is varying slowly across a given frame of data but varies independently from frame to frame. To simplify subsequent notation, we assume that we have a frame with a single symbol; thus, we let $k = 1$ and suppress the k subscripts.

At the receiver, an $M \times M_r$ linear equalizer matrix $\mathbf{G}$ is applied to $\mathbf{y}$, and then, the components of $\mathbf{G}\mathbf{y}$ are detected independently. For the ZF linear decoder, $\mathbf{G} = (\mathbf{H}\mathbf{W}_{M,p})^{\dagger}$, whereas for the minimum mean square error (MMSE) linear decoder, $\mathbf{G} = [\mathbf{W}_{M,p}^H\mathbf{H}^H\mathbf{H}\mathbf{W}_{M,p} + \gamma_0^{-1}M\mathbf{I}_M]^{-1}\mathbf{W}_{M,p}^H\mathbf{H}^H$, where $\gamma_0 = E_s/N_o$. Note that we address the application of nonlinear receivers in Section VII.

In this paper, we assume that a zero-delay limited capacity feedback link is available from the receiver to the transmitter. Consequently, the parameters M $(1 \leq M \leq M_t)$ and p $(1 \leq p \leq \binom{M_t}{M})$ of $\mathbf{W}_{M,p}$ can be chosen using exact knowledge of $\mathbf{H}$. The presence of a limited capacity feedback link is reasonable in any nonbroadcast wireless system. Zero delay is not always a good assumption; however, the amount of delay that can be tolerated requires extensive investigation on its own (see, e.g., [36]); thus, we choose to defer this to future work.

Another assumption is that the channel matrix $\mathbf{H}$ is known perfectly at the receiver. Estimation methods such as ML and MMSE techniques discussed in [37] would have to be used to obtain knowledge of the channel at the receiver. This means that an $M_t \times T$ with $T \geq M_t$ symbol matrix will have to be transmitted each time the channel changes. Note that this means we require a training preamble of length $T \geq M_t$ that is, in general, larger than the preamble of length M that is required for detection. In reality, estimation errors are always present and will degrade performance. The analytical effects of estimation error is beyond the scope of this paper.

III. Preliminary Analysis and Motivation

In this section, we provide background and motivation for combining spatial multiplexing and selection diversity. First, we consider spatial multiplexing performance. We review diversity order results that show how it degrades with larger M. Then, we consider antenna selection, where $M = 1$, and the best transmit antenna is selected. In contrast to random selection, it is possible to obtain a diversity order of M_tM_r. Finally, we illustrate the substantial variation in error rate with both approaches to prompt optimal selection of both the antenna subset and the number of substreams.

A. *Performance With Spatial Multiplexing Without Dynamic Selection*

The performance of spatial multiplexing with linear receivers is a function of the effective SNR_k for each stream $k = 1, 2, \ldots, M_t$ after linear processing. From [7], for the ZF case

$$\mathrm{SNR}_k^{(\mathrm{ZF})} = \gamma_0 \frac{1}{[\mathbf{H}^H\mathbf{H}]_{k,k}^{-1}} \tag{2}$$

whereas for the MMSE case

$$\mathrm{SNR}_k^{(\mathrm{MMSE})} = \gamma_0 \frac{1}{[\mathbf{H}^H\mathbf{H} + \gamma_0^{-1}M\mathbf{I}_M]_{k,k}^{-1}} - 1 \tag{3}$$

respectively, where $\mathbf{A}_{k,k}^{-1}$ is entry (k,k) of $\mathbf{A}^{-1}$.

In this paper, we are interested in the probability of vector symbol error, that is, the probability that at least one of the R bits in the transmission is in error. Let $d_{\min}(M,R)$ denote the minimum distance of the constellation used for transmission on one of the M substreams. Let $N_e(M,R)$ denote the average number of nearest neighbors of this constellation [38], [39]. The dependence on R is used to remind us that the constellation has $2^{R/M}$ points, whereas the dependence on M is used to recall the normalization by M. Given $\mathbf{H}$ and using the nearest neighbor union bound (NNUB) results from [7], we can bound the conditional vector symbol error rate (VSER), which is the probability that at least one transmitted symbol is in error, as

$$P_e(\mathbf{H}) \leq 1 - \left(1 - N_e(M,R) \times Q\left(\sqrt{\mathrm{SNR}_{\min}\frac{d_{\min}^2(M,R)}{2}}\right)\right)^M \tag{4}$$

where $\mathrm{SNR}_{\min} = \min_k \mathrm{SNR}_k$, and SNR_k depends on if the receiver is ZF or MMSE. The performance, as measured by the average probability of vector symbol error, is typical in fading channels. Based on (4)

$$\begin{aligned} P_e &= \mathcal{E}_{\mathbf{H}}[P_e(\mathbf{H})] \\ &\leq 1 - \mathcal{E}_{\mathrm{SNR}_{\min}}\left(1 - N_e(M,R) Q\left(\sqrt{\mathrm{SNR}_{\min}\frac{d_{\min}^2(M,R)}{2}}\right)\right)^M. \end{aligned}$$

The diversity performance, which is the slope of the average error rate curve plotted on the log scale [40], depends on the statistics of the minimum SNR.

In the case where $M_t = M_r$, it has been proven that $\mathrm{SNR}_k^{(\mathrm{ZF})}$ is chi-squared distributed with two degrees of freedom [5], [41]. Thus, the minimum SNR is also chi-squared with two degrees of freedom (which follows since the chi-squared distribution with two degrees of freedom is equivalent to the exponential distribution). Note, however, that the post-processing SNR for a single-transmit single-receive antenna system is also chi-squared with two degrees of freedom. Thus, with $M_t = M_r$, the diversity performance for the linear receiver is exactly the same as that of a single antenna communication link! Therefore, the price of simple receiver decoding is a reduction in the diversity order.

This is in contrast to the more complex ML receiver for which the diversity order is always M_r [3].

In the case where $M_r > M_t$, it can be shown that the diversity order is $M_r - M_t + 1$ in general for the ZF receiver [5], [41]. This has been proven for the symbol error rate in [5], but a proof for the VSER does not seem to be available. The MMSE receiver is found to slightly outperform the ZF receiver though the diversity order through simulations has been found to be similar.

B. Performance With Selection Diversity

Now, consider a system with $M = 1$ that uses optimal subset selection with the ZF receiver. In this case, $\mathcal{W}_1$ is simply the set of columns of the identity matrix, and the selection problem corresponds to ideal transmit antenna selection. For $M = 1$, the equivalent channel is a vector, and it is apparent from the definition of the pseudo-inverse that $\mathbf{G} = (\mathbf{HW}_{1,p})^H / \|\mathbf{HW}_{1,p}\|$, which is just maximum ratio combining (normalized to preserve detection regions). Therefore, the post-combining SNR (with $\mathbf{h}_p$ representing the pth column of $\mathbf{H}$) is

$$\mathrm{SNR}_p^{(\mathrm{ZF})} = \gamma_0 \|\mathbf{HW}_{1,p}\|^2 = \gamma_0 \|\mathbf{h}_p\|^2 \tag{5}$$

which is a function of the transmit antenna p. The most common selection algorithm is the one that chooses p such that the post-processing SNR is maximized [42]. Let p^* denote the maximizer. Then, it is possible to approximate the conditional VSER using the NNUB as

$$P_e\left(\mathrm{SNR}_{p^*}^{(\mathrm{ZF})}\right) \le N_e(1,R) Q\left(\sqrt{\mathrm{SNR}_{p^*}^{(\mathrm{ZF})} \frac{d_{\min}^2(1,R)}{2}}\right)$$

and the average VSER as

$$\begin{aligned} P_e &= \mathcal{E}_{\mathrm{SNR}_{p^*}^{(\mathrm{ZF})}}\left[P_e\left(\mathrm{SNR}_{p^*}^{(\mathrm{ZF})}\right)\right] \\ &\le N_e(1,R)\mathcal{E}_{\mathrm{SNR}_{p^*}^{(\mathrm{ZF})}}\left[Q\left(\sqrt{\mathrm{SNR}_{p^*}^{(\mathrm{ZF})} \frac{d_{\min}^2(1,R)}{2}}\right)\right]. \end{aligned}$$

The key to evaluating (6) is to recognize that the $\mathrm{SNR}_{p^*}^{(\mathrm{ZF})}$ is the maximum of M_t chi-squared random variables, each with $2M_r$ degrees of freedom. Using the results from [43], it is possible to expand this expression into an integral (see [43, 8] for details). The result of this analysis is that the diversity order of transmit antenna selection with optimum combining at the receiver can be shown to be $M_t M_r$. In contrast, randomly selecting a transmit antenna provides only a diversity order of M_r. The large diversity order of selection diversity shows the importance of selection diversity as a transmission mode.

