after a rough estimation of the value of P_p since two curves of the conventional scheme and the fixed linear one meet only once around P_p of 0.4. We need to determine whether or not the value of P_p is greater than 0.4. The second choice of Fig. 10(c) is to select one result after two parallel decoding processes of the conventional scheme and the fixed linear scheme. We will choose either decoding output without block error notification such as cyclic redundancy check (CRC) error. This would require more power due to parallel decoding but no estimation of the P_p value.

V. Conclusion

An OCHM scheme [1], [2] has been proposed as a novel statistical-multiplexing scheme for orthogonal downlink to accommodate more low-activity bursty users than the number of orthogonal downlink codewords. OCHM can cause perforations among symbols, which degrade the performance of channel decoding when the perforation probability is high. We propose new LLR conversion schemes that improve the decoding performance in perforation environments. The schemes require a simple conversion function between channel demodulator output and channel decoder input. We propose several types of LLR conversion functions. The proposed schemes reduce the required E_b/N_0 value by up to 10 dB when the value of P_p is high. The piecewise linear (suboptimum) conversion scheme yields performance similar to the exact (optimum) scheme and reduces computational complexity. In these LLR conversion schemes, estimation of the perforation probability is required. Therefore, several estimation methods are proposed. The fixed linear LLR conversion scheme without estimation of the perforation probability is proposed to avoid accurate estimation. Combination of the conventional and fixed linear conversion schemes does not require an exact estimation of the perforation probability. However, performance is similar to the (suboptimum) linear conversion scheme.

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Limited Feedback Diversity Techniques for Correlated Channels

David J. Love and Robert W. Heath, Jr.

***Abstract*—Employing multiple antennas at the transmitter is a well-established technique for providing diversity advantage in wireless systems. Transmit beamforming relies on the assumption of current channel knowledge at the transmitter, which is unrealistic when the forward and reverse links are separated in frequency. One solution to this problem is for the receiver to send a small number of feedback bits that convey channel information to the transmitter. Feedback design techniques have been proposed over the past few years, but they were derived using the assumption of spatially uncorrelated Rayleigh fading. This correspondence addresses the design of limited feedback beamformers when the channel is correlated.**

***Index Terms*—Beamforming, diversity methods, limited feedback, multiple antennas.**

I. Introduction

Multiantenna transmitters can provide reduced error rates and increased capacities by mitigating the effects of multipath fading caused by the channel. Closed-loop techniques obtain these benefits by adapting the transmitted signal using forward-link channel knowledge.

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Unfortunately, obtaining knowledge of the current channel at the transmitter is unrealistic in systems using frequency division duplexing because the forward and reverse links are separated in frequency.

One solution to this problem is the use of limited feedback, where the receiver conveys current channel conditions to the transmitter over a limited capacity feedback channel [1]–[10]. Limited feedback beamforming algorithms work by using a codebook of beamforming vectors that is designed offline, fixed for all channel realizations, and known at both the transmitter and the receiver. Codebook design techniques were derived in [4] and [5] by thinking of the codebook vectors as points in the Grassmann manifold. This analysis is specific to the assumption of spatially uncorrelated Rayleigh fading models, however, and does not extend to the correlated case. This correspondence proposes techniques for the design of limited feedback beamformers in correlated Rayleigh models. We propose a codebook design technique that uses a modified Grassmannian line packing codebook. Grassmannian line packing codebooks were derived in [4] and [5] for spatially uncorrelated channels. A similar result was presented independently in [11].

We will use the following notation. We use * to denote conjugate transposition, $|\cdot|$ to denote absolute value, $\|\cdot\|$ to denote the vector two norm, $E[\cdot]$ to denote expectation, $\arg\min$ ($\arg\max$) to represent a function that returns a global minimizer (maximizer), $\mathbb{R}$ to denote the set of real numbers, $\mathbb{C}^{M_t}$ to denote the set of M_t length complex column vectors, Ω to denote the set of M_t-dimensional unit vectors, $\mathrm{Re}\{\cdot\}$ to denote a function that returns the real part of a complex number, and $\mathcal{CN}(0,1)$ to represent the complex Gaussian distribution where the real and imaginary portions are independent and identically distributed (i.i.d.) $\mathcal{N}(0, 1/2)$.

