

Limited Feedback Unitary Precoding for Orthogonal Space-Time Block Codes

David J. Love, *Member, IEEE*, and Robert W. Heath, Jr., *Member, IEEE*

Abstract—Orthogonal space-time block codes (OSTBCs) are a class of easily decoded space-time codes that achieve full diversity order in Rayleigh fading channels. OSTBCs exist only for certain numbers of transmit antennas and do not provide array gain like diversity techniques that exploit transmit channel information. When channel state information is available at the transmitter, though, precoding the space-time codeword can be used to support different numbers of transmit antennas and to improve array gain. Unfortunately, transmitters in many wireless systems have no knowledge about current channel conditions. This motivates limited feedback precoding methods such as channel quantization or antenna subset selection. This paper investigates a limited feedback approach that uses a codebook of precoding matrices known *a priori* to both the transmitter and receiver. The receiver chooses a matrix from the codebook based on current channel conditions and conveys the optimal codebook matrix to the transmitter over an error-free, zero-delay feedback channel. A criterion for choosing the optimal precoding matrix in the codebook is proposed that relates directly to minimizing the probability of symbol error of the precoded system. Low average distortion codebooks are derived based on the optimal codeword selection criterion. The resulting design is found to relate to the famous applied mathematics problem of subspace packing in the Grassmann manifold. Codebooks designed by this method are proven to provide full diversity order in Rayleigh fading channels. Monte Carlo simulations show that limited feedback precoding performs better than antenna subset selection.

Index Terms—Diversity methods, Grassmannian subspace packing, MIMO systems, orthogonal space-time block coding, quantized precoding, Rayleigh channels.

I. INTRODUCTION

TO allow untethered communication in rich scattering environments, suitable sources of diversity need to be exploited to resist the detrimental effects of fading. Spatial diversity, which is obtained through the use of sufficiently spaced antennas, is one promising source of diversity. In particular,

Manuscript received August 10, 2003; revised December 29, 2003. D. J. Love was supported by a Continuing Graduate Fellowship and a Cockrell Doctoral Fellowship through The University of Texas at Austin. This material is based in part on work supported by the Texas Advanced Technology Program under Grant 003658-0614-2001, the National Instruments Foundation, the SBC Foundation, and the Samsung Advanced Institute of Technology. This paper was presented in part at the IEEE Global Telecommunications Conference, Dallas, TX, November 29–December 3, 2004. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Zhi Ding.

D. J. Love was with the Wireless Networking and Communications Group, Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712 USA. He is now with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907 USA (email: djlove@ecn.purdue.edu).

R. W. Heath, Jr. is with the Wireless Networking and Communications Group, Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, TX 78712 USA (e-mail: rheath@ece.utexas.edu).

Digital Object Identifier 10.1109/TSP.2004.838928

transmit diversity is attractive because the antennas can be located at the base station and the benefits shared by all users. Obtaining the benefits of multiple transmit antennas requires the use of special space-time signaling schemes such as space-time codes (see, for example, [1]–[3] and the references in [4] and [5]). Space-time codes are generally referred to as “open-loop” because they operate without the benefit of channel state information at the transmitter. As evidence of their practical importance, space-time codes are used in third-generation cellular networks [6] and are likely to be used in next generation wireless local area networks (LANs) [7].

Orthogonal space-time block codes (OSTBCs) are perhaps the most practical space-time codes due to the simplicity of optimal decoding [2], [3]. OSTBCs exist only for certain numbers of transmit antennas, though, and this limits the scope of their potential application. Further, compared with low-complexity closed-loop diversity techniques such as beamforming, OSTBCs do not fare well due to a significant array gain penalty [8]. Solving both of these problems by applying “closed-loop fixes” to OSTBCs has been the subject of recent work on closed-loop space-time block coding [9]–[21]. A convenient way to unify the presentation in these papers is through the construct of linear precoding. Linear precoding is the technique of applying a linear transformation to the spatio-temporal codeword before transmission [22]. By restricting the linear transformation, it is possible to include other closed-loop methods such as transmit antenna selection (see, for example, [14] and [15]).

There are various kinds of closed-loop methods that can be classified based on the type of channel information available at the transmitter: the instantaneous channel, statistics of the channel, and partial or quantized channel state information. Assuming true channel state information [9], [17] leads to bounds on performance but is not practical in frequency division duplexing (FDD) systems where the channel state information must be conveyed over a limited-rate feedback control channel. Knowledge of the statistics of the channel [20], [21] is practical when only the long-term statistics can be tracked, but this does not provide the benefits of near-instantaneous channel tracking. Partial or quantized channel state information [10]–[16] is the most practical of the three methods for FDD wireless systems since the amount of feedback can be tailored to the data rate available on the limited-rate feedback control channels. Developing limited feedback techniques for OSTBCs is the subject of this paper.

Previous work on limited feedback space-time coding has focused primarily on the case where the linear precoding matrix chooses subsets of the available transmit antennas [14], [15],

[23]–[25]. Performance improves, however, by allowing the space-time code's linear precoding matrix to have a more general form [10], [11], [13], [26], [27]. These ideas were explored in [10] and [11] using precoders designed with quantized versions of the forward-link channel. Quantizing the channel directly is a computationally complex problem that often yields too much feedback, and the quantization noise can adversely affect the computation of optimal precoders. For this reason, it is of interest to use limited feedback techniques that directly attack the problem of quantizing the set of precoding matrices.

In this paper, we propose a new form of limited feedback precoding using the general principles of limited feedback precoder selection, as in [13], and limited feedback covariance selection, as in [28]. Before transmission, an M antenna OSTBC matrix is multiplied by a linear precoding matrix and transmitted over M_t antennas, where $M_t > M$. The linear precoding matrix is chosen from a finite cardinality codebook of possible precoding matrices that is designed off-line and available to both the transmitter and receiver. We address the two main problems associated with this system model: i) choosing the optimal precoding matrix from the codebook and ii) designing the codebook. We propose a criterion for choosing the optimal precoding matrix from the codebook by adapting the criterion in [9] and [14]. Because we use a finite codebook, the optimal precoding matrix is selected by searching over all possible precoders in the codebook. Using the optimal codeword selection criterion, we derive a criterion for finding codebooks with low average distortion. We show that the codebook design problem relates directly to packing subspaces in the Grassmann manifold using the chordal distance metric. Thus, the subspace packing design methods available in, for example, [29]–[31], can be used to design high-performance limited feedback precoders. We furthermore prove that these design methods yield limited feedback precoders that obtain the full diversity order proportional to the product of the number of transmit and receive antennas.

Compared with prior work, our approach to limited feedback precoding offers numerous benefits. Unlike the antenna selection techniques, our proposed work allows a tradeoff between feedback rate and performance. This allows *more feedback* to be used when *lower bit error rates* are required. Our method is also *resilient to quantization error*, unlike direct channel quantization. Because the receiver chooses the precoder matrix using perfect channel information, the eigenstructure of the matrix channel is not compromised during the quantization. These benefits also come with *lower feedback rates*. For example, an 8×8 precoding system using only two bits of feedback per complex entry (i.e., one bit for the real dimension and one bit for the imaginary dimension) would require 128 bits of feedback per channel realization.