C. Comparison of Multiplexing and Transmit Selection Diversity

The fact that R is fixed makes it is possible to fairly compare the error rate performance of spatial multiplexing with M streams and transmit antenna selection. To provide motivation for the subsequent development in the paper, in Fig. 2, we provide a VSER comparison of different signaling schemes with $R = 8$ bits. The ZF receiver with $M = 1, 2, 4$ using 256-quadrature amplitude modulation (QAM), 16-QAM, and 4-QAM modulation is compared with selection diversity (where $M = 1$) using a 256-QAM constellation. Notice the difference in diversity order for each approach. Note that in all cases except for selection diversity, $M = M_t$.

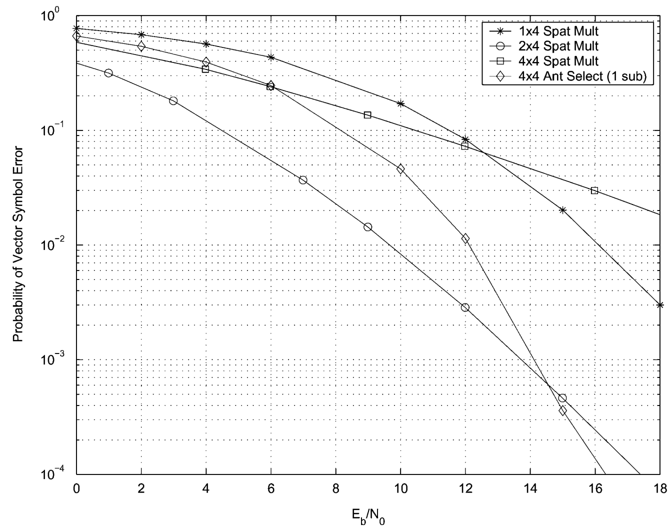

Fig. 2. Vector symbol error rate comparison of different signaling schemes for various numbers of transmit antennas with $R = 8$ bits (QAM constellations).

As expected, selection diversity outperforms all other approaches in terms of diversity advantage. Its performance, however, lags the ZF receiver at low SNR. The reason is that diversity is a high SNR measure of error rate performance. Spatial multiplexing uses multidimensional constellations with larger average distance between the points compared with a dense two-dimensional constellation.

A stochastic approach to optimizing system performance would be to choose $M_t = 2$ and $M_r = 4$ spatial multiplexing for E_b/N_o of less than $14dB$ and transmit antenna selection otherwise. This would give both the coding gain of spatial multiplexing and diversity advantages of selection, each in its preferred region.

Given instantaneous channel knowledge, we should be able to do much better. First, performance should improve by dynamically selecting either spatial multiplexing or diversity, based on the instantaneous channel realization. This is the subject of Section IV. Second, we have not considered the effect of optimal subset selection, which is known to further improve diversity performance with linear receivers [7]. Allowing both antenna subset selection and selection of M dynamically based on the channel is the subsect of Section V.

IV. Dual-Mode: Diversity or Multiplexing?

Consider the system of Fig. 1 with two modes of operation: spatial multiplexing and single antenna selection diversity. The optimal precoding matrix is chosen by the receiver from either $\mathcal{W}_1$ or $\mathcal{W}_{M_t} = \{\mathbf{I}_{M_t}\}$ and conveyed back using $\lceil \log_2(M_t+1) \rceil$ feedback bits. In this section, we consider dynamic approaches for selection either diversity and multiplexing based on the VSER, the effective SNR, and the condition number.

A. Vector Symbol Error Rate Based Selection

Given an instantaneous channel realization $\mathbf{H}$, the optimal fixed-rate selection algorithm is to choose the mode that delivers the lowest probability of vector symbol error for that given channel. Clearly, two selections are required. First, the optimal element of $\mathcal{W}_1$ must be chosen. Second, the optimal number of substreams (either $M=1$ or $M=M_t$) must be determined. The motivation for complicating the transmission scheme is the substantial improvements in diversity order as given by the following theorem.

Theorem 1: Optimal choice of either spatial multiplexing or selection diversity provides full diversity advantage.

Proof: Selection diversity provides full diversity advantage on the order of M_tM_r (cf. [43]). Optimal mode selection can only be better than choosing single antenna selection diversity for all channel realizations. □

Because we do not always have exact expressions for the VSER, we will use NNUBs [38] on the symbol error rates to compute the optimal subset and number of substreams. These bounds are tight at high SNR, and we find they provide good selection performance.

The NNUB requires the notions of post-processing SNR given an antenna subset. Let $\mathrm{SNR}_k(M,p)$ denote the post-processing SNR for the kth stream, given precoding matrix $\mathbf{W}_{M,p}$. This could be obtained from either (2) or (3), depending on the receiver criterion. These SNR results allow closed-form bounds to be derived. For selection diversity, compute $P_e^{(s)}(\mathbf{H})$

$$\begin{aligned}&\leq N_e(1,R)\min_{1\leq p\leq M_t} Q\left(\sqrt{\mathrm{SNR}_1(1,p)\frac{d^2_{\min}(1,R)}{2}}\right)\\ &= N_e(1,R)Q\left(\sqrt{\max_{1\leq p\leq M_t}\frac{E_s}{N_0}\|\mathbf{h}_p\|^2\frac{d^2_{\min}(1,R)}{2}}\right) \quad (6)\end{aligned}$$

and for spatial multiplexing, compute

$$\begin{aligned}P_e^{(m)}(\mathbf{H}) &\leq 1-(1-N_e(M_t,R)\\ &\times Q\left(\sqrt{\min_{1\leq k\leq M_t}\mathrm{SNR}_k(M_t,1)\frac{d^2_{\min}(M,R)}{2}}\right)\Bigg)^{M_t}. \quad (7)\end{aligned}$$

Note that we have used the modified notation, where $\mathrm{SNR}_k(M,p)$ is the SNR of the kth substream of the effective channel $\mathbf{W}_{M,p}$.

The NNUB-based VSER selection criterion is given as the following.

Selection Criterion 1—VSER Based Selection: Choose multiplexing if $P_e^{(m)}(\mathbf{H}) < P_e^{(s)}(\mathbf{H})$; otherwise, choose diversity transmission from the best transmit antenna.

Based on the tightness of the NNUB, we anticipate full diversity advantage from VSER-based selection. For implementation, $N_e(1,R), N_e(M_t,R), d^2_{\min}(1,R)$, and $d^2_{\min}(M_t,R)$ should be precalculated. The Q function can be implemented as a lookup table or approximated using an accurate polynomial method; see, e.g., [44]. The cost function can also be simplified using the approximations $1-(1-\eta)^M \approx M\beta$ for small β and $Q(x)\approx(1/\sqrt{2\pi}x)\exp(-0.5x^2)$ for large x.

B. Post-Processing SNR-Based Selection

Although *Selection Criterion 1* is close to optimal, its computation makes it difficult to motivate in real-time implementation. Further, it does not provide intuition about the role played by the "quality" of the channel. Thus, we also investigate alternative selection criteria with lower complexity.

A useful approximation that provides a selection criterion directly as a function of the post-processing SNR is obtained by neglecting the nearest neighbors terms in (6) and (7). Then, the performance criterion can be posed in terms of the arguments of the Q function. Remembering that the Q function is decreasing with increasing arguments, we propose the following.

Selection Criterion 2—SNR Based Selection: Choose multiplexing if

$$d^2_{\min}(M_t,R)\min_{1\leq k\leq M_t}\mathrm{SNR}_k(M_t,1) \geq d^2_{\min}(1,R)\max_{1\leq p\leq M_t}\mathrm{SNR}_1(1,p).$$

Otherwise, choose diversity transmission from the best transmit antenna.