This correspondence is organized as follows. Section II reviews the beamforming framework. Section III is based on Grassmannian line packing. Monte Carlo beamforming simulations are presented in Section IV. We conclude in Section V.

II. System Overview

Consider a beamforming M_t antenna transmitter that sends an M_t-dimensional complex vector

$$\mathbf{x}_k = \mathbf{f} s_k$$

at the kth channel usage, where s_k is a single-dimensional real ($s_k \in \mathbb{R}$) or complex ($s_k \in \mathbb{C}$) symbol and $\mathbf{f}$ is an M_t-dimensional beamforming vector. Because the receiver is assumed to have one antenna, the beamforming vector $\mathbf{f}$ can be thought of as a transmit version of the combining vectors studied for receive diversity in [12]. Omitting the channel use index k, this yields a baseband input–output relationship of

$$y = |\mathbf{h}^*\mathbf{f}| s + n \tag{1}$$

where $\mathbf{h}$ is an $M_t \times 1$ channel vector and n is a noise term distributed according to $\mathcal{CN}(0, N_0)$. We will assume that the transmitted constellation is normalized such that $E_{s_k}[|s_k|^2] = \mathcal{E}$.

The channel $\mathbf{h}^*$ will be modeled as a correlated Rayleigh channel, where

$$\mathbf{h} = \mathbf{R}\mathbf{g}$$

with $\mathbf{g}$ being an $M_t \times 1$ vector with i.i.d. $\mathcal{CN}(0,1)$ entries and $\mathbf{R}\mathbf{R}^*$ denoting the spatial correlation. We restrict the minimum singular value $\lambda_{\min}$ of $\mathbf{R}$ to be greater than zero and the maximum singular value $\lambda_{\max}$ of $\mathbf{R}$ to be less than or equal to 1. We will assume that the receiver has perfect knowledge of $\mathbf{h}$ and that the transmitter has no knowledge of the current realization of $\mathbf{h}$ or $\mathbf{R}$ other than the quantized channel information carried by a low-rate feedback channel from the receiver to the transmitter.

In order to use this feedback channel, the vector $\mathbf{f}$ will be restricted to lie in a codebook $\mathcal{F} = \{\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_N\}$. The receiver will choose $\mathbf{f}$ from $\mathcal{F}$ using knowledge of $\mathbf{h}$ and convey the vector back to the transmitter using $\log_2 N$ bits of feedback. We will assume that the feedback is designed offline and that the correlation matrix $\mathbf{R}$ is static. This will allow the codebook to be designed as a function of the correlation matrix $\mathbf{R}$. This would also allow the codebook to be adapted to the channel correlation in the event that $\mathbf{R}$ is known to the transmitter.

The beamforming vector will be chosen to maximize the average receive signal-to-noise ratio (SNR) given by

$$\mathrm{SNR} = \frac{|\mathbf{h}^*\mathbf{f}|^2\mathcal{E}}{N_0} = \frac{\gamma_r^2\mathcal{E}}{N_0}. \tag{2}$$

This corresponds to choosing

$$\mathbf{f} = \arg\max_{\mathbf{f}' \in \mathcal{F}} |\mathbf{h}^*\mathbf{f}'|^2. \tag{3}$$

Note that the average transmitted symbol power conditioned on $\mathbf{f}$ is given by $\|\mathbf{f}\|^2\mathcal{E}$. In order to constrain the transmit power, we will require that $\|\mathbf{f}\| = 1$. This means that the codebook $\mathcal{F}$ is a finite set of unit vectors.

Note also that γ_r is phase invariant. This follows because $|\mathbf{h}^*\mathbf{f}| = |e^{j\theta}\mathbf{h}^*\mathbf{f}|$ for any $\theta \in [0, 2\pi)$. For any unit vector $\mathbf{f}$, $\gamma_r \le \|\mathbf{h}\|$. This bound on γ_r is achieved with $\mathbf{f}_{\mathrm{opt}} = \mathbf{h}/\|\mathbf{h}\|$. The optimal solution, however, is not unique because of the phase invariance.