The paper is organized as follows. Section II provides a general overview of the system design under consideration. A selection criterion for limited feedback OSTBC precoding is derived in Section III. The codebook design problem is solved in Section IV. Section V shows that the designed codebooks provide full diversity order in Rayleigh fading channels. We present Monte Carlo simulation results in Section VI and conclude in Section VII.

II. SYSTEM OVERVIEW

An M_t transmit antenna and M_r receive antenna limited feedback precoded OSTBC system is illustrated in Fig. 1. A block of n_s symbols $s_1, s_2, \dots, s_{n_s}$ taken from a constellation $\mathcal{S}$ is fed into an OSTBC encoder producing an $M \times T$ (with $M < M_t$) space-time code matrix $\mathbf{C}(k) = [\mathbf{c}_1(k) \ \mathbf{c}_2(k) \ \dots \ \mathbf{c}_T(k)]$ at the k th transmission. Note that for any codeword¹ [4]

$$\mathbf{C}(k)\mathbf{C}(k)^* = \left(\sum_{l=1}^{n_s} |s_l|^2 \right) \mathbf{I}_M. \quad (1)$$

For power constraint reasons, we will assume that $E_{s_l} [|s_l|^2] = 1$.

Before transmission, the space-time codeword is premultiplied by an $M_t \times M$ matrix $\mathbf{F}$. The matrix $\mathbf{F}$ is chosen by a function $f : \mathbb{C}^{M_r \times M_t} \rightarrow \mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N\}$ evaluated using the current channel conditions and then conveyed to the transmitter using $\lceil \log_2 N \rceil$ bits of feedback. The set $\mathcal{F}$ can be thought of as a codebook and functions similarly to those found in the standard source coding literature (see, for example, [32]). This codebook is designed offline and known to both the transmitter and receiver. We will impose a peak power limit for all $\mathbf{F} \in \mathcal{F}$ by requiring that

$$\max_{\mathbf{s} \in \mathcal{S}^M} \frac{\|\mathbf{F}\mathbf{s}\|_2}{\|\mathbf{s}\|_2} \leq 1$$

for any constellation $\mathcal{S} \subseteq \mathbb{C}$. Because $\mathcal{S}$ is nonempty and $0 \notin \mathcal{S}$, this constraint corresponds to the assumption that $\lambda_1\{\mathbf{F}\} \leq 1$ for all $\mathbf{F} \in \mathcal{F}$. We will further restrict the form of the matrices in $\mathcal{F}$, but the justification will be presented in Section III.

We assume that the transfer function between the transmitter and receiver can be modeled as a spatially uncorrelated, memoryless linear channel that is constant over several codeword transmissions before independently taking on a new value. Assuming optimal pulse shaping, match-filtering, and sampling, the received signal can thus be written as

$$\mathbf{Y} = \sqrt{\frac{\rho}{M}} \mathbf{H} \mathbf{F} \mathbf{C} + \mathbf{W} \quad (2)$$

where $\mathbf{H}$ is the $M_r \times M_t$ channel matrix with independent entries distributed as $\mathcal{CN}(0, 1)$, $\mathbf{W}$ is an $M_r \times T$ noise matrix with independent entries distributed according to $\mathcal{CN}(0, 1)$, and ρ is the signal-to-noise ratio (SNR). $\mathbf{F}$ corresponds to the evaluation of the mapping function for the current channel realization $\mathbf{H}$, i.e.,

$$\mathbf{F} = f(\mathbf{H}).$$

¹We use $i = \sqrt{-1}$, $\mathbb{C}^{k \times l}$ to denote the vector space of $k \times l$ complex matrices, $\mathbb{C}^k$ to denote the k -dimensional complex vector space, $\mathbb{R}^k$ to denote the k -dimensional real vector space, $\mathbb{Z}^k$ to denote the k -fold cartesian product set of the ring of integers, $\mathbf{I}_k$ to denote the $k \times k$ identity matrix, $E_y[\cdot]$ to denote expectation with respect to y , $*$ to denote matrix conjugate transposition, $\arg \max$ to denote a function that returns a single global maximizer, $\lambda_i\{\mathbf{A}\}$ to denote the i th largest singular value of $\mathbf{A}$, $\text{tr}(\cdot)$ to denote the trace of a matrix, $\|\cdot\|_2$ to denote the matrix two-norm, $|\cdot|$ to denote the absolute value, $\|\cdot\|_F$ to denote the matrix Frobenius norm, $\lceil \cdot \rceil$ to denote the ceiling function, $(\mathbf{A})_k$ to denote the k th column of $\mathbf{A}$, $\mathbf{A}^K = \prod_{k=1}^K \mathbf{A}$, $\text{vec}(\mathbf{A})$ to denote a vector obtained by stacking the columns of a matrix $\mathbf{A}$, $\mathbf{0}$ to denote a matrix of all zeros, $\mathcal{L}(M_t, M)$ to denote the set of $M_t \times M$ matrices whose maximum singular value is less than or equal to one (i.e., $\mathcal{L}(M_t, M) = \{\mathbf{F} \in \mathbb{C}^{M_t \times M} \mid \lambda_1\{\mathbf{F}\} \leq 1\}$), and $\mathcal{U}(M_t, M)$ to denote the set of $M_t \times M$ matrices with orthonormal columns.

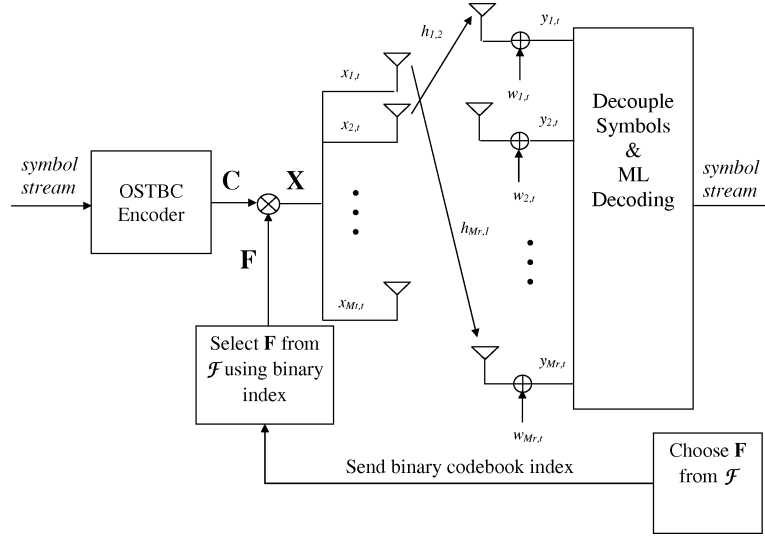


Fig. 1. Block diagram of a limited feedback precoding system.