Selection Criterion 2 reveals the striking difference in the performance dependence of each modulation scheme. Spatial multiplexing is a function of the worst substream, whereas diversity transmission is a function of the column of $\mathbf{H}$ with the largest norm. Of course, this comparison is interesting because it is not just a function of the channel; it is also a function of the data rate and constellation through $d^2_{\min}(M,R)$ and $d^2_{\min}(1,R)$.

Effectively, the selection in *Selection Criterion 2* is made only by the argument of the Q function. Since the number of points in the vector constellation is the same, this is in fact equivalent to selection based on the union bound on the vector symbol error probability [10]. Thus, the quality of selections based on the SNR are essentially related to how well the union bound is a good predictor of performance. Although there is often a substantial gap to the true probability of vector symbol error, it was found in [10] in similar circumstances that the relative performance between different bounds was good enough for prediction.

C. Condition Number Based Selection

The selection in *Selection Criterion 2* requires the computation of the post-processing SNR for each symbol stream in each case of diversity and multiplexing. It would be nice to discover the characteristics of the channel that make it good for either multiplexing or diversity to obtain some insight on the diversity-multiplexing tradeoff as well as to further simplify the selection criterion. Since we are using linear receivers, intuitively, the invertibility of the channel plays a role. However, how does the required data rate fit in?

To answer this question, we consider the case of spatial multiplexing with the ZF receiver. Let $\{\lambda_i(\mathbf{H})\}_{i=1}^{M_t}$ denote the nonzero singular values of $\mathbf{H}$ in decreasing order. Then, from [7]

$$\begin{aligned}\max_{1\leq k\leq M_t}[\mathbf{H}^H\mathbf{H}]^{-1}_{kk} &= \max_{1\leq k\leq M_t}\mathbf{e}_k^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{e}_k \quad (8)\\ &\leq \max_{\|x\|^2=1}\mathbf{x}^H[\mathbf{H}^H\mathbf{H}]^{-1}\mathbf{x}\\ &= \lambda_{\max}([\mathbf{H}^H\mathbf{H}]^{-1})\\ &= \lambda^{-2}_{\min}(\mathbf{H}) \quad (9)\end{aligned}$$

where $\mathbf{e}_k$ is the kth column of $\mathbf{I}_M$, and (9) follows from the Rayleigh–Ritz theorem [45]. Equality occurs when the right singular vector corresponding to the minimum singular value is a multiple of one column of the identity vector. Using (9), it follows that

$$\min_{1\le k\le M_t} \mathrm{SNR}_k^{(\mathrm{ZF})}(M_t,1) \ge \frac{E_s}{N_0}\lambda_{\min}^2(\mathbf{H}). \tag{10}$$

Thus, the performance of the ZF receiver depends on the minimum singular value of the channel (for other dependencies on the minimum singular value, see [46]). Similarly, it can be shown that the minimum SNR performance of a linear MMSE receiver also is lower bounded by the channel's minimum singular value [11].

In a similar manner, let us now bound the performance with selection diversity. Let $\mathbf{h}_k$ be the kth column of $\mathbf{H}$. Observe that

$$\begin{aligned}\max_{1\le k\le M_t} \|\mathbf{h}_k\|^2 &= \max_{1\le k\le M_t} \mathbf{e}_k^H[\mathbf{H}^H\mathbf{H}]\mathbf{e}_k \\ &\le \max_{\|x\|^2=1} \mathbf{x}^H[\mathbf{H}^H\mathbf{H}]\mathbf{x} \\ &= \lambda_{\max}^2(\mathbf{H})\end{aligned} \tag{11}$$

through a similar argument as before. Thus, we have

$$\max_{1\le p\le M_t} \mathrm{SNR}_1^{(\mathrm{ZF})}(1,p) \le \frac{E_s}{N_0}\lambda_{\max}^2(\mathbf{H}). \tag{12}$$

Note that the beamformer that achieves equality in (11) is the right singular vector corresponding to the maximum singular value of $\mathbf{H}$. Therefore, (12) is achieved with optimal combining at the transmitter. We could approach this bound as close as desired by increasing the number of vectors in $\mathcal{W}_1$ using the quantization-based approach of [25].

Now, we have tight bounds in (11) and (12) of the opposite direction. This is not really a problem because we can use the bounds in the following way. Suppose that we select multiplexing only when the worst possible minimum SNR is better than the best possible maximum SNR achieved through selection diversity. This is a conservative selection policy since we will not choose multiplexing as often as with optimal selection. It does, however, afford a nice simplification. Using (11) and (12), substituting into *Selection Criterion 2* and simplifying allows us to propose the following elegant criterion.

Selection Criterion 3—Condition Number-Based Selection: Let $\kappa(\mathbf{H}) = \lambda_{\max}(\mathbf{H})/\lambda_{\min}(\mathbf{H})$ be the regular condition number of the channel $\mathbf{H}$. Choose multiplexing if

$$\kappa(\mathbf{H}) \le \frac{d_{\min}(M_t,R)}{d_{\min}(1,R)}. \tag{13}$$

Otherwise, choose diversity transmission from the best transmit antenna.

Numerically stable algorithms for computing $\kappa(\mathbf{H})$ are available in the matrix computation literature (see, e.g., [47] for details).

The beauty of *Selection Criterion 3* is that we can relate the quality of the channel, in this case the condition number, directly to the minimum distance of the constellations as a function of the rate. This provides substantial intuition about dual-mode selection. Specifically, allowing a diversity mode in the system improves robustness to ill-conditioning of the channel. The reason is that the diversity mode depends only on the power that can be received by the best transmit antenna and is invariant to the condition number of the channel. A similar result, although it uses a different condition number, was obtained in [9] by examining the tradeoff between transmit diversity and spatial multiplexing with maximum likelihood receivers.

It is interesting to question what happens to the condition number threshold in (13) for a fixed M_t, except that the rate R increases. To do this, we need to understand how the ratio $d_{\min}^2(M_t,R)/d_{\min}^2(1,R)$ behaves as R increases. For this calculation, we adopt the approach of [10] and suppose that both diversity and multiplexing use a constellation family with a minimum distance that is only a function of the rate, i.e., the number of points in the constellation. Let $g(R)$ be the squared distance of the constellation family as a function of the data rate, and suppose that the constellation energy is normalized to one. Therefore, $d_{\min}^2(1,R) = g(R)$, and $d_{\min}^2(M_t,R) = g(R/M_t)/M_t$. The reason for the factor of M_t in the latter case is that we assumed that the per-antenna constellation was normalized so that the total transmit energy across all the transmit antennas is one.

If the dimensionality of the constellation family is fixed, then the minimum distance between points decreases as the number of points in the constellation increases. Thus, $g(R/M_t)/g(R)$ is increasing since $g(R) \le g(R/M_t)$. For example, for the QAM constellation family

$$g(R) = \frac{6}{(2^R-1)}. \tag{14}$$

Therefore, for large R, $g(R/M_t)/g(R) \approx 2^{R(1-1/M_t)}$, which is clearly increasing as R increases. In conclusion, when the rate increases, the channels do not need to be "as invertible." Thus, spatial multiplexing is used more often. This is reasonable since the capacity of a MIMO system is fundamentally larger than the capacity of a system with antenna selection.

V. Multimode Transmission

In the previous section, we restricted ourselves to choosing between selection diversity and spatial multiplexing for a given channel realization. In this section, we allow selection from the complete set of subsets $\{\mathcal{W}_m\}_{m=1}^{M_t}$. This allows the freedom to adopt approaches that are in between single antenna selection diversity and full spatial multiplexing. We propose various optimal and suboptimal selection schemes based on the error rate as well as an eigenmode analysis.

A. Multimode Selection From Error Probability Considerations

In the general case of multimode selection, the receiver must select both the number of data streams M as well as optimal precoding matrix within $\mathcal{W}_M$. The total number of distinct feedback messages that are required is

$$\sum_{m=1}^{M_t}\binom{M_t}{m} = 2^{M_t}-1 \tag{15}$$

which can be implemented using M_t bits of the feedback control channel. Thus, the number of bits of feedback that are required scales linearly with the number of transmit antennas. In contrast, consider quantizing the channel and conveying the coefficients back to the transmitter. If k bits are used to quantize each real or imaginary component, a total of $2M_tM_rk$ bits are required for feedback. Sending back only the sign (one bit per coefficient) still requires $2M_tM_r$ bits—a significant penalty even with an extremely coarse, and potentially not worthwhile, quantization. Dual-mode selection requires only $\lceil \log_2(M_t+1) \rceil$ bits of feedback.