The generalized Lloyd algorithm is now a standard tool in feedback codebook design and has been successfully used for the design of limited feedback in spatially uncorrelated channels [1], [9]. The Lloyd algorithm can be applied to correlated channel feedback design by designing a codebook $\mathcal{F}$ to minimize the average distortion

$$d_{\mathrm{Lloyd}}(\mathcal{F}) = E\left[\|\mathbf{h}\|^2 - \max_{1\le i\le N} |\mathbf{h}^*\mathbf{f}_i|^2\right]. \tag{4}$$

The algorithm can be easily implemented by generating Q_{test} test channels and iteratively redefining the codebook.

III. Grassmannian Codebook Design

While the Lloyd algorithm is fairly simple, it does not give a concrete criterion as to the properties that the codebook should possess. This section will present a simple codebook design that allows the codebook to be easily modified when $\mathbf{R}$ changes.

Note that an optimal beamforming vector is given by

$$\mathbf{f}_{\mathrm{opt}} = \frac{e^{j\theta}\mathbf{R}\mathbf{e}_g}{\|\mathbf{R}\mathbf{e}_g\|} \tag{5}$$

where $\theta \in [0, 2\pi)$ and $\mathbf{e}_g = \mathbf{g}/\|\mathbf{g}\|$. Because the matrix $\mathbf{R}$ is fixed, the only random parameter that the optimal beamforming vector depends on is the vector $\mathbf{e}_g$. Similarly, we can represent the codebook vectors in $\mathcal{F}$ as

$$\mathcal{F} = \left\{\frac{\mathbf{R}\mathbf{c}_1}{\|\mathbf{R}\mathbf{c}_1\|}, \ldots, \frac{\mathbf{R}\mathbf{c}_N}{\|\mathbf{R}\mathbf{c}_N\|}\right\} \tag{6}$$

where $\mathbf{c}_i \in \Omega$ for all i. Therefore, we will design the codebook in this scenario to minimize the average loss in SNR given by

$$d_{\text{grass}}(\mathcal{F}) = E\left[\min_{1\le i\le N}\left(\left|\mathbf{g}^*\mathbf{R}^*\frac{\mathbf{R}\mathbf{e}_g}{\|\mathbf{R}\mathbf{e}_g\|}\right|^2 - \left|\mathbf{g}^*\mathbf{R}^*\frac{\mathbf{R}\mathbf{c}_i}{\|\mathbf{R}\mathbf{c}_i\|}\right|^2\right)\right]. \tag{7}$$

This expression can be bounded as in (8)–(11), shown at the bottom of the page, where (8) uses the fact 1) that $(a^2 - b^2) = (a+b)(a-b)$ for real numbers a and b, and 2) $|\mathbf{g}^*\mathbf{R}^*\mathbf{v}| \le \|\mathbf{R}\mathbf{g}\|$ for any unit vector $\mathbf{v}$, (10) follows from expanding (8) and using

$$\frac{\|\mathbf{R}\mathbf{e}_g\|^2 + \|\mathbf{R}\mathbf{c}_i\|^2}{\|\mathbf{R}\mathbf{e}_g\|\|\mathbf{R}\mathbf{c}_i\|} \ge 2$$

and (11) uses the fact that $\|\mathbf{R}\mathbf{g}\| \le \|\mathbf{g}\|$.

The minimization over θ in (11) takes into account the phase invariant properties of γ_r. The distortion in (11) is further bounded as

$$\begin{aligned} d_{\text{grass}}(\mathcal{F}) &\le 2M_t\lambda_{\max}\lambda_{\min}^{-1}E\left[\min_{1\le i\le N}\min_{\theta}\|e^{j\theta}\mathbf{e}_g - \mathbf{c}_i\|\right] \\ &\le 2M_t\lambda_{\max}\lambda_{\min}^{-1}\left(2 - 2E\left[\max_{1\le i\le N}\left|\mathbf{e}_g^*\mathbf{c}_i\right|\right]\right)^{\frac{1}{2}} \end{aligned} \tag{12}$$

where $\lambda_{\max}$ is the maximum singular value of $\mathbf{R}$.