Note that we have suppressed the temporal parameter k in $\mathbf{C}(k)$ because of our channel model and interest in the use of codeword-by-codeword detection. After reception, the receiver performs maximum likelihood (ML) decoding on the OSTBC using the received matrix $\mathbf{Y}$ and the effective channel $\mathbf{H}\mathbf{F}$.

Problem Statement: Given a system illustrated by Fig. 1 and described by (2), two main problems arise. The first problem we address is the selection of an optimal precoder for given channel from a codebook of matrices $\mathcal{F}$. The second problem is how to design the set $\mathcal{F}$ given the chosen selection criterion. Solving these problems is the objective of this paper.

III. SELECTION CRITERION

The first issue with limited feedback unitary precoding we address is the design of the precoder selection mapping $f(\cdot)$. We assume an arbitrary set $\mathcal{F} \subset \mathcal{L}(M_t, M)$ and ML decoding.

Our goal in this paper is to minimize the symbol error rate (SER) given $\mathbf{H}$, $\Pr(\text{ERROR} \mid \mathbf{H})$. Closed-form expressions for the SER would be extremely difficult to obtain because they would be a function of the effective channel $\mathbf{H}\mathbf{F}$ that is in general no longer matrix Rayleigh fading and the selection function $f(\cdot)$.

We will therefore take the approach of [3], [9], [13], and [14] and use a bound on the probability of error. Using the ML detection properties of OSTBCs, it can be shown that

$$\Pr(\text{ERROR} \mid \mathbf{H}) \leq \exp(-\gamma \|\mathbf{H}\mathbf{F}\|_F^2) \quad (3)$$

where γ is a function that depends on M , ρ , and $\mathcal{S}$ [4]. Because γ is fixed, minimizing the bound in (3) requires the maximization of $\|\mathbf{H}\mathbf{F}\|_F$. Note that this maximization is equivalent to choosing the matrix $\mathbf{F} \in \mathcal{F}$ that maximizes the receive minimum distance

$$d_{\min} = \min_{\mathbf{C}_k \neq \mathbf{C}_l} \|\mathbf{H}\mathbf{F}(\mathbf{C}_k - \mathbf{C}_l)\|_F \quad (4)$$

because of the orthogonality of the OSTBC codewords in (1). Therefore, we can state the following selection criterion.

Selection Criterion: Choose the limited feedback precoder according to

$$\mathbf{F} = f(\mathbf{H}) = \arg \max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F. \quad (5)$$

This selection criterion can be implemented by computing a matrix multiplication and Frobenius norm for each of the N codebook matrices. Ties in distance between codeword matrices are broken arbitrarily by selecting the precoding matrix with the lowest index because they occur with zero probability.

To bound the performance with quantized precoding, it is of interest to characterize an optimal unquantized precoder over the set $\mathcal{L}(M_t, M)$. Note that this matrix $\mathbf{F}_{\text{opt}}$ will not be unique over $\mathcal{L}(M_t, M)$ because for any $\mathbf{U} \in \mathcal{U}(M, M)$, $\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F = \|\mathbf{H}\mathbf{F}_{\text{opt}}\mathbf{U}\|_F$. Let the singular value decomposition of $\mathbf{H}$ be given by

$$\mathbf{H} = \mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^* \quad (6)$$

where $\mathbf{V}_L \in \mathcal{U}(M_r, M_r)$, $\mathbf{V}_R \in \mathcal{U}(M_t, M_t)$, and $\mathbf{\Sigma}$ is an $M_r \times M_t$ diagonal matrix with $\lambda_j\{\mathbf{H}\}$ at entry (j, j) . Let $\bar{\mathbf{V}}_R$ be the matrix formed from the first M columns of $\mathbf{V}_R$. The following lemma shows an optimal matrix in this unquantized scenario.

Lemma 1: An optimal unquantized precoder $\mathbf{F}_{\text{opt}}$ over $\mathcal{L}(M_t, M)$ that maximizes $\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F$ is given by $\mathbf{F}_{\text{opt}} = \bar{\mathbf{V}}_R$.

Proof: Let $\mathbf{F}$ have singular value decomposition $\mathbf{F} = \mathbf{U}_L \mathbf{\Gamma} \mathbf{U}_R^*$, where $\mathbf{U}_L \in \mathcal{U}(M_t, M_t)$, $\mathbf{U}_R \in \mathcal{U}(M, M)$, and $\mathbf{\Gamma}$ is an $M_t \times M$ diagonal matrix with $\lambda_j\{\mathbf{F}\}$ at entry (j, j) . It then follows that

$$\begin{aligned} \|\mathbf{H}\mathbf{F}\|_F &= \|\mathbf{\Sigma} \mathbf{V}_R^* \mathbf{U}_L \mathbf{\Gamma}\|_F \\ &\leq \left\| \mathbf{\Sigma} \mathbf{V}_R^* \mathbf{U}_L [\mathbf{I}_M \quad \mathbf{0}]^T \right\|_F \end{aligned} \quad (7)$$

$$\begin{aligned} &\leq \sqrt{\sum_{j=1}^M \lambda_j \left\{ \mathbf{U}_L^* \mathbf{V}_R \mathbf{\Sigma}^T \mathbf{\Sigma} \mathbf{V}_R^* \mathbf{U}_L \right\}} \\ &= \|\bar{\mathbf{\Sigma}}\|_F \end{aligned} \quad (8)$$

by using the invariance of the Frobenius norm to unitary transformation [33] in (7) and the singular values bounds of $\mathbf{F}$, where

$\mathbf{0}$ is the $M \times (M_t - M)$ matrix of zeros and $\bar{\Sigma}$ is the matrix consisting of the first M columns of Σ . Equality in (8) is obtained when $\mathbf{F} = \bar{\mathbf{V}}_R$. ■

Lemma 1 is interesting because it says that not only is $\lambda_1\{\mathbf{F}_{\text{opt}}\} = 1$ but $\lambda_j\{\mathbf{F}_{\text{opt}}\} = 1$ for $1 \leq j \leq M$. Intuitively, this means that we should *always* transmit at full power on the precoder effective channel modes when performing optimal precoding. This result would be expected given the mean squared error results for precoded spatial multiplexing in [22]. We are, however, interested in suboptimal precoders constructed using a limited feedback path. The following lemma shows that *all* precoding matrices should have full power in the singular values.

Lemma 2: If $\mathbf{F} \in \mathcal{L}(M_t, M)$ has a singular value decomposition $\mathbf{F} = \mathbf{U}_L \mathbf{\Gamma} \mathbf{U}_R^*$ and $\lambda_M\{\mathbf{F}\} < 1$, then $\tilde{\mathbf{F}} = \mathbf{U}_L [\mathbf{I}_M \mathbf{0}]^T$ satisfies $\|\mathbf{H}\mathbf{F}\|_F \leq \|\mathbf{H}\tilde{\mathbf{F}}\|_F$.