The optimal fixed rate selection metric given perfect channel knowledge is to choose the mode that provides the lowest probability of error. The following theorem on the achievable diversity order in Rayleigh fading channels, which follows directly from Theorem 1, is provided for completeness.

Theorem 2: Selection of the optimal M^* and p^* such that $\mathbf{W}_{M^*,p^*} \in \mathcal{W}_{M^*}$ minimizes the conditional probability of error provides full diversity advantage.

Proof: Selection diversity provides full diversity advantage on the order of M_tM_r (cf. [43]). Since selection diversity is included among the subsets, optimal selection can only be better than using single-antenna selection diversity for all channel realizations. □

Both dual-mode and multimode selection ideally provide full diversity advantage. We will show that the difference in performance can be characterized as an array gain in Section VIII.

To obtain a close but computable approximation to optimal selection, we rely on the NNUB on the symbol error rate to obtain a form of optimal selection that does not require significant numerical integration. Remembering the definition of $\mathrm{SNR}_k(M,p)$, we propose an approximate VSER criterion by exploiting the generality of (7).

Selection Criterion 4—Complete VSER-Based Selection: Choose M^* and p^* that solve

$$\arg \min_{1\le M\le M_t, 1\le p\le \binom{M_t}{M}} 1-\left(1-N_e(M,R) \times Q\left(\sqrt{\min_{1\le k\le M} \mathrm{SNR}_k(M,p)\frac{d^2_{\min}(M,R)}{2}}\right)\right)^M$$

where $M=1,2,\ldots,M_t$, and $p=1,2,\ldots,\binom{M_t}{M}$.

The minimization can be carried out with a brute-force search. For implementation, $N_e(M,R)$ and $d^2_{\min}(M,R)$ should be precalculated for all M and the Q function approximated using an accurate polynomial method (see, e.g., [44]). Again, the selection could be simplified using the approximations to the NNUB and Q function.

When complexity is an issue, it is more direct to make the selection directly in terms of post-processing SNR and the minimum distance of the constellation by neglecting the effects of the nearest neighbors on the constellation. In this case, we propose the following suboptimal selection based on the worst post-processing SNR.

Selection Criterion 5—Complete SNR-Based Selection: Choose M^* and p^* that solve

$$\arg \max_{1\le M\le M_t, 1\le p\le \binom{M_t}{M}} \min_{1\le k\le M} d^2_{\min}(M,R)\mathrm{SNR}_k(M,p).$$

For the brute force search, a total of 2^M-2 comparisons need to be made. As before, the effectiveness of SNR-based selection depends on the ability of the NNUB to provide an accurate relative prediction of vector symbol error probability. Since the NNUB is only reasonably tight at high SNR, we expect that SNR-based selection will yield better results as the SNR grows large.

B. Eigenmode-Based Selection

In the previous section, it was possible to relate dual-mode selection to the condition number of the matrix channel. The intuition in the case of multimode selection is that the "array gain" should be improved, in a sense, by providing more tolerance to ill-conditioned channels. The reason is that by allowing more flexibility in how we map data to the antennas, we should be able to better couple energy into the eigenmodes of the channel. To understand this intuition, we study the impact on the improvement of the eigenmodes of the equivalent channel and generate a suboptimal but lower complexity selection criterion as a byproduct.

Consider the ZF receiver. The performance is determined by the smallest singular value of $\mathbf{HW}_{M,p}$. For a given M, the question is how to bound the best of the worst singular values. It turns out that we can solve this problem by leveraging a result from matrix theory, and the general result is summarized in the following theorem.

Theorem 3: Let $\lambda_1(\mathbf{H}), \lambda_2(\mathbf{H}), \ldots, \lambda_{M_t}(\mathbf{H})$ be the singular values of $\mathbf{H}$ arranged in decreasing order. Given M, the following is true:

$$\lambda^2_M(\mathbf{H}) \ge \lambda^2_M(\mathbf{HW}_{M,p}) \ge \lambda^2_{M_t}(\mathbf{H}) \tag{16}$$

for all $p \in \{1,\ldots,M_t\}$.

Proof: The proof follows from the Poincare Separation Theorem [45, p. 190] by recognizing the fact that $\mathbf{W}_{M,p} \in \mathcal{W}_M$ is a subset of the set of all $M_t \times M$ unitary matrices. Equality on the left occurs when $\mathbf{W}_{M,p}$ is a permutation of the M dominant right singular vectors of $\mathbf{H}$. Equality on the right occurs when $\mathbf{W}_{M,p}$ is a permutation of the M least dominant right singular vectors of $\mathbf{H}$, assuming the two largest singular values are unique. □

The upper bound in Theorem 3 is the one of interest for the present discussion. This is indicates how well we could improve the worst singular value if it were possible to select the best unitary matrix of dimension $M_t \times M$ with unit norm columns. The upper bound is achievable when $\mathbf{W}_{M,p}$ consists of the M columns of the right singular vector matrix of $\mathbf{H}$ that correspond to the M largest singular values. Similarly, when $\mathbf{W}_{M,p}$ equals the M singular vectors corresponding to the M smallest singular values of $\mathbf{H}$,, the lower bound is achieved.

Practically, the restriction to the columns of the identity matrix will degrade performance relative to optimum in much the same way that selection diversity is worst than maximum ratio combining. We can decrease the gap by increasing the number of elements of $\mathcal{W}_M$ (see [27] for details). Thus, in what follows, we use $\lambda_M(\mathbf{H})$ to approximate the performance from selection of the optimal subset of $\mathcal{W}_M$.

A straightforward eigenmode selection rule would solve for the $\{M^*, p^*\}$ that maximize the minimum singular value. This requires computing $\lambda_{\min}(\mathbf{HW}_{M,p})$ for all possible $\mathbf{W}_{M,p} \in \mathcal{W}_M$. This is arguably less complex than the SNR- or NNUB-based approaches but can still be quite intensive even for moderate M_t. Using the upper bound in (16), however, we propose a suboptimal, low-complexity selection criterion by solving first for M^* using Theorem 3 and then for the $p^* \in \{1, 2, \ldots, M^*\}$ that most closely achieves the optimum.

Selection Criterion 6—Eigenmode Based Selection: Choose M^* such that

$$M^* = \arg \max_{1 \leq M \leq M_t} \lambda_M^2(\mathbf{H}) d_{\min}^2(M, R) \tag{17}$$

and then, find the p^* that solves

$$p^* = \arg \max_{1 \leq p \leq \binom{M_t}{M^*}} \lambda^2(\mathbf{H}\mathbf{W}_{M^*,p}). \tag{18}$$

The real advantage of *Selection Criterion 6* is that only the singular value decomposition of $\mathbf{H}$ and, at most, $\binom{M_t}{\lfloor M_t/2 \rfloor}$ minimum singular values are needed instead of all $2^{M_t}-1$ minimum singular values. The tradeoff is, of course, accuracy since the assumption is that $\mathcal{W}_M$ is sufficiently dense, which may break down in the cases considered.

Selection of M based on the singular values of $\mathbf{H}$ provides intuition about multimode selection in low-rank channels. Specifically, if $\mathbf{H}$ is of rank r, there is no reason to consider any $M > r$ since the resulting matrix will be low rank and at least one substream lost in the null space. Notice, however, that depending on the distribution of the remaining singular values, $M < r$ may be optimal, depending on R and the constellations according to *Selection Criterion 6*.

VI. Probabilistic Analysis

The selection criteria derived in Sections IV and V are based on the conditional error probability and, thus, are valid for any channel distribution, as long as the channel is flat-fading. In this section, we use the complex Gaussian assumption about $\mathbf{H}$ to study the probabilistic behavior of systems with dual-mode and multimode selection. We will study the dual-mode selection probability, which is the average number of times diversity or multiplexing is used as a function of rate. Then, we will consider the solution of the optimum M^* as a function of rate in the multimode case. We use exact and asymptotic results from the theory of random matrices where needed.