It was shown in [5] that the modified correlation $|\mathbf{e}_g^*\mathbf{c}_i|$ is a subspace correlation because it only depends on the column space of $\mathbf{e}_g$ and $\mathbf{c}_i$. The column space of a vector is a one-dimensional (1-D) subspace or line. Thus, just as in [4]–[6], we should think of $\mathcal{F}$ as a codebook of lines rather than a set of vectors. The set of lines in the M_t-dimensional complex vector space is known as the Grassmann manifold $\mathcal{G}(M_t, 1)$ [5]. A distance between points on $\mathcal{G}(M_t, 1)$ can be defined by the sine of the angle between the corresponding lines in $\mathbb{C}^{M_t}$ [13]. For example, if $\mathcal{P}_1$, $\mathcal{P}_2 \in \mathcal{G}(M_t, 1)$ with corresponding unit vectors $\mathbf{v}_1$, $\mathbf{v}_2 \in \mathbb{C}^{M_t}$, then

$$\delta(\mathcal{P}_1, \mathcal{P}_2) = \sqrt{1 - |\mathbf{v}_1^*\mathbf{v}_2|^2}. \tag{13}$$

Line packing is a different problem than traditional spherical coding (see, for example, [14]). While our codebook can be viewed as a collection of points on the sphere, these points are actually only representative vectors of lines (i.e., we are interested in the vector column space rather than the vector itself).

The reformulation in (6) is actually using a form of companding to compress $\mathbf{R}\mathbf{e}_g$. The basic idea of companding is to transform a source for quantization and then transform it back before usage [15]. Our compander works by designing a quantizer for $\mathbf{e}_g$. It was shown in [4] and [5] that the bound in (12) can be minimized by designing $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$ using Grassmannian line packing. Grassmannian line packings would design a codebook $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$ such that $\min_{k\ne l}\delta(\mathcal{P}_{\mathbf{c}_k}, \mathcal{P}_{\mathbf{c}_l})$ (where $\mathcal{P}_{\mathbf{c}_k}$ is the column space of $\mathbf{c}_k$) is maximized. A line packing can be designed using numerical [16], [17] or analytical [18], [19] techniques. The single-antenna noncoherent code designs in [16] lead to a particularly simple design algorithm.

A companding approach would then use the codebook obtained from multiplying each element of $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$ by $\mathbf{R}$. This approach, however, does not maintain our unit vector power constraint requirement. Thus, we will normalize our codebook to enforce the unit vector requirement.

Therefore, to perform the codebook design, we will take a two-step approach of designing a codebook for a spatially uncorrelated source and then rotating the codebook. This leads to the following criterion.

Correlated Grassmannian Beamforming: Design $\mathcal{F}$ by picking $\{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_N\}$ that maximize

$$\delta_{\min} = \min_{1\le k<l\le N}\sqrt{1 - |\mathbf{c}_k^*\mathbf{c}_l|^2}$$

and set

$$\mathbf{f}_i = \frac{\mathbf{R}\mathbf{c}_i}{\|\mathbf{R}\mathbf{c}_i\|}.$$

This criterion actually allows the system to adjust the limited feedback beamforming to current correlation conditions. The transmitter spatial correlation matrix can sometimes be estimated at the transmitter without any additional feedback. This would allow only one beamforming codebook, which is designed using the Grassmannian beamforming criterion in [5], to be stored in a system because the same codebook could be adapted to any transmit correlation structure.