Proof: Assume $\mathbf{F}$ has a singular value decomposition as shown in the first lemma. Then

$$\|\mathbf{H}\mathbf{F}\|_F = \|\mathbf{H}\mathbf{U}_L \mathbf{\Gamma}\|_F \leq \left\| \mathbf{H}\mathbf{U}_L [\mathbf{I}_M \mathbf{0}]^T \right\|_F = \|\mathbf{H}\tilde{\mathbf{F}}\|_F$$

using the invariance of the Frobenius norm to unitary transformation [33], the singular value bound for $\mathbf{F}$, and $\tilde{\mathbf{F}} = \mathbf{U}_L [\mathbf{I}_M \mathbf{0}]^T$. ■

Lemma 2 demonstrates the significant effect that the power restriction has on optimal precoders. The sum power constraint results in [15], [22], and [34] all yield optimal precoders with unequal power allocation among the precoder singular values. In contrast, unequal power pouring when using a maximum singular value constraint only reduces the *total transmit power*.

Lemma 2 reveals that the precoder should *always* be designed to have singular values that are as large as possible. Because of the maximum singular value restriction, the precoder should be chosen from the set $\mathcal{U}(M_t, M)$ rather than $\mathcal{L}(M_t, M)$. For this reason, we will restrict $\mathcal{F} \subset \mathcal{U}(M_t, M)$ and, thus, *only* design our precoders over $\mathcal{U}(M_t, M)$.

IV. CHORDAL DISTANCE PRECODING: MOTIVATION AND CODEBOOK DESIGN

In this section, we derive a codebook design criterion that follows directly from the Section III codeword selection result. This criterion is based on a distortion function that we define in order to minimize the average SER. The criterion turns out to relate to the famous applied mathematics problem of Grassmannian subspace packing.

A. Distortion Function

To design a codebook, we will propose a distortion measure that is a function of the channel and then find a codebook that minimizes the average distortion. This distortion function must differ, however, from the distortion functions commonly used in vector quantization such as mean squared error (see for example [32]) because *we are interested in improving system performance rather than improving the quality of the estimated precoder at the transmitter*.

Essentially, we would like the quantized equivalent channel $\mathbf{H}\mathbf{F}$ to provide SER performance close in some sense to that provided by the optimal precoded equivalent channel $\mathbf{H}\mathbf{F}_{\text{opt}}$,

where $\mathbf{F} \in \mathcal{F}$ is the “best” precoding matrix in the codebook. Along these lines, consider the total effective power $\|\mathbf{H}\mathbf{F}\|_F^2$, which according to (3) relates to the SER. The loss in received channel power is expressed as

$$\min_{\mathbf{F}' \in \mathcal{F}} (\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F^2 - \|\mathbf{H}\mathbf{F}'\|_F^2). \quad (9)$$

We propose to design the codebook by considering, as a measure of distortion, the average of the loss in received channel power given by

$$E_{\mathbf{H}} \left[\min_{\mathbf{F}' \in \mathcal{F}} (\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F^2 - \|\mathbf{H}\mathbf{F}'\|_F^2) \right]. \quad (10)$$

Recall that $\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F \geq \|\mathbf{H}\mathbf{F}\|_F$ for all $\mathbf{F} \in \mathcal{U}(M_t, M)$ so that the function will always be non-negative. Notice also that minimizing this distortion function relates directly to minimizing the bound on the SER in (3). Thus, the distortion has *physical meaning* in terms of the SER, as opposed to a mean squared error distance function.

The distortion function in (10) yields little insight into the codebook design problem directly; thus, we will derive a tight upper bound on the distortion. Using Lemma 1 and the definition of $\bar{\Sigma}$, the distortion for this arbitrary matrix is written as

$$\begin{aligned} & \min_{\mathbf{F}' \in \mathcal{F}} (\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F^2 - \|\mathbf{H}\mathbf{F}'\|_F^2) \\ &= \min_{\mathbf{F}' \in \mathcal{F}} (\text{tr}(\bar{\Sigma}\bar{\Sigma}^T) - \text{tr}(\Sigma\mathbf{V}_R^* \mathbf{F}' \mathbf{V}_R^* \Sigma^T)) \\ &\leq \min_{\mathbf{F}' \in \mathcal{F}} (\text{tr}(\bar{\Sigma}\bar{\Sigma}^T) - \text{tr}(\bar{\Sigma}\bar{\mathbf{V}}_R^* \mathbf{F}' \bar{\mathbf{V}}_R^* \bar{\Sigma}^T)) \end{aligned} \quad (11)$$

$$= \min_{\mathbf{F}' \in \mathcal{F}} \text{tr}(\bar{\Sigma}^T \bar{\Sigma} (\mathbf{I}_M - \bar{\mathbf{V}}_R^* \mathbf{F}' \bar{\mathbf{V}}_R^*)) \quad (12)$$

$$\leq \lambda_1^2\{\mathbf{H}\} \min_{\mathbf{F}' \in \mathcal{F}} \text{tr}(\mathbf{I}_M - \bar{\mathbf{V}}_R^* \mathbf{F}' \bar{\mathbf{V}}_R^*) \quad (13)$$

$$= \lambda_1^2\{\mathbf{H}\} \min_{\mathbf{F}' \in \mathcal{F}} \frac{1}{2} \|\bar{\mathbf{V}}_R \bar{\mathbf{V}}_R^* - \mathbf{F}' \mathbf{F}'^*\|_F^2 \quad (14)$$

where (11) follows from zeroing the least $M_t - M$ singular values of $\mathbf{H}$, (13) follows by substituting $\lambda_1\{\mathbf{H}\}$ for the other nonzero singular values in (12), and (14) uses the alternative representation for subspace distance [35].

To find a good codebook for many channel realizations, we are interested in the *average* distortion of our codebook given by

$$E_{\mathbf{H}} \left[\min_{\mathbf{F}' \in \mathcal{F}} (\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F^2 - \|\mathbf{H}\mathbf{F}'\|_F^2) \right]. \quad (15)$$

Using (14) and the independence of Σ and $\mathbf{V}_R$ [36]

$$\begin{aligned} & \left[\min_{\mathbf{F}' \in \mathcal{F}} (\|\mathbf{H}\mathbf{F}_{\text{opt}}\|_F^2 - \|\mathbf{H}\mathbf{F}'\|_F^2) \right] \leq E_{\mathbf{H}} \\ & E_{\mathbf{H}} [\lambda_1^2\{\mathbf{H}\}] E_{\mathbf{H}} \left[\min_{\mathbf{F}' \in \mathcal{F}} \frac{1}{2} \|\bar{\mathbf{V}}_R \bar{\mathbf{V}}_R^* - \mathbf{F}' \mathbf{F}'^*\|_F^2 \right]. \end{aligned} \quad (16)$$

Thus, by bounding the distortion function, we can think of the limited feedback performance as being characterized by two different terms: one related to the distribution of the maximum channel singular value and another representing the “quality” of the codebook $\mathcal{F}$.