A. Dual-Mode Selection Probability

Consider the case of dual-mode transmission as described in Section IV. Suppose that the condition number based selection method is used as described in *Selection Criterion 3*. For this criterion, we would like to compute the probability that each mode is selected. This provides useful information for the system designer about how often each mode will be used in the system and, thus, the frequency of feedback.

Expressed formally, the objective is to compute $\Pr\{\kappa(\mathbf{H}) \leq (d_{\min}(M_t, R))/(d_{\min}(1, R))\}$ based on our complex Gaussian assumption about the statistics of $\mathbf{H}$. In the case of $M_t = 2$, we can use an exact expression for the probability density function of $\kappa(\mathbf{H})$ from [48, p. 72] given by

$$f(x) = 2\frac{\Gamma(2M_r)}{\Gamma(M_r)\Gamma(M_r-1)} \frac{x^{2M_r-3}(x^2-1)^3}{(x^2+1)^{2M_r}}$$

to calculate $\Pr\{\kappa(\mathbf{H}) \leq (d_{\min}(M_t, R))/(d_{\min}(1, R))\}$ exactly in terms of gamma and hypergeometric functions. The resulting equation is long; thus, we omit the final functional form (it can be evaluated with Mathematica).

Beyond $M_t = 2$, the exact distribution for the condition number does not seem to be available. For reasonably sized M_t, it is possible to use Monte Carlo simulation methods to numerically estimate this distribution and then compute the corresponding probability.

Example 1: Consider the phase shift keying (PSK) family of constellations with the ratio $d_{\min}(M_t, R)/d_{\min}(1, R) = \sin(\pi/2^{R/M_t})/\sqrt{M_t}\sin(\pi/2^R)$. In Fig. 3, we illustrate the switching probability as a function of R for various choices of $M_t = M_r$ obtained via Monte Carlo simulation, assuming phase shift keying (PSK) constellations. While the minimum distance functions for a given constellation family are only defined for integer rates, we plot the rate variation as a smooth function for illustrative purposes. We see another interesting indication of the effect of rich scattering MIMO Rayleigh fading channels when the transmit and receive array sizes increase. In particular, the expected value of the condition number grows larger and larger as M_t and M_r increase. The 0.4 crossing point shifts from 4 bits per channel use for $M_t = M_r = 2$ to 7 bits per channel use for $M_t = M_r = 8$.

For the purposes of developing intuition, we use asymptotic results that are available from the random matrix literature. First, consider the case where $M_t = M_r$ and M_t grows large. Then, based on [48]

$$Pr\left\{\frac{\kappa(\mathbf{H})}{M_t} < x\right\} \approx e^{-4/x^2}. \tag{19}$$

Therefore

$$\Pr\left\{\frac{\kappa(\mathbf{H})}{M_t} < \frac{d_{\min}(M_t, R)}{M_t d_{\min}(1, R)}\right\} \approx e^{\frac{-4M_t^2 d_{\min}^2(1,R)}{d_{\min}^2(M_t,R)}}. \tag{20}$$

Now, to obtain some insight, we again assume that the QAM constellation family with squared minimum distance $g(R)$ is used for transmission. Then, the ratio $d_{\min}^2(1, R)/d_{\min}^2(M_t, R) = M_t g(R)/g(R/M_t)$, and

$$\Pr\left\{\frac{\kappa(\mathbf{H})}{M_t} < \frac{d_{\min}(M_t, R)}{M_t d_{\min}(1, R)}\right\} \approx e^{-4M_t^3 g(R)/g(R/M_t)}. \tag{21}$$

The expression in (21) provides one very important point. *As M_t increases, the probability of choosing spatial multiplexing decreases*, assuming that R is fixed. This follows because $M_t^3 g(R)/g(R/M_t)$ is increasing as a function of M_t under some general assumptions [49]. In fact, this is what we would expect, given the results in Fig. 3 and Example 1. As the number of entries in the matrix channel grows, the largest singular value will grow. Thus, we would expect the matrix channel found between large antenna arrays to be highly accommodating to selection diversity.

Now, for the case where $M_r > M_t$, suppose that $M_t = \alpha M_r$, where $0 < \alpha < 1$. Then, asymptotically, it can be shown [48] that $\kappa(\mathbf{H})$ converges almost surely to $(1+\sqrt{\alpha})/(1-\sqrt{\alpha})$. Using this result, we can derive, for illustrative purposes, an equivalent deterministic selection criteria as a function of α and

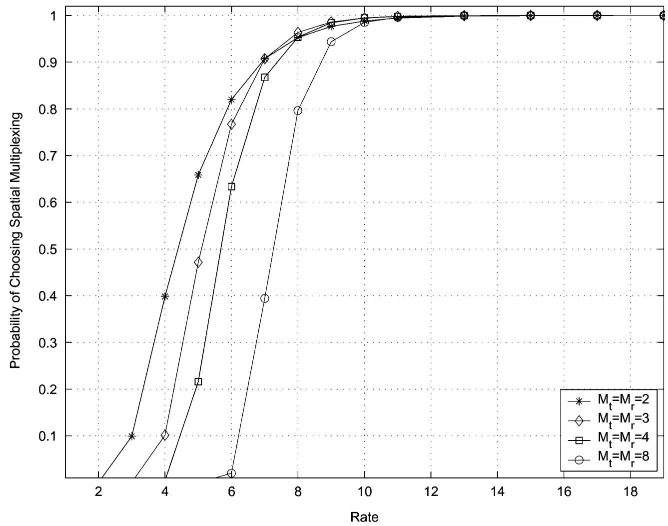

Fig. 3. Plot of the probability of selecting spatial multiplexing for the PSK constellation family with $\mathrm{R} = 2M_t$ and $M_t = M_r = 2, 3, 4, 8$.

$d_{\min}(M_t, R)/d_{\min}(1, R)$. With this asymptotic selection criteria, spatial multiplexing is used if

$$\frac{1+\sqrt{\alpha}}{1-\sqrt{\alpha}} \leq \frac{d_{\min}(M_t, R)}{d_{\min}(1, R)}. \tag{22}$$

This rule simplifies to choosing spatial multiplexing if

$$\frac{\frac{d_{\min}(M_t,R)}{d_{\min}(1,R)} - 1}{\frac{d_{\min}(M_t,R)}{d_{\min}(1,R)} + 1} \geq \sqrt{\alpha}. \tag{23}$$

This result provides the intuition that for small M_t/M_r, we are more likely to choose spatial multiplexing than for M_t/M_r close to one when M_t is large. Thus, the receive size of the antenna array plays an important role in the choice of an optimal transmission scheme.

B. Multimode Selection Probability

Now, consider the case of multimode transmission as described in Section V. Suppose that we use eigenmode selection as described in *Selection Criterion 6*. For this criterion, we would like to compute the probability that mode M^* from a possible $M = 1, 2, \ldots, M_t$ modes is selected. This provides a measure of utilization of each mode of the system. In effect, it describes the balance between diversity, multiplexing, and the modes in between. The behavior of this probability as M_t grows large is interesting in its own right since it gives a more granular twist on the multiplexing versus diversity issue.

Let $f(\mu_1, \mu_2, \ldots, \mu_{M_t})$ be the joint distribution of the ordered eigenvalues of $\mathbf{H}^H\mathbf{H}$ (with respect to the Lebesgue measure on $\mathbb{R}^{M_t}$) arranged in increasing order. Recall that we assume $M_t \leq M_r$. For the complex Gaussian case distribution, relying on results from [50] and [51]

$$\begin{aligned} f(\mu_1, \mu_2, \ldots, \mu_{M_t}) &= \frac{1}{c}\left[\prod_{1\leq i<j\leq M_t}(\mu_i-\mu_j)^2\right] \\ &\quad\times\left[\prod_{j=1}^{M_t}\mu_j\right]^{M_r-M_t} e^{-\sum_{j=1}^{M_t}\mu_j} 1_{+}(\mu_1, \mu_2, \ldots, \mu_{M_t}) \end{aligned}$$

where

$$\begin{aligned} &1_{+}(\mu_1, \mu_2, \ldots, \mu_{M_t}) \\ &\quad = \{(\mu_1, \mu_2, \ldots, \mu_{M_t}) \in \mathbb{R}^n \mid 0 \leq \mu_1 \leq \mu_2 \cdots \leq \mu_{M_t}\} \end{aligned}$$

and $c = [\prod_{j=1}^{M_t}(M_t-j)!(M_r-j)!]$.