One interesting byproduct is the relationship to traditional statistical beamforming [20]. In statistical beamforming, the beamforming vector $\mathbf{f}$ would always be set equal to the dominant singular vector of $\mathbf{R}$ denoted by $\mathbf{f}_{sv}$. This can provide excellent performance in highly

$$d_{\text{grass}}(\mathcal{F}) \le E\left[\min_{1\le i\le N}\min_{\theta}\left(2\|\mathbf{R}\mathbf{g}\|^2\left\|\frac{e^{j\theta}\mathbf{R}\mathbf{e}_g}{\|\mathbf{R}\mathbf{e}_g\|} - \frac{\mathbf{R}\mathbf{c}_i}{\|\mathbf{R}\mathbf{c}_i\|}\right\|\right)\right] \tag{8}$$

$$= E\left[\min_{1\le i\le N}\left(2\|\mathbf{R}\mathbf{g}\|^2\left(2 - 2\left|\frac{\mathbf{e}_g^*\mathbf{R}^*\mathbf{R}\mathbf{c}_i}{\|\mathbf{R}\mathbf{e}_g\|\|\mathbf{R}\mathbf{c}_i\|}\right|\right)^{\frac{1}{2}}\right)\right] \tag{9}$$

$$\le E\left[\min_{1\le i\le N}\left(2\|\mathbf{R}\mathbf{g}\|^2\lambda_{\min}^{-1}\left(\|\mathbf{R}\mathbf{e}_g\|^2 + \|\mathbf{R}\mathbf{c}_i\|^2 - 2\|\mathbf{R}\mathbf{e}_g\|\|\mathbf{R}\mathbf{c}_i\|\left|\frac{\mathbf{e}_g^*\mathbf{R}^*\mathbf{R}\mathbf{c}_i}{\|\mathbf{R}\mathbf{e}_g\|\|\mathbf{R}\mathbf{c}_i}\right|\right)^{\frac{1}{2}}\right)\right] \tag{10}$$

$$= E\left[\min_{1\le i\le N}\left(2\|\mathbf{R}\mathbf{g}\|^2\lambda_{\min}^{-1}\left(\|\mathbf{R}\mathbf{e}_g\|^2 + \|\mathbf{R}\mathbf{c}_i\|^2 - 2\left|\mathbf{e}_g^*\mathbf{R}^*\mathbf{R}\mathbf{c}_i\right|\right)^{\frac{1}{2}}\right)\right]$$

$$= E\left[2\|\mathbf{R}\mathbf{g}\|^2\lambda_{\min}^{-1}\min_{1\le i\le N}\min_{\theta}\|e^{j\theta}\mathbf{R}\mathbf{e}_g - \mathbf{R}\mathbf{c}_i\|\right]$$

$$\le 2M_t\lambda_{\min}^{-1}E\left[\min_{1\le i\le N}\min_{\theta}\|e^{j\theta}\mathbf{R}\mathbf{e}_g - \mathbf{R}\mathbf{c}_i\|\right] \tag{11}$$

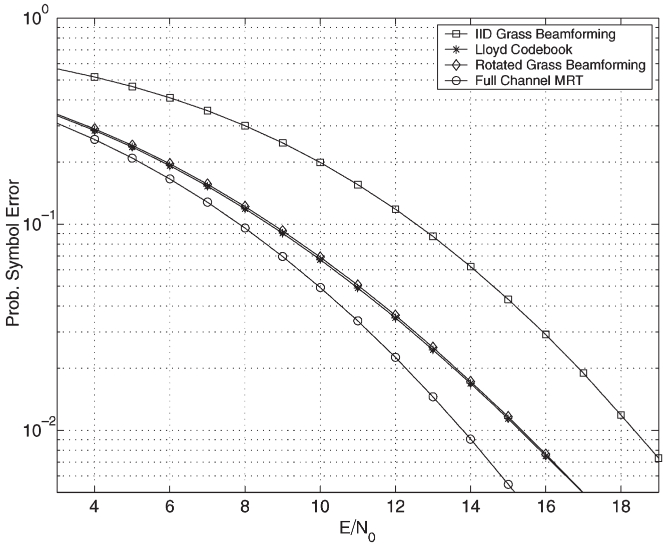

Fig. 1. Probability of symbol error performance comparison on an 8 × 1 correlated channel system.