B. Codebook Design Criterion

Minimizing (16) requires finding the distribution of $\bar{\mathbf{V}}_R$. Recall that a matrix $\mathbf{V}$ is *isotropically distributed* on $\mathcal{U}(M_t, M)$ if for any $\Theta \in \mathcal{U}(M_t, M_t)$, $\Theta^* \mathbf{V} \stackrel{d}{=} \mathbf{V}$ with $\stackrel{d}{=}$ denoting equivalence in distribution [37]. The following lemma gives this distribution.

Lemma 3: The optimal precoding matrix $\mathbf{F}_{\text{opt}} = \bar{\mathbf{V}}_R$ for a memoryless, i.i.d. Rayleigh channel $\mathbf{H}$ is isotropically distributed on $\mathcal{U}(M_t, M)$.

Proof: First, note that $\mathbf{F}_{\text{opt}} = \bar{\mathbf{V}}_R = \mathbf{V}_R [\mathbf{I}_M \quad \mathbf{0}]^T$. Since $\mathbf{V}_R$ is isotropically distributed [36], it is easily seen that $\Theta^* \mathbf{F}_{\text{opt}} \stackrel{d}{=} \mathbf{F}_{\text{opt}}$. ■

To propose a design for $\mathcal{F}$, let us review some common properties of finite subsets of $\mathcal{U}(M_t, M)$. These properties are found in the Grassmannian subspace packing literature (for example, [29], [31], [35], [38], and [39]). The set $\mathcal{U}(M_t, M)$ defines the Stiefel manifold [39]. Each matrix in $\mathcal{U}(M_t, M)$ generates an M -dimensional subspace of the complex M_t -dimensional vector space $\mathbb{C}^{M_t}$. We will adopt Grassmannian packing notation and define the set of all column spaces of the matrices in $\mathcal{U}(M_t, M)$ to be the complex Grassmann manifold $\mathcal{G}(M_t, M)$. Thus, if $\mathbf{F}_1, \mathbf{F}_2 \in \mathcal{U}(M_t, M)$, then the column spaces of $\mathbf{F}_1$ and $\mathbf{F}_2$, $\mathcal{P}_{\mathbf{F}_1}$ and $\mathcal{P}_{\mathbf{F}_2}$, respectively, are contained in $\mathcal{G}(M_t, M)$. A normalized invariant measure μ is induced on $\mathcal{G}(M_t, M)$ by the Haar measure on $\mathcal{U}(M_t, M)$. This measure allows the computation of volumes within $\mathcal{G}(M_t, M)$. Subspaces within the Grassmann manifold can be related by their distance from each other. The *chordal distance* between the two subspaces $\mathcal{P}_{\mathbf{F}_1}$ and $\mathcal{P}_{\mathbf{F}_2}$ is given by

$$d(\mathbf{F}_1, \mathbf{F}_2) = \frac{1}{\sqrt{2}} \|\mathbf{F}_1 \mathbf{F}_1^* - \mathbf{F}_2 \mathbf{F}_2^*\|_F. \quad (17)$$

Let $\mathcal{V} = \{\mathcal{P}_{\mathbf{F}_1}, \mathcal{P}_{\mathbf{F}_2}, \dots, \mathcal{P}_{\mathbf{F}_N}\}$ be the set of column spaces where $\mathcal{P}_{\mathbf{F}_k}$ is the column space of $\mathbf{F}_k$. This set $\mathcal{V} \subset \mathcal{G}(M_t, M)$ is a *packing* of subspaces in $\mathcal{G}(M_t, M)$. A packing can be described by its minimum distance

$$\delta = \min_{1 \leq k < l \leq N} d(\mathbf{F}_k, \mathbf{F}_l). \quad (18)$$

The Grassmannian subspace packing problem is the problem of finding the set of N subspaces in $\mathcal{G}(M_t, M)$ such that δ is as large as possible.

Consider the open ball in $\mathcal{G}(M_t, M)$ of radius $\delta/2$ defined as

$$\mathcal{B}_{\mathbf{F}_k} \left(\frac{\delta}{2} \right) = \left\{ \mathcal{P}_{\mathbf{U}} \in \mathcal{G}(M_t, M) \mid d(\mathbf{U}, \mathbf{F}_k) < \frac{\delta}{2} \right\}. \quad (19)$$

Notice that the balls are disjoint by the triangle inequality of metrics [35]. This observation allows the *density* of a chordal subspace packing to be defined as

$$\Delta(\mathcal{F}) = \mu \left(\bigcup_{k=1}^N \mathcal{B}_{\mathbf{F}_k} \left(\frac{\delta}{2} \right) \right) = \sum_{k=1}^N \mu \left(\mathcal{B}_{\mathbf{F}_k} \left(\frac{\delta}{2} \right) \right). \quad (20)$$

Using the density, we find that the probability of the isotropically distributed $\bar{\mathbf{V}}_R$ falling in one of the sets $\mathcal{B}_{\mathbf{F}_k}(\delta/2)$ can be expressed as

$$\Pr \left(\bar{\mathbf{V}}_R \in \bigcup_{k=1}^N \mathcal{B}_{\mathbf{F}_k} \left(\frac{\delta}{2} \right) \right) = \Delta(\mathcal{F}). \quad (21)$$

For large M_t , it has been shown in [35] that

$$\Delta(\mathcal{F}) \approx N \left(\frac{\delta}{2\sqrt{M}} \right)^{2M_t M + o(M_t)}. \quad (22)$$

We can thus bound the “codebook quality” term in (16) as

$$\begin{aligned} E_{\mathbf{H}} \left[\min_{\mathbf{F}' \in \mathcal{F}} \frac{1}{2} \left\| \bar{\mathbf{V}}_R \bar{\mathbf{V}}_R^* - \mathbf{F}' \mathbf{F}'^* \right\|_F^2 \right] \\ \leq \frac{1}{4} \delta^2 \Delta(\mathcal{F}) + M(1 - \Delta(\mathcal{F})) \\ \approx M + N \left(\frac{\delta}{2\sqrt{M}} \right)^{2M_t M + o(M_t)} \left(\frac{1}{4} \delta^2 - M \right). \end{aligned} \quad (23)$$

Differentiating (23) and noting $\delta < \sqrt{M}$ easily shows that the bound is a decreasing function of δ when $2M_t M + o(M_t) > 2/3$. We know from the probabilistic analysis in [40] that this is always satisfied for any M_t when $M = 1$ because $o(M_t) = -2$. In addition, we also know that this assumption is also satisfied for large M_t because of the definition of the little-o Landau symbol. We have experimentally verified this assumption for many other M and conjecture that it is always true. Thus, practically, (23) is minimized by maximizing δ . We have now established that designing low distortion codebooks is equivalent to packing subspaces in the Grassmann manifold using the chordal distance metric.

Therefore, we now understand how to design codebooks for limited feedback precoding. Maximizing the minimum subspace distance between any pair of codebook column spaces approximately minimizes our distortion bound. Thus, we state the following design method for creating limited feedback precoding codebooks.