Now, remembering that the results in Section V-B were stated in terms of the singular values, the probability mass function of interest $\Pr_{R,M_t}\{M\}$, the probability of M given R, and M_t, are given by

$$\Pr\left\{\lambda_M^2 d_{\min}^2(M,R) \geq \lambda_k^2 d_{\min}^2(k,R) \quad \text{for } k = 1, 2, \ldots, M_t, k \neq M\right\}$$

which is equal to

$$\Pr\left\{\lambda_M^2(\mathbf{H}) \geq \lambda_k^2(\mathbf{H}) \frac{d_{\min}^2(k,R)}{d_{\min}^2(M,R)} \quad \text{for } k = 1, 2, \ldots, M_t, k \neq M\right\}.$$

Evaluating this probability requires integrating, over the appropriate subspace, (24), shown at the bottom of the page. Beyond special cases, it is difficult to compute (24) in closed form. Therefore, we resort to numerical methods to evaluate the integral or preferably Monte Carlo methods to estimate the probability mass function.

Example 2: Consider the QAM family of constellations. In Fig. 4, we illustrate $Pr_{R,M_t}\{M\}$ for various choices of R and M_t with $M_r = M_t$. The QAM minimum distance expression in (14) and Selection Criterion 6 were used. This allows the selection to be plotted independently of the SNR.

First, notice, as was similarly observed in Example 1, that for a fixed M_t, the number of most probable substreams increases as the rate increases. This effect is illustrated for $M_t = 4$ and for $M_t = 8$. We conjecture that as $R \to \infty$, $M \to M_t$. This would relate, in some sense, to Telatar's outage probability conjecture in [52].

In addition, note that for a fixed rate, the number of most probable substreams increases as M_t increases. This effect is a direct result of our assumption of a rich scattering environment. This, however, is not an indication that $M \to M_t$ as $M_t \to \infty$.

$$\int_{\lambda_M^2=0}^{\infty}\int_{\lambda_1^2=0}^{\lambda_M^2\frac{d_{\min}^2(M,R)}{d_{\min}^2(1,R)}} \cdots \int_{\lambda_{M_t}^2=0}^{\lambda_M^2\frac{d_{\min}^2(M,R)}{d_{\min}^2(M_t,R)}} f(\mu_1, \mu_2, \ldots, \mu_{M_t}) d\mu_M d\mu_1 \cdots d\mu_{M_t}. \tag{24}$$

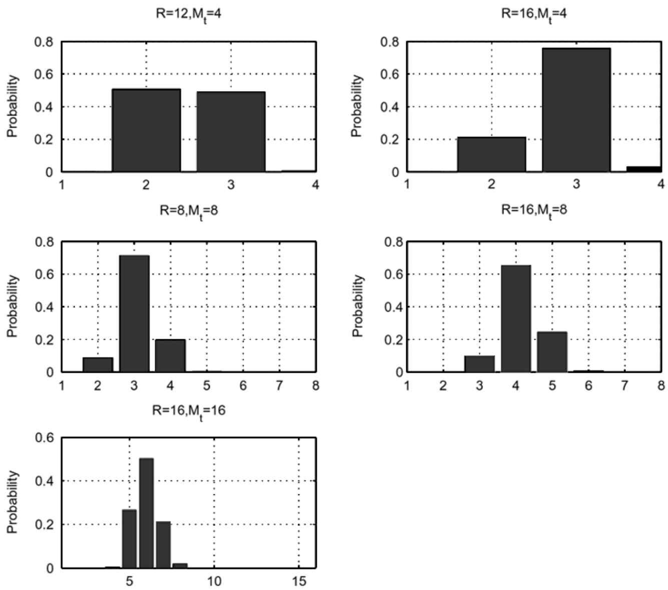

Fig. 4. Plot of the multimode selection probability for the QAM constellation family and $M_t = M_r = 4, 8, 16$ for various choices of R. Notice how the probability clusters around a subset of available modes.

Another important observation is that each of the plots have modes that have approximately zero selection probabilities. This is an indication that it may often be more practical to restrict M to a subset of $\{1, 2, \ldots, M_t\}$. This would reduce the require amount of feedback and the transmitter complexity.

C. Reduced-Complexity Mode Selection

The probabilistic results in Sections VI-A and B can be used for complexity reduction in many situations. In this case, the possible modes can be restricted to lie in a set $\mathcal{M} \in \{1, 2, \ldots, M_t\}$. The optimal substream to antenna mapping would be selected from within $\bigcup_{m \in \mathcal{M}} \mathcal{W}_m$.

Restricting the mode to lie in a subset would especially be beneficial in spatially correlated situations such as those being studied in the IEEE 802.11N work group [53]. In general, a spatially correlated channel will bias the modal probability distribution to a subset of $\{1, 2, \ldots, M_t\}$. Using probabilistic techniques, feedback and implementation complexity can thus be dramatically reduced.

VII. Application to Nonlinear Receivers

While the results presented in Sections IV and V were tailored for use with linear receivers, dual-mode and multimode antenna selection are also applicable to nonlinear receivers.

The V-BLAST decoder is a successive cancellation decoder that can provide error rate improvements over linear receivers without a substantial complexity penalty [54]. Just as with linear receivers, the performance of V-BLAST decoding is a function of the minimum substream SNR denoted by $\mathrm{SNR}_{\min}^{(\mathrm{VB})}$. It was shown in [11] that the minimum SNR can be bounded as

$$\mathrm{SNR}_{\min}^{(\mathrm{VB})}(M, p) \geq \frac{E_s}{N_0} \lambda_{\min}^2(\mathbf{H}\mathbf{W}_{M,p}). \qquad (25)$$

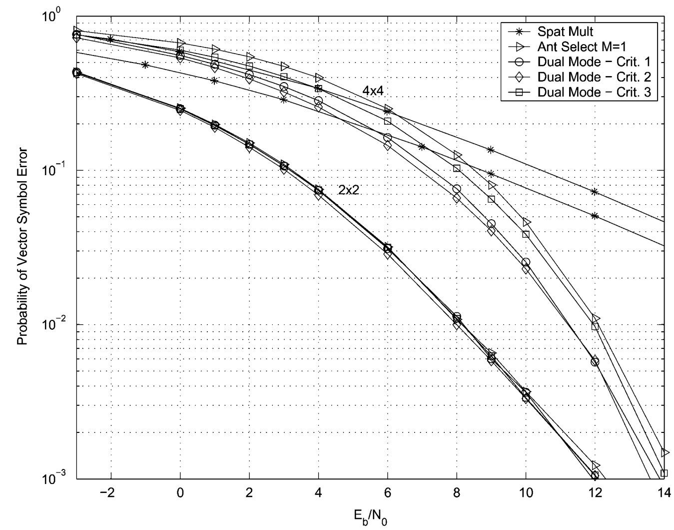

Fig. 5. Error rate performance of dual-mode selection for a $M_r = M_t = 2 (R = 4)$ and a $M_r = M_t = 4 (R = 8)$ system compared with single transmit antenna selection and spatial multiplexing.

To maximize this lower bound, we can use *Selection Criterion 3* for dual-mode and *Selection Criterion 3* for multimode antenna selection.

In contrast to V-BLAST and linear receiver decoders, joint ML decoding across all substreams provides optimal performance at a possibly severe complexity penalty. ML performance is primarily a function of the receive minimum distance, which is defined as

$$d_{\min,\mathrm{rec}}(M, p) = \min_{\mathbf{s}' \neq \mathbf{s}''} \|\mathbf{H}\mathbf{W}_{M,p}(\mathbf{s}' - \mathbf{s}'')\|_2. \qquad (26)$$

This can be lower bounded by the case when the minimum error vector $(\mathbf{s}' - \mathbf{s}'')$ is collinear to the minimum singualar vector direction of $\mathbf{H}\mathbf{W}_{M,p}$, yielding

$$d_{\min,\mathrm{rec}}^2(M, p) \geq \frac{E_s}{N_0} \lambda_{\min}^2(\mathbf{H}\mathbf{W}_{M,p}) d_{\min}^2(M, R). \qquad (27)$$

The bound in (27) can thus be maximized using *Selection Criterion 3* with dual-mode antenna selection and *Selection Criterion 6* with multimode antenna selection.