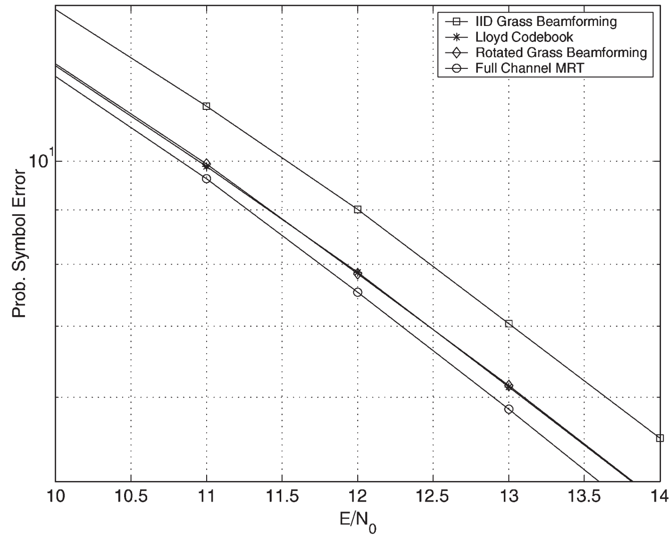

Fig. 2. Probability of symbol error performance comparison on a 4 × 1 correlated channel system.

correlated environments. Interestingly, our technique can be designed such that it always provides performance equal to or better than statistical beamforming. This follows from the fact that the minimum distance of the line packing $\tilde{\mathcal{C}} = \{\tilde{\mathbf{c}}_1, \tilde{\mathbf{c}}_2, \ldots, \tilde{\mathbf{c}}_N\}$ is equal to the minimum distance of the line packing $\mathcal{C} = \{\mathbf{U}\tilde{\mathbf{c}}_1, \mathbf{U}\tilde{\mathbf{c}}_2, \ldots, \mathbf{U}\tilde{\mathbf{c}}_N\}$, where $\mathbf{U}$ is an arbitrary $M_t \times M_t$ unitary matrix. Thus, we can design our codebook by obtaining a line packing $\tilde{\mathcal{C}}$ and then setting $\mathcal{C}$ equal to the unitary rotated codebook with $\mathbf{U}$ designed such that $\mathbf{U}\tilde{\mathbf{c}}_1 = \mathbf{f}_{sv}$.

IV. Simulations

Three experiments were performed to measure the benefits of taking correlation into account when designing limited feedback beamformers.

Experiment 1: This experiment addresses the probability of symbol error for an eight transmit antenna beamforming system. The correlation was computed from [21] using three clusters and a 5° angular spread. The results are shown in Fig. 1. The limited feedback codebooks were restricted to 6 bits of feedback. Codebooks were designed using the spatially uncorrelated Grassmannian beamforming design (from [4] and [5]), the Lloyd algorithm, and by rotating the spatially uncorrelated Grassmannian beamformer as in Section III. The Lloyd algorithm beamformer and the rotated Grassmannian beamformer both perform approximately the same, more than 3 dB better than the spatially uncorrelated Grassmannian beamformer. The full channel knowledge beamformer (i.e., maximum ratio transmission [12], [22], [23]) is shown for comparison.

Experiment 2: The second simulation, shown in Fig. 2, demonstrates the probability of symbol error performance of a four transmit antenna limited feedback beamforming system. In this simulation, the correlation matrix $\mathbf{R}$ was taken from the transmit "Micro Correlated" measurements in [24]. Sixty-four (i.e., six feedback bits) vector codebooks were designed using the Lloyd algorithm and the correlated Grassmannian beamforming algorithm. For comparison, a 6-bit spatially uncorrelated codebook (designed from [4] and [5]) and perfect channel knowledge maximum ratio transmission were also simulated. Note that taking into account the correlation gives more than a 0.5-dB gain. The limited feedback-correlated beamformers perform within 0.2 dB of full channel knowledge maximum ratio transmission. Once again, the Lloyd algorithm and the correlated Grassmannian technique perform approximately the same.

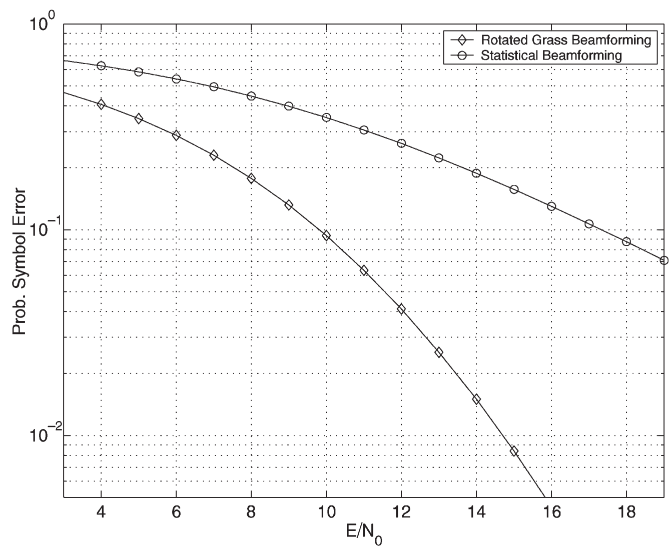

Fig. 3. Comparison between correlated Grassmannian beamforming and statistical beamforming on a 4 × 1 correlated channel system.

Experiment 3: Fig. 3 presents the probability of symbol error performance for correlated Grassmannian beamforming and statistical beamforming on a four transmit antenna limited feedback beamforming system. The correlation matrix was designed to have singular values of 1, 0.9, 0.8, and 0.7. Four feedback bits were used for the limited feedback scheme. Note that in this scenario, statistical beamforming performs dramatically worse than the limited feedback technique. This is because the channel is only weakly correlated in space. If the correlation were rank one, the performance of both schemes would be identical.

V. Summary and Conclusion

We proposed a limited feedback beamforming technique for spatially correlated Rayleigh fading channels. This new method can use codebooks designed from a simple modification to the existing limited feedback beamforming (or Grassmannian beamforming) method discussed in [4] and [5]. Monte Carlo simulation results show that this new approach provides substantial performance gains over limited feedback beamformers designed for uncorrelated channels.

It is of interest to find efficient ways to search over these subspace codebooks. Currently, a brute force search is used by computing the beamforming gain for each possible codebook vector. It might be possible to use other coding techniques to localize the beamforming vector required for feedback to a small search sphere in the Grassmann manifold.

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Error Probability of Bit-Interleaved Coded Modulation in Wireless Environments

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***Abstract*—The bit-interleaved coded modulation (BICM) method is efficient in mitigating multipath fading by providing time diversity. In this paper, union bounds on the bit and packet error probabilities of the BICM are derived. In the derivation, the authors assume the uniform interleaving of coded bits prior to mapping them onto the signal constellation. This results in a random distribution of the error bits in a codeword over the transmitted symbols. This distribution is evaluated, and the corresponding pairwise error probability is derived. Union bounds are functions of the distance spectrum of the channel code and the signal constellation used in the BICM system. The authors consider BICM systems operating over additive white Gaussian noise (AWGN), Rayleigh, Rician, and Nakagami fading channels. Results show that the new bounds are tight to simulation curves for different channel models. The proposed bounds are general for any coding scheme with a known distance spectrum.**

***Index Terms*—Additive white Gaussian noise (AWGN), bit interleaved, bit-interleaved coded modulation (BICM), coded modulation, convolutional codes, fading channels, generalized fading, Nakagami, Rayleigh, Rician, turbo codes, union bound.**

I. Introduction

The growing demand for data communications require bandwidth-efficient transmission techniques. A serious challenge to reliable communication in wireless systems is the time-varying multipath fading environment, which causes the received SNR to vary randomly. The fading distribution depends on the environment. For example, if a line of site (LOS) exists between the transmitter and the receiver in addition to the multipath reception, the fading process can be modeled by a Rician distribution [1]. Another popular fading model is the Nakagami distribution [2], which provides a family of distributions that match measurements in different propagation environments [3].

Coding and diversity techniques are methods used to mitigate the effects of multipath fading. Coded modulation [4] jointly considers error control coding and modulation to achieve high transmission rates

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