Codebook Design Summary: The codebook $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N\}$ should be designed such that $\delta = \min_{1 \leq k < l \leq N} d(\mathbf{F}_k, \mathbf{F}_l)$ is as large as possible.

C. Practical Codebook Designs

Finding good precoder codebooks from Grassmannian packings for arbitrary M_t , M , and N is actually quite difficult [29], [31], [38]. For instance, in the simplest case of $M = 1$ where the Rankin lower bound on line packing correlation [38] can be employed, packings that achieve equality with the lower bound are often impossible to design. A comprehensive tabulation of real packings can be found on [41]. The most practical method for generating these packings is to use codebooks designed from the noncoherent space-time modulation designs in [29] and [30].

The search algorithm in [30] can be very easily implemented and yields codebooks with large minimum distances. The algorithm works by considering codebooks of the form

$$\mathcal{F} = \left\{ \mathbf{F}_{\text{DFT}}, \Theta \mathbf{F}_{\text{DFT}}, \dots, \Theta^{N-1} \mathbf{F}_{\text{DFT}} \right\}$$

where $\mathbf{F}_{\text{DFT}}$ is an $M_t \times M$ matrix with $(1/\sqrt{M_t})e^{i(2\pi/M_t)kl}$ at entry (k, l) , and Θ is a diagonal matrix given by

$$\Theta = \begin{bmatrix} e^{i(2\pi/N)u_1} & 0 & \dots & 0 \\ 0 & e^{i(2\pi/N)u_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{i(2\pi/N)u_{M_t}} \end{bmatrix}$$

where

$$0 \leq u_1, \dots, u_{M_t} \leq N - 1.$$

The values for $u_1, u_2, \dots, u_{M_t}$ are chosen according to the entries of the vector $\mathbf{u} = [u_1 \ u_2 \ \dots \ u_{M_t}]^T$ from the set $\mathcal{Z} = \{\mathbf{u} \in \mathbb{Z}^{M_t} \mid \forall k, 0 \leq u_k \leq N-1\}$ given by

$$\mathbf{u} = \arg \max_{\mathcal{Z}} \min_{1 \leq l \leq N-1} d(\mathbf{F}_{\text{DFT}}, \Theta^l \mathbf{F}_{\text{DFT}}).$$

Thus, there are N^{M_t} different possibilities for Θ that must be checked. For small transmit antenna arrays and/or low feedback rates, it is possible to search over all possible values of $\mathbf{u}$ in $\mathcal{Z}$. In general, however, random search methods must be employed to design $\mathcal{F}$. These methods optimize the cost function by randomly testing values of $\mathbf{u}$ using a uniform distribution on $\mathcal{Z}$.

Codebooks designed from [30] also have the added benefit of easy memory storage. The codebook $\mathcal{F}$ can be stored at the transmitter/receiver using $\lceil M_t \log_2 N \rceil$ bits because only the numbers $u_1, u_2, \dots, u_{M_t}$ need to be stored. The chosen codeword matrix $\mathbf{F}$ can then be easily calculated by computing $\Theta^{l-1} \mathbf{F}_{\text{DFT}}$ for the chosen binary codeword index l . Note that the matrix $\mathbf{F}_{\text{DFT}}$ can be either stored or computed at each codebook update.

V. PERFORMANCE ANALYSIS

Because of the difficulty in deriving closed-form SER expressions, we will characterize the diversity of our limited feedback precoders. A signaling scheme is said to obtain diversity order d if [42]

$$d = - \lim_{\rho \rightarrow \infty} \frac{\log \Pr(\text{ERROR})}{\log \rho}.$$

We can bound the asymptotic performance of limited feedback precoding given a channel using the SER bound in (3). Thus in order to understand the diversity performance of limited feedback precoding, we will bound $\max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F$.

It follows from the Poincaré separation theorem [33, pp. 190] that

$$\max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F \leq \|\mathbf{H}\|_F.$$

Note that $\|\mathbf{H}\|_F$ is the post-processing channel gain of an M_t antenna OSTBC that is known to obtain a diversity order of $M_t M_r$ [3]. Therefore, the diversity order of our limited feedback precoders is less than or equal to $M_t M_r$. Thus, all that is needed is a lower bound on diversity order.

Let $\mathbf{f}_{k,l}$ denote the l th column of codebook matrix $\mathbf{F}_k$. The following lemma will prove useful in lower bounding $\max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F$.

Lemma 4: Let $\tilde{\mathcal{F}} = \{\tilde{\mathbf{F}}_1, \tilde{\mathbf{F}}_2, \dots, \tilde{\mathbf{F}}_N\}$ be a codebook with packing minimum distance $\tilde{\delta}$ and $N \geq M_t/M$. If there exists a vector $\mathbf{v} \in \mathbb{C}^{M_t}$ such that $\tilde{\mathbf{F}}_k^* \mathbf{v} = \mathbf{0}$ for all $1 \leq k \leq N$, then we have the following.

- 1) There exists (k', l') such that $\tilde{\mathbf{f}}_{k', l'} = \sum_{j=1}^m \alpha_j \tilde{\mathbf{f}}_{k_j, l_j}$, where k_j and l_j are indexing sequences with $(k_j, l_j) \neq (k', l')$, $0 < |\alpha_j| < 1$, and $1 \leq m < M_t$,
- 2) A new codebook $\mathcal{F}$ with

$$\mathbf{f}_{k,l} = \begin{cases} \tilde{\mathbf{f}}_{k,l}, & \text{if } (k,l) \neq (k', l') \\ \mathbf{v}, & \text{if } (k,l) = (k', l') \end{cases}$$

has minimum distance $\delta \geq \tilde{\delta}$.

Proof: Part 1 follows from the fact that a basis for the column space of the matrix $\tilde{\mathbf{E}} = [\tilde{\mathbf{F}}_1 \ \tilde{\mathbf{F}}_2 \ \dots \ \tilde{\mathbf{F}}_N]$ can be formed from $m < M_t$ columns of the matrix $\tilde{\mathbf{E}}$. Part 2 is a result of the fact that for $k \neq k'$, $\|\tilde{\mathbf{F}}_k^* \tilde{\mathbf{F}}_{k'}\|_F = \|\tilde{\mathbf{F}}_k^* \tilde{\mathbf{F}}_{k'}\|_F \geq \|\tilde{\mathbf{F}}_k^* \mathbf{v}\|_F$, where

$$\mathbf{F}_{k'} = [\tilde{\mathbf{f}}_{k',1} \ \dots \ \tilde{\mathbf{f}}_{k',l'-1} \ \mathbf{v} \ \tilde{\mathbf{f}}_{k',l'+1} \ \dots \ \tilde{\mathbf{f}}_{k',M}].$$

By applying Lemma 4 repeatedly, any $N \geq M_t/M$ matrix codebook $\tilde{\mathcal{F}}$ with minimum distance $\tilde{\delta}$ and columns $\{\tilde{\mathbf{f}}_{k,l}\}$ that do not span $\mathbb{C}^{M_t}$ can be trivially modified to a codebook $\mathcal{F}$ with columns $\{\mathbf{f}_{k,l}\}$ that span $\mathbb{C}^{M_t}$ and minimum distance $\delta \geq \tilde{\delta}$. Therefore, we can state the following theorem.

Theorem 1: If $N \geq M_t/M$ and the columns $\{\mathbf{f}_{k,l}\}$ span $\mathbb{C}^{M_t}$, then $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \dots, \mathbf{F}_N\}$ provides full diversity order.

Proof: First note that the channel power is given by $\max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F^2$. Notice that

$$\begin{aligned} \max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F^2 &\geq \max_{k,l} \|\mathbf{H}\mathbf{f}_{k,l}'\|_1^2 \\ &\geq \frac{1}{M_r} \max_{k,l} \|\mathbf{H}\mathbf{f}_{k,l}'\|_1^2 \\ &= \frac{1}{M_r} \|\mathbf{H}\mathbf{E}\|_1^2 \end{aligned}$$

where $\mathbf{E} = [\mathbf{F}_1 \ \mathbf{F}_2 \ \dots \ \mathbf{F}_N]$. Because the columns $\{\mathbf{f}_{k,l}\}$ span $\mathbb{C}^{M_t}$, we can find $\mathbf{U}_{E,L} \in \mathcal{U}(M_t, M_t)$, $\mathbf{U}_{E,R} \in \mathcal{U}(NM, NM)$, and Φ with diagonal entries $\phi_1 \geq \phi_2 \geq \dots \geq \phi_{M_t} > 0$ such that $\mathbf{E} = \mathbf{U}_{E,L} \Phi \mathbf{U}_{E,R}^*$. Then, using the invariance of complex normal matrices to unitary transformation [36] and the bounds in [40]

$$\begin{aligned} \max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{F}'\|_F^2 &\stackrel{d}{=} \max_{\mathbf{F}' \in \mathcal{F}} \|\mathbf{H}\mathbf{U}_{E,L}^* \mathbf{F}'\|_F^2 \\ &\geq \frac{1}{NMM_r} \|\mathbf{H}\Phi\|_1^2 \\ &\geq \frac{\phi_{M_t}^2}{NMM_r} \|\mathbf{H}\|_1^2 \\ &\geq \frac{\phi_{M_t}^2}{NMM_t^2 M_r} \|\mathbf{H}\|_F^2. \end{aligned}$$

The lower bound $(\phi_{M_t}^2 / NMM_t^2 M_r) \|\mathbf{H}\|_F^2$ is the channel gain of an M_t antenna OSTBC with an array gain shift $(\phi_{M_t}^2 / NMM_t^2 M_r)$. Systems with effective channel gains of this form are known to obtain a diversity order of $M_t M_r$ [3]. We can conclude now that the limited feedback precoders designed using the chordal distance obtain full diversity order.

Because limited feedback precoding maintains full diversity performance, this allows the generalization of OSTBCs to transmission over any transmit antenna configuration with full diversity. This lemma generalizes the result in [14] that proves antenna selection achieves full diversity order.

VI. SIMULATIONS

Precoded OSTBCs using chordal distance limited feedback precoders have been simulated in the following experiments. The precoder codebooks were designed using the criterion proposed in Section IV and implemented with packings designed from [30]. For comparison, OSTBCs with antenna selection have also been simulated [14].

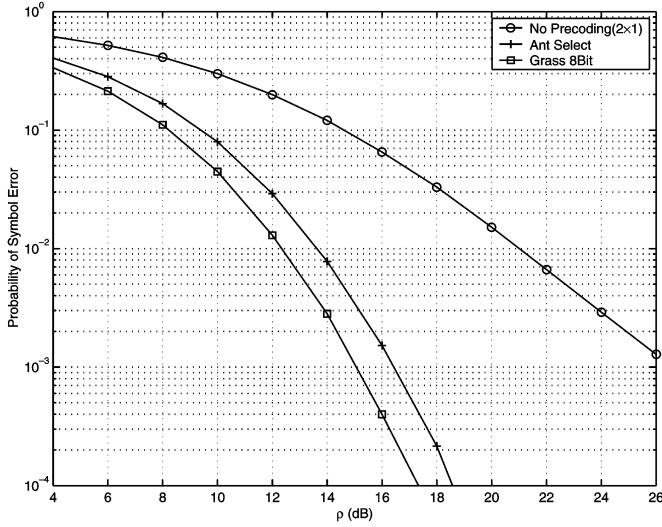


Fig. 2. SER comparison of various OSTBC precoding schemes for a two-substream 8×1 system using 16-QAM.

Note that by our power scaling definitions, ρ is the ratio of the total transmitted energy to noise power for each transmission. For example, if a two antenna Alamouti code were transmitted, then

$$\mathbf{X} = \sqrt{\frac{\rho}{2}} \mathbf{F} \begin{bmatrix} s_1 & s_2^* \\ s_2 & -s_1^* \end{bmatrix}$$

with $E_{s_1} [|s_1|^2] = E_{s_2} [|s_2|^2] = 1$.

Experiment 1: The first experiment compares antenna selection and eight-bit chordal distance precoding for a precoded Alamouti code on an 8×1 wireless system using 16-quadrature amplitude modulation (QAM). The SER performance for a 2×1 Alamouti code is shown for comparison. The results are shown in Fig. 2. Note that antenna selection adds approximately a 7.5-dB gain over the unprecoded system at an error rate of 10^{-2} . Using chordal distance precoding provides approximately a 1.4-dB gain over antenna selection.

Experiment 2: Fig. 3 considers a 4×2 system using a precoded Alamouti code with 4-QAM. Antenna selection provides a 3-dB array gain compared with an unprecoded OSTBC. Chordal distance precoding with a three-bit codebook provides over a 0.3-dB array gain and with a six-bit codebook provides over a 0.7-dB array gain over antenna selection. Interestingly, antenna selection requires $\lceil \log_2 \binom{4}{2} \rceil = 3$ bits of feedback. Thus, by simply lifting the restriction of $\mathbf{F}$ having columns of $\mathbf{I}_{M_t}$ adds a 0.3-dB gain.

Experiment 3: The third experiment addresses the performance of a 5×4 wireless system using the OSTBC

$$\mathbf{C} = \begin{bmatrix} s_1 & 0 & s_2 & -s_3 \\ 0 & s_1 & s_3^* & s_2^* \\ -s_2^* & -s_3 & s_1^* & 0 \end{bmatrix} \quad (24)$$

obtained from [4] with 16-QAM. The results are shown in Fig. 4. Using antenna selection adds around a 1.5-dB improvement over a 3×4 OSTBC. An array gain of approximately 0.4 dB is added by using six bit chordal distance precoding instead of antenna selection. Chordal distance precoding with

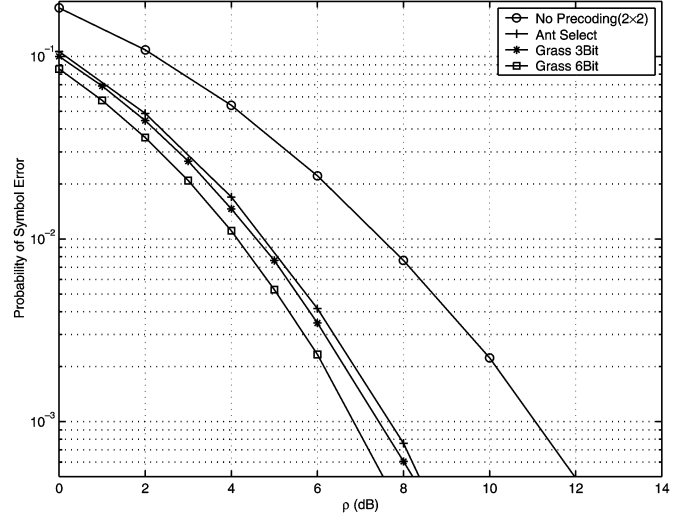


Fig. 3. SER comparison of various OSTBC precoding schemes for a two-substream 4×2 system using 4-QAM.

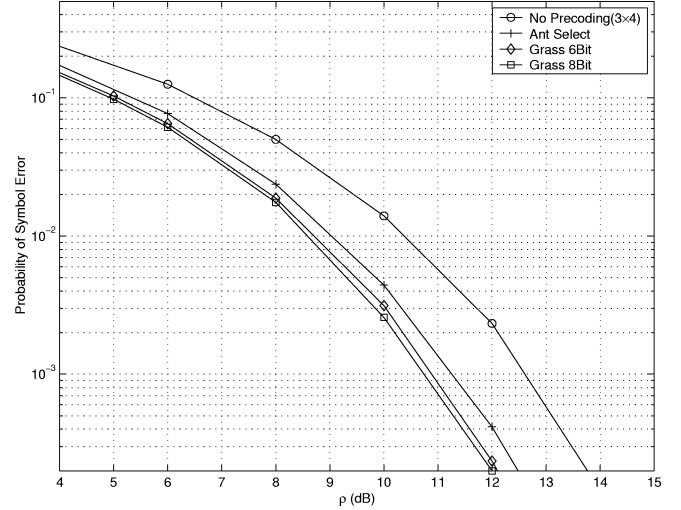


Fig. 4. SER comparison of various OSTBC precoding schemes for a three-substream 5×4 system using 16-QAM.

an eight bit codebook outperforms antenna selection by more than 0.5 dB.

Experiment 4: The fourth experiment, which is shown in Fig. 5, addresses the performance of a precoded OSTBC using the code in (24) over a 6×3 wireless with 16-QAM. Six- and eight-bit chordal distance precoding have array gains of approximately 2.1 and 2.3 dB, respectively, compared with an unprecoded 3×3 OSTBCs. Note that six bits provides most of the array gain available with eight bits; thus, a careful simulation analysis must be done in any practical system before choosing a given feedback rate.

Experiment 5: The effect of channel estimation error during precoder selection for an 8×1 Alamouti system is addressed in Fig. 6. Antenna selection and eight-bit chordal distance precoding are simulated for various channel correlations. We simulate estimation error by assuming that the receiver chooses $\mathbf{F} = f(\mathbf{H}_{est})$ using knowledge of a matrix $\mathbf{H}_{est}$, where

$$\mathbf{H}_{est} = \alpha \mathbf{H} + \sqrt{1 - \alpha^2} \mathbf{H}_{error} \quad (25)$$

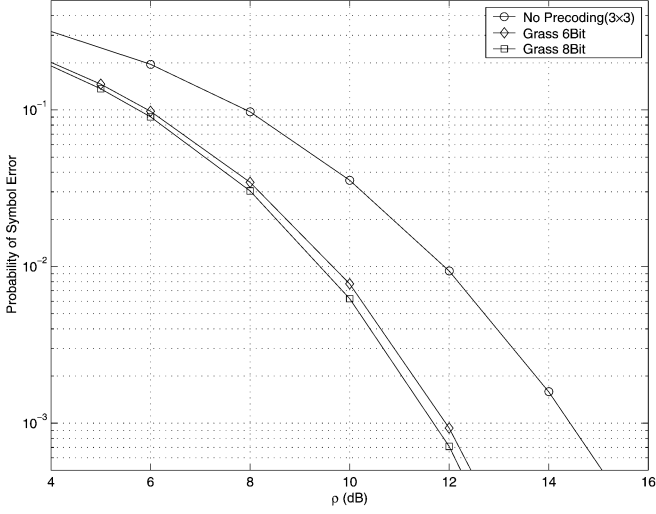


Fig. 5. SER comparison of various OSTBC precoding schemes for a three-substream 6×3 system using 16-QAM.

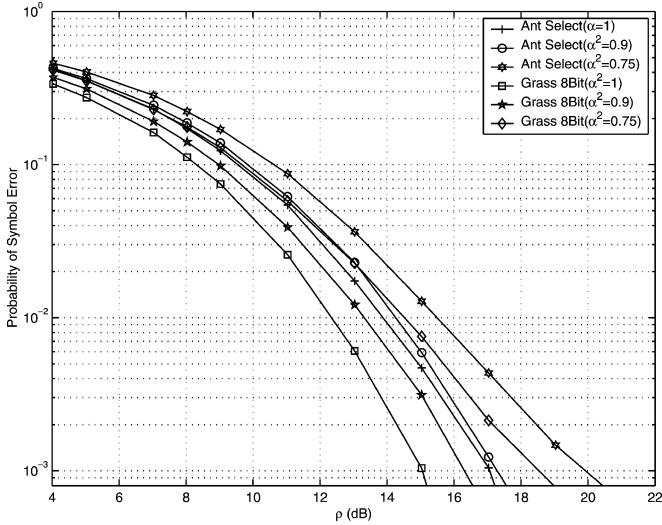


Fig. 6. SER comparison of various OSTBC precoding schemes with channel estimation error for a two substream 8×1 system using 16-QAM.

where the entries of $\mathbf{H}_{\text{error}}$ are independent and distributed according to $\mathcal{CN}(0, 1)$. Notice that by our definition for α , this means $E[\text{vec}(\mathbf{H}_{\text{est}})\text{vec}(\mathbf{H})^*] = \alpha \mathbf{I}_{M_r M_t}$. Note that $\mathbf{H}_{\text{est}}$ is a *biased estimate* of $\mathbf{H}$. This is to emphasize that the channel estimate is not only a *noisy estimate* but also a *dated estimate*. Thus, the precoder was designed based on an outdated channel estimate.

It is assumed that the ML decoder has perfect knowledge of $\mathbf{H}\mathbf{F}$ in order to isolate the effect of precoder mismatch. Interestingly, eight-bit chordal distance precoding still outperforms perfect channel knowledge antenna subset selection for $\alpha^2 = 0.9$. When $\alpha^2 = 0.75$, chordal distance precoding performs approximately the same as $\alpha^2 = 0.9$ antenna selