LIMITED FEEDBACK POWER LOADING FOR OFDM

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is a practical technique for communicating over broadband channels in multi-path fading environments. It is well known that varying the power allocation from frequency tone-to-frequency tone, commonly called power loading, can improve capacity or error rate performance. Unfortunately, the implementation of power loading is complicated by the fact that the transmitter must have knowledge of the forwardlink channel. This assumption is often unrealistic, particularly in systems using frequency division duplexing (FDD). In this paper we present a limited feedback approach for power loading OFDM symbols. In this approach, the receiver, which is assumed to have full forwardlink channel knowledge, designs a power loading vector and conveys it to the transmitter over a limited rate feedback channel. The technique uses a codebook of power loading vectors known to both the transmitter and receiver. We design limited feedback schemes for error rate optimized and capacity optimized power loading. We show how to design quantizers using iterative optimization techniques from the theory of vector quantization for the case of error rate selection. In the capacity case, we characterize optimal feedback schemes for asymptotic signal-to-noise ratio (SNR) scenarios and present a simple method called multi-mode power loading that uses a codebook that switches subcarriers off and on. Simulation results show that the limited feedback techniques provide performance close to perfect channel knowledge power loading.

I. INTRODUCTION

Multicarrier modulation (MCM) is a powerful technique for combating inter-symbol interference in multi-tap communication channels. One popular form of MCM, called orthogonal frequency division multiplexing (OFDM), has been or is expected to be included in wireless standards such as IEEE 802.16.3 [1], IEEE 802.11a [2], and IEEE 802.20. In OFDM, a broadband channel is divided into a large number of narrowband channels or tones. The signal is constructed in frequency, converted to a time signal via an orthogonal transformation, and then appended with a cyclic prefix before transmission.

Power loading, where the frequency tones are weighted subject to a total power constraint before transmission, can be used to optimize the capacity [3]–[7], error rate performance [8]–[12] and transmit power [13] when perfect channel knowledge is available at the transmitter. Power loading can also be combined with rate adaptation to optimize some performance criterion [3]–[7], [9], [11]–[13].

In time division duplexing (TDD) systems, it may be possible for the transmitter to use the reverselink channel estimate for designing forwardlink power loading [14], [15]. Problems arise, however, when power loading is implemented in frequency division duplexing (FDD) systems because the forward and reverselink channels fade independently. There are two methods for working around this problem. Linear precoding techniques (see [16]–[18]) provide excellent performance without any form of transmitter channel knowledge. Alternatively, power allocation can be done using a low rate feedback channel where some form of channel information is carried from the receiver to the transmitter. Limited feedback techniques have previously been studied in the multiple antenna literature for use with channel quantization feedback [19], [20], limited feedback beamforming [21]–[23], and limited feedback precoding [24], [25]. Unlimited feedback OFDM power loading was studied.
in [26] with emphasis on channel estimation and delay imperfections.

In this paper, we present a limited feedback approach to power loading in OFDM wireless systems. We consider limited feedback schemes using an error rate criterion or capacity criterion. We show that directly quantizing the vector of narrowband channel responses across the OFDM symbol is ineffective because it does not efficiently capture the salient features needed for power loading. We present a technique for designing limited feedback power loading using a codebook of power loading vectors designed offline and known to both the transmitter and receiver. The receiver can then choose one of the codebook vectors using its channel estimate and convey the vector to the transmitter over a low-rate feedback channel. When using error rate optimized loading, we design the codebook using the well-known Lloyd algorithm [27]. For the capacity criterion, we characterize asymptotically optimal feedback strategies.

We also present a capacity optimizing limited feedback technique called multi-mode power loading. Multi-mode power loading operates similarly to the algorithm proposed by Leke and Cioffi in [7]. In multi-mode power loading, a codebook of power loading vectors is stored at both the transmitter and receiver, and the receiver chooses the optimal vector from this codebook. The codebook is designed by choosing all possible on-off tone configurations. This means that the codebook for an $M$ tone OFDM system consists of $2^M-1$ unit vectors constructed by normalizing all possible non-zero vector combinations of 1’s and 0’s. This technique was previously considered for efficient precoding and adaptive modulation in multiple antenna wireless systems [28], [29].

This paper is organized as follows. Section II overviews power loading OFDM systems. Power loading for use with an error rate criterion is discussed in Section III. Section IV proposes methods for power loading when using a capacity performance criterion. Section V presents Monte Carlo simulation results. Section VI presents some conclusions and discusses future areas of research.

### II. System Overview

Consider the $M$ tone OFDM system shown in Fig. 1. Assuming perfect pulse shaping, sampling, and synchronization, the frequency domain input-output relationship for the $i$th tone is given by

$$y_i = h_i x_i + n_i$$  \hspace{1cm} (1)

where $x_i$ is the transmitted signal, $h_i$ is the complex channel response, $n_i$ is additive complex Gaussian noise, and $y_i$ is the post-processing received signal. The tone-by-tone expressions from (1) can be collected in vector form as

$$
y = H x + n$$  \hspace{1cm} (2)

where $\mathbf{y} = [y_1 \ y_2 \ \cdots \ y_M]^T$, $\mathbf{x} = [x_1 \ x_2 \ \cdots \ x_M]^T$, $\mathbf{n} = [n_1 \ n_2 \ \cdots \ n_M]^T$, and $H = \text{diag}(h_1, h_2, \ldots, h_M)$. We will assume that the noise is distributed according to $CN(0, N_0)$ for all $i$. Thus the noise statistics are assumed fixed across frequency. We will also assume that $E[n_i^* n_j] = 0$ when $i \neq j$.

In power loading systems, $x_i$ is expressed as

$$x_i = a_i s_i$$  \hspace{1cm} (3)

where $a_i$ is a weight on the $i$th tone and $s_i$ is the transmitted symbol. The key is to design the vector $\mathbf{a} = [a_1 \ a_2 \ \cdots \ a_M]^T$ as a function of the channel responses. We will assume that $\mathbf{a} \in \mathbb{R}^M$. This follows from the fact that, given our assumptions, the capacity

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1We use $^T$ to denote transposition, $\text{diag}(h_1, h_2, \ldots, h_M)$ to denote a function that returns a matrix with $h_1, h_2, \ldots, h_M$ on the diagonal, $E[\cdot]$ to denote expectation, $\| \cdot \|_2$ to denote the vector two-norm, and $\text{card}(\cdot)$ to denote the cardinality of a set.
and error rate performance are independent of the phase on each subcarrier. We will assume that $E \left[ |s_i|^2 \right] = \mathcal{E}_s$.

Given $a$, we will require that the system maintain a total power constraint of

$$E \left[ \sum_{i=1}^{M} |a_i s_i|^2 \right] = \sum_{i=1}^{M} |a_i|^2 E \left[ |s_i|^2 \right] = \mathcal{E}_s. \quad (4)$$

This constraint simplifies to $\|a\|_2 = 1$.

The vector $a$ will be designed at the receiver and fed back over a low rate feedback channel. We will assume that $a$ is restricted to lie in a finite set, or codebook, $\mathcal{A} = \{a_1, a_2, \ldots, a_N\}$. This codebook will be fixed for all transmissions. The receiver will choose one of the vectors in $\mathcal{A}$ and convey it back to the transmitter using $B = \lceil \log_2 N \rceil$ bits of feedback.

**Problem Statement:** There are two fundamental problems that we solve in this paper. First, it must be determined how the receiver chooses the optimal $a$ given the codebook $\mathcal{A}$. We will leverage existing work from perfect channel knowledge power loading. Second, methods for designing $\mathcal{A}$ must be determined. We will solve this problem depending on the selection criterion chosen.

### III. Minimum Error Rate Power Loading

In this section, we will design $a$ based on minimizing the average error rate. We will make the assumption that all subcarriers carry the same number of bits. Therefore, we assume that for all $1 \leq i \leq M$, $s_i \in \mathcal{S}$ where $\mathcal{S}$ is some constellation.

Let $P_e(\gamma)$ denote the symbol error rate of a system transmitting symbols from $\mathcal{S}$ over an additive white Gaussian noise (AWGN) channel with average signal-to-noise ratio (SNR) $\gamma = \mathcal{E}_s/N_0$. The nearest neighbor union bound (NNUB) on the symbol error rate is given by [30]

$$P_e(\gamma) \leq N_e Q \left( \sqrt{\frac{1}{2} d_{\text{min}}^2 \gamma} \right) \quad (5)$$

where $N_e$ is the average number of nearest neighbors to points in $\mathcal{S}$, $Q(\cdot)$ is the Q-function, and $d_{\text{min}}$ is the minimum distance of $\mathcal{S}$.

Given a power loading allocation, a bound on the vector symbol error probability (i.e. the probability that at least one symbol is in error) is given by

$$1 - \prod_{i=1}^{M} \left( 1 - P_e(|a_i h_i|^2 \gamma) \right) \leq 1 - \prod_{i=1}^{M} \left( 1 - N_e Q \left( \sqrt{\frac{1}{2} d_{\text{min}}^2 |a_i h_i|^2 \gamma} \right) \right). \quad (6)$$

We will thus choose $a$ to minimize the bound in (13) using the following selection criterion.

**Error Rate Criterion:** Choose $a \in \mathcal{A}$ using

$$a = \arg \min_{a' \in \mathcal{A}} \left[ 1 - \prod_{i=1}^{M} \left( 1 - N_e Q \left( \sqrt{\frac{1}{2} d_{\text{min}}^2 |a'_i h_i|^2 \gamma} \right) \right) \right]. \quad (7)$$

It was shown in [10] that

$$a_i = \left[ \frac{|h_i|^2 d_{\text{min}}^2 \gamma}{1 + \left( |h_i|^2 d_{\text{min}}^2 \gamma \right)^2} \left( \sum_{k=1}^{M} |h_k|^2 d_{\text{min}}^2 \gamma \right)^{-1} \right]^{-1} \quad (8)$$

obtains excellent error rate performance compared to other power loading techniques. This expression is derived as an approximate solution to the optimal power loading scheme which requires $M$ transcendental equations to be solved. Interestingly, when $\gamma \to \infty$, (8) approaches

$$a_i = \frac{|h_i|^{-1}}{\sqrt{\sum_{k=1}^{M} |h_k|^{-2}}} \quad (9)$$

The asymptotic expression in (9) is known as equalized power loading [8]. In this case the goal, is to force all tones to encounter what looks like an AWGN channel. This comes at the expense, however, of full transmitter channel knowledge.

The design of $a$ only depends on the channel magnitudes; thus, there is no reason to feedback channel phase information to the transmitter. Using the high SNR power loading, we will design a codebook that only depends on

$$g = \left[ |h_1|^{-1} |h_2|^{-1} \cdots |h_M|^{-1} \right]^T.$$
The high SNR expression in (9) tells us that we would like to maximize \( \frac{1}{\sqrt{\sum_{k=1}^{M} |h_k|^2}} |a_i/h_i| \) for all \( i \). Therefore, we will attempt to maximize the correlation

\[
\frac{1}{\sqrt{\sum_{k=1}^{M} |h_k|^2}} \sum_{i=1}^{M} |a_i/h_i| = a^T g/\|g\|_2
\]

of our power loading vector with the normalized inverse channel weights.

Note that

\[
\|a - g/\|g\|_2\|_2^2 = 2 - 2a^T g/\|g\|_2.
\]

Maximizing (10) on average is therefore equivalent to minimizing

\[
D(A) = E \left[ \min_{a \in A} \|a - g/\|g\|_2\|_2^2 \right]. \tag{11}
\]

Using the average distortion in (11), we can design the codebook using the Lloyd algorithm [27, p. 188]. This algorithm works by generating a large number of channels and a starting codebook. The algorithm then repartitions the channel responses into Voronoi regions and regenerates the codebook vectors. This process is repeated iteratively until a fixed point is reached.

IV. CAPACITY MAXIMIZING POWER LOADING

Unlike error rate power loading, capacity based power loading is usually combined with bit loading in order to match the per-subcarrier capacity. In this section, we will design power loading algorithms without regard to bit loading. This design is applicable because we are only trying to optimize the capacity of the effective parallel subcarrier channels after power loading.

The capacity of the OFDM system with power loading is given by

\[
C(a) = \sum_{i=1}^{M} \log_2 \left( 1 + \gamma |a_i h_i|^2 \right) \tag{12}
\]

where \( \gamma = E_s/N_0 \). In order to maximize this given a codebook \( A \) and a channel realization, we will use the following design criterion.

**Capacity Criterion:** Choose \( a \in A \) using

\[
a = \arg\max_{a' \in A} C(a'). \tag{13}
\]

The optimal unquantized solution for power loading design is waterfilling [4], [31]. In general waterfilling requires a computationally complex optimization, but this optimization simplifies asymptotically. The following lemma summarizes this result.

**Lemma 1:** Bliss et al [32] As \( \gamma \to \infty \), the optimal power loading scheme is to set \( a_i = 1/\sqrt{M} \) for all \( i \). As \( \gamma \to 0 \), the optimal power loading scheme is to set

\[
a_i = \begin{cases} 1 & \text{if } i = i_0; \\ 0 & \text{else} \end{cases} \tag{14}
\]

where \( i_0 = \arg\max_{1 \leq i \leq M} |h_i|^2 \).

Lemma 1 tells us two important design criteria for extreme cases. At high SNR, no feedback is needed. This matches similarly with the high SNR MIMO results cited in the spatial multiplexing literature [33]. It also allows us to characterize the optimal feedback scheme at low SNR. We will now discuss feedback design for the three cases: high SNR, low SNR, and non-extreme SNR.

**High SNR**

Using the results of Lemma 1, we can see the equal power allocation scheme first proposed by Bingham in [8] is optimal. Thus, feedback at high SNR is unnecessary. Therefore set \( B = 0 \).

**Low SNR**

Low SNR systems, in contrast, desperately need feedback. Pouring power on smaller gain channels wastes valuable resources and causes a capacity loss. Luckily, Lemma 1 points out a simple, optimal feedback technique.

**Optimal Low SNR Feedback Design:** At low SNR, use \( B = \lceil \log_2 M \rceil \) bits of feedback and set

\[
a = \{e_1, e_2, \ldots, e_M\} \tag{15}
\]

where \( e_i \) is the \( i \)th column of the \( M \times M \) identity matrix. Then feedback

\[
a = e_{i_0} \tag{16}
\]

where \( i_0 = \arg\max_{1 \leq i \leq M} |h_i|^2 \).

At low SNR the optimal solution is to transmit all data on the tone with the best channel. This leads to an efficient codebook design that can be easily stored in deployed OFDM systems. The optimal vector can be obtained using a brute force search over the \( M \) subcarrier tones.

**Non-Extreme SNR**

When the SNR is not an extreme value, the feedback design is not as simple. Previous work in [7] motivated turning off tones that had channel gains that yielded a negative waterfilling value. We will take a modified
version of this approach that we call multi-mode power loading.

**Multi-Mode Power Loading:** Let $\mathcal{J}$ be the set of all non-empty subsets of $\{1, 2, \ldots, M\}$. Design the codebook $A$ as

$$A = \left\{ \frac{1}{\sqrt{\text{card}(I)}} \sum_{i \in I} e_i \mid \forall I \in \mathcal{J} \right\}. \quad (17)$$

The receiver then chooses

$$a = \arg\max_{a' \in A} \sum_{i=1}^{M} \log_2 \left(1 + \gamma |a'_i h_i|^2\right) \quad (18)$$

and conveys the power loading vector to the transmitter using $M$ bits of feedback.

It is important to note that multi-mode power loading will become impractical for large $M$. Both the brute force search over the codebook and the storage requirements will grow exponentially with $M$. For large $M$, vector quantization schemes such as the Lloyd algorithm could possibly be employed.

V. SIMULATIONS

Monte Carlo simulations were performed using the independent Rayleigh fading channel assumptions. The simulations tested the error rate and capacity criteria limited feedback schemes.

**Experiment 1:** This experiment simulated limited feedback power loading on a 32 tone OFDM system using the error rate criterion and binary phase shift keying (BPSK) modulation. The vector symbol error rate (SER), the probability that at least one subcarrier symbol is in error, results are shown in Fig. 2. Four bit and eight bit codebooks were designed using the Lloyd algorithm. Uniform power loading, equalized power loading, and the algorithm from [10] were simulated for comparison. Note that four bits of feedback provides a 3.4dB improvement over uniform power allocation at an error rate of $10^{-3}$. Eight bit limited feedback power loading provides a 4.4dB gain over the approximately optimal scheme in [10] at an error rate of $10^{-3}$. This is a dramatic performance advantage that comes with only a few bits of feedback, while the subcarrier equalization and the power loading in [10] require perfect channel knowledge.

**Experiment 2:** The second simulation looks at the performance of the asymptotic feedback schemes for a 16 tone OFDM system. Fig. 3 presents the performance when the capacity of the asymptotic allocations are normalized by the perfect channel knowledge water-filling capacity. The high SNR assumption technique, which uses no feedback, obtains excellent performance at high SNR but dips below 80% of the full channel knowledge capacity at $E_s/N_0 = 10$dB. In contrast, sending $\lceil \log_2 16 \rceil = 4$ bits of feedback with the low SNR feedback scheme is optimal at very low SNRs. It falls below 80% of the capacity for SNRs above 0dB.

**Experiment 3:** The final simulation, see Fig. 4, shows the capacity performance of the asymptotic feedback
techniques and the multi-mode feedback technique for an eight tone OFDM system. Once again, the mutual informations were normalized by the perfect channel knowledge capacity. The 80% cross-over points are now 7dB for the zero-feedback high SNR assumption technique and 1.5dB for the three bit feedback low SNR technique. Note that the multi-mode technique, which requires eight bits of feedback, obtains more than 98.5% of the waterfilling capacity for all SNRs. This high performance comes at the expense of only switching the subcarriers off and on.

VI. Conclusions

In this paper, we proposed limited feedback power loading for OFDM wireless systems. The approach uses a codebook of power loading vectors known to both the transmitter and receiver. The receiver can then choose an optimal power loading vector from the codebook and convey the vector to the transmitter over a low rate feedback channel. We discussed power loading using an error rate criterion and a capacity criterion. In the error rate case, we use vector quantization techniques to design the codebook. We characterized asymptotically optimal codebooks for capacity selection. We also proposed a technique called multi-mode precoding for use with capacity selection.

This paper serves only as an introduction to the applications of limited feedback in OFDM. Much work has been published over the last few years about the application of limited feedback to multi-antenna wireless systems (see for example [21], [22], [24], [34]–[36]), and we conjecture that there is also a wide array of performance enhancing applications of limited feedback in OFDM. In particular, the application of limited feedback to multi-antenna OFDM systems remains an open problem.

References