VIII. Simulations

In this section, we provide some Monte Carlo simulations of the vector symbol error rate for dual-mode and multimode antenna selection under different selection criteria. We used an i.i.d. Rayleigh fading model, as discussed earlier. The plots use the average per bit SNR $E_b/N_0 = E_s/(R \cdot N_0)$.

Experiment 1: In this experiment, we simulated dual-mode antenna selection for both 2×2 and 4×4 systems with ZF decoding using the various criteria proposed. For the 2×2 system, we fixed $R = 4$ bits per channel use, whereas for the 4×4 system, we fixed $R = 8$ bits per channel use. The results, which are shown in Fig. 5, provide insight into how each criteria performs. For $M_t = 2$,, there is little difference between the performance between the three different criteria. However, the $M_t = 4$ system shows a pronounced difference between *Selection Criterion 1-2* and *Selection Criterion 3*. The difference is almost 1 dB

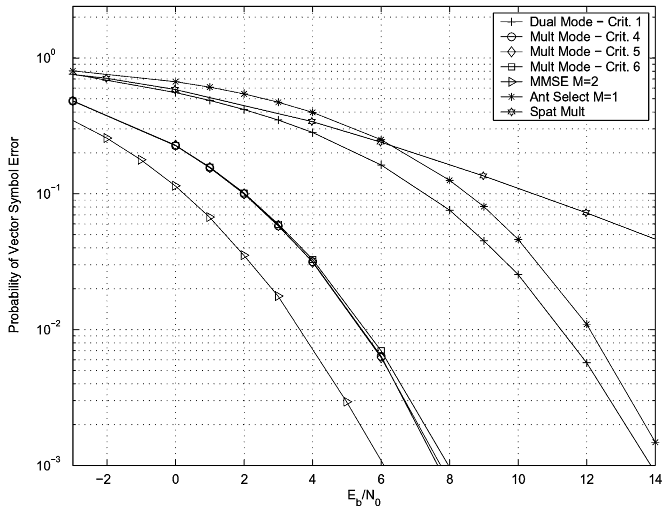

Fig. 6. Error rate performance of multimode selection for a $M_r = M_t = 4 (R = 8)$ system compared with dual-mode selection, single transmit antenna selection, spatial multiplexing, and optimal MMSE precoding proposed by Scaglione *et al.* Multimode selection significantly outperforms dual-mode selection and approaches the performance obtained with optimal precoding.

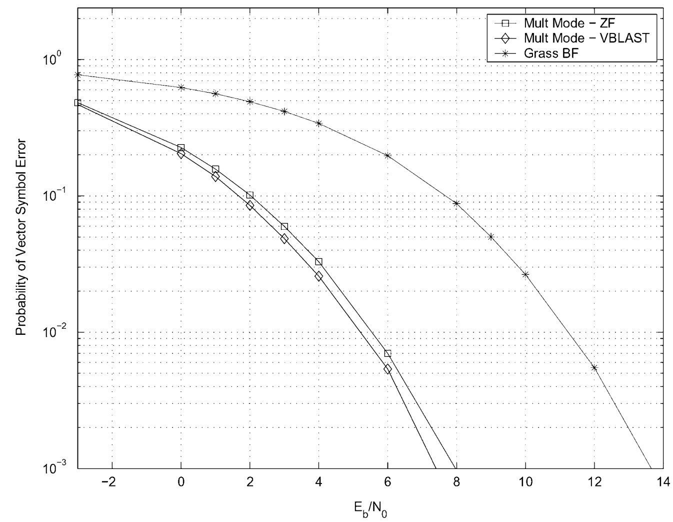

Fig. 7. Error rate performance comparison of multimode selection for a $M_r = M_t = 4 (R = 8)$ system with a ZF linear receiver and a V-BLAST receiver and performance of limited feedback beamforming.

between them, whereas the difference between the first two is much smaller. This is indicative of the fact that *Selection Criterion 3* is only an approximate minimizer of the probability of vector symbol error. Compared with plain spatial multiplexing, it is clear that dual-mode selection substantially improves the diversity order with a minimal amount of feedback with a gain of about 6 and 3.5 dB at 10^{-1} for $M_t = 2$ and $M_t = 4$, respectively.

Selection diversity is plotted to show the diversity order of the dual-mode scheme. All three criteria provide the full diversity order of $M_t M_r$, verifying that the results of Theorem 1 apply even with our suboptimal selection criteria. Note that the benefit of dual-mode antenna selection is not fully evident in the $M_t = 2$ system because the probability of choosing selection diversity is so high in this scenario. The benefit of dual-mode antenna selection is more pronounced for the $M_t = 4$ system, where the gain provided by dual-mode antenna selection is approximately 1.5 dB over just transmit selection.

This is also an important indication that *the performance of dual-mode antenna selection is highly dependent on the number of transmit antennas*. We expect the dual-mode array gain over selection diversity to continue to increase as M_t increases. This would be expected because the most probable modes usually strictly satisfy $1 < M < M_t$.

Experiment 2: In this experiment, we considered *Selection Criteria 4–6* for multimode antenna selection on a 4×4 system using ZF decoding with $R = 8$ bits per channel use. We used the mode set $\{1, 2, 4\}$ with 256-QAM, 16-QAM, and 4-QAM. The results are shown in Fig. 6. For comparison, we show single transmit antenna selection with $M = 1$ as well as spatial multiplexing with $M = 4$ and a ZF receiver. By comparing with the transmit antenna selection curve, we see that all the selection criteria provide full 16th-order diversity, verifying that we can apply the results of Theorem 2 to our suboptimal selection strategies. Of particular interest is that *Selection Criteria 4-6* all yield approximately identical probability of vector symbol error performance. Thus, we can choose the lowest complexity solution with little degradation in error rate performance. The diversity gain gives multimode selection a substantial 8-dB improvement over standard spatial multiplexing at 10^{-1}.

For comparison with dual mode transmission, we plot the results of using *Selection Criterion 1*. This is the best dual-mode criterion, but it still performs more than 5 dB away from any of the three multimode selections. This shows, as we conjectured, that multimode selection would provide additional array gain.

To see how multimode antenna selection performs in comparison with an optimal infinite precision feedback system, MMSE precoding using an MMSE linear receiver with the trace cost function (as proposed by Scaglione *et al.* in [23]) was simulated. This system uses an MMSE receiver and a transmit precoder that assumes perfect channel knowledge at the transmitter. Multimode selection performs within 2 dB of two-substream MMSE precoding. We chose the best number of substreams for the MMSE to provide the most fair comparison. This difference illustrates the tradeoff between limited feedback antenna selection and unlimited feedback precoding. The 2-dB penalty comes with a system architecture that is substantially easier to implement. This result is quite striking, given that MMSE precoding i) uses *perfect* channel knowledge, ii) allows waterfilling among substreams, and iii) uses a more complex linear receiver.

Experiment 3: The performance of $R = 8$ bit multimode antenna selection using *Selection Criterion 6* with a V-BLAST receiver and $M_t = M_r = 4$ is examined in this experiment. For comparison, 256-QAM limited feedback beamforming from [25] using a 4-bit codebook and an $R = 8$ bit multimode antenna selection with a ZF receiver are shown. The results are presented in Fig. 7. Multimode precoding with a linear receiver provides more than a 5.5-dB improvement over beamforming. Employing a V-BLAST receiver instead of a linear ZF receiver adds another 0.5 dB. The reason that the V-BLAST receiver does not provide a substantial improvement in performance is that multimode precoding already provides the linear ZF receiver with the full diversity available. Thus, V-BLAST cannot provide additional diversity advantage and further suffers from error propagation, unlike the ZF receiver. This experiments

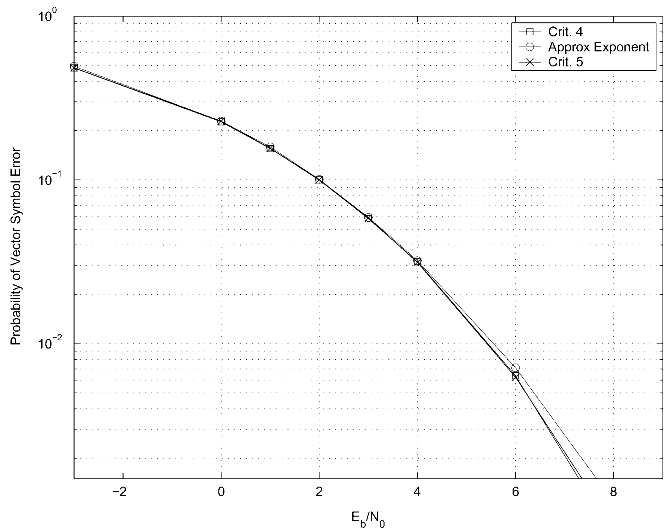

Fig. 8. Error rate performance comparison of multimode selection with various approximations to error rate selection.

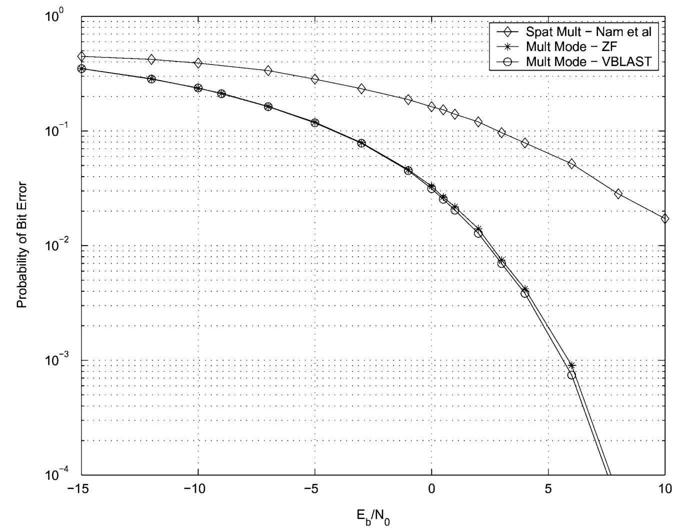

Fig. 10. Bit error rate performance comparison of multimode selection with the power loading algorithm in [57].

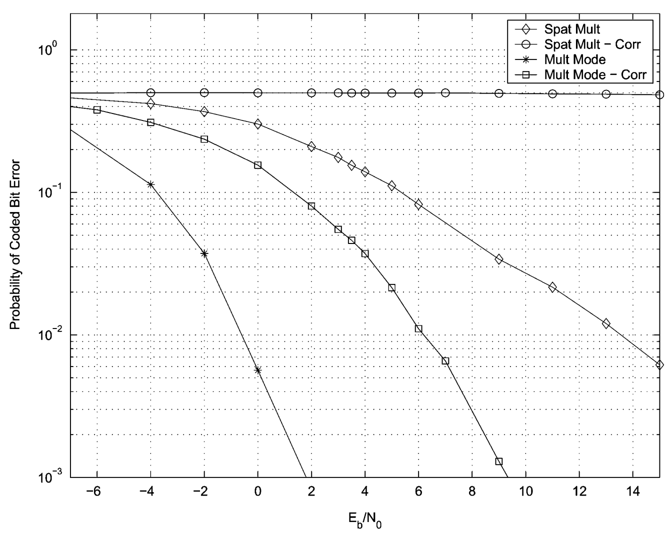

Fig. 9. Coded bit error rate performance comparison of multimode selection and spatial multiplexing using a convolutional outer code.

indicates the validity of *Selection Criterion 6* for use with nonlinear receivers.

Experiment 4: Although *Selection Criterion 5* is an approximation to *Selection Criterion 4*, there are other approximations that could be used. Fig. 8 shows the performance with the approximation $1-(1-\beta)^M \approx M\beta$. This rids *Selection Criterion 4* of the exponent computation. All three criteria perform approximately the same. Interestingly, *Selection Criterion 5* actually outperforms the exponential approximation at high SNR.

Experiment 5: The analysis in this paper was geared primarily to optimizing the uncoded vector SER. Real systems, however, use outer codes to account for variation across the eigenmodes. Fig. 9 demonstrates the coded bit error rate performance of 4×4 multimode antenna selection using *Selection Criterion 6* and 4×4 spatial multiplexing. The outer code was a constraint length two convolutional code with an ideal random interleaver. A ZF linear receiver was used with a soft Viterbi decoder. The system was simulated on the spatially uncorrelated Rayleigh fading channel and on the transmit-receive Rayleigh correlated channel considered for IEEE 802.11N [55]. The transmit and receive correlations were taken from the "micro correlated" model in [56]. With a spatially uncorrelated channel, the multimode system provides approximately a 14-dB improvement over spatial multiplexing. The improvement is even more dramatic with the realistic correlated channel model. In the correlated case, spatial multiplexing exhibits a bit error rate of approximately 0.5 for all SNR values. This is demonstrative of the problems encountered by spatial multiplexing in low rank channels.

Experiment 6: This experiment, which is shown in Fig. 10, compares 4×4 multimode antenna selection with $R = 8$ bits and *Selection Criterion 6* to power loaded spatial multiplexing, as proposed in [57]. The multimode performance is shown with a linear ZF receiver and a V-BLAST receiver. The system in [57] assumes a V-BLAST detector. At an error rate of 10^{-2}, the multimode systems provide more than an 8-dB improvement over the algorithm in [57]. As in Experiment 3, the gain of V-BLAST over the ZF receiver is marginal for the same reasons as noted in Experiment 3. Note that the power loading algorithm *is not* a limited feedback algorithm since perfect knowledge of the substream powers is assumed. Thus, multimode antenna selection can be implemented in a limited feedback scenario with a large gain.

IX. Summary and Conclusions

We proposed multimode antenna selection where both the number of substreams and the mapping from substreams to antennas was optimally chosen based on the channel and conveyed from receiver to transmitter via a low-rate feedback link. Our results were tailored to spatial multiplexing systems using linear receivers under the assumptions of a flat-fading channel and a zero-delay, zero-error feedback link. We found that both dual-mode and multimode selection improved the diversity gain dramatically over simple spatial multiplexing. Multimode showed higher array gain over simple dual-mode selection, however. We derived optimal and suboptimal selection criteria and related the selection problem and potential performance improvements to the eigenstructure of the channel.

One of the main assumptions we made was that the overall data rate was fixed. Thus, the selection criterion was tasked with reducing the error probability for a given data rate. In effect, this is a scheme for fixed-rate spatial adaptation since only the spatial dimension is adapted. Of course, if a feedback channel is available, some applications may benefit from adapting the data rate with the goal of maximizing throughput for a target error rate. The proposed criteria naturally extend to allow the rate to be adaptive. In brief, the data rate threshold needs to be translated to a selection threshold. Then, for a set of candidate rates, the optimum multimode solution can be derived, and the largest rate that meets the threshold can be selected for transmission.

An interesting byproduct of our statistical analysis for Rayleigh matrix channels is the frequency of use of each mode. Based on these results, for a given data rate, it may not be necessary to have the option of selecting all modes since only a subset of the available modes are chosen with high probability (cf. Fig. 4). This has the potential to reduce the amount of feedback and yet still maintain the performance of multimode selection. Of course, more work is needed to study these cases to ensure that these low-probability events are not big contributors to the probability of error.

An important point that is not addressed in this paper is the effect of delay and errors in the feedback channel. This will lead to a degradation of the bit error rate performance compared with ideal channels. A detailed analysis, along the lines of [36], is a possible avenue of future work. Further, the statistical analysis provided was only for Rayleigh fading matrix channels, which are known to be ideal in practice. Studying the probability of mode selection in different environments, as a function of the propagation parameters, using characterizations of measured MIMO channels [58] is an interesting topic for future research.

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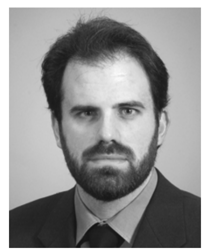

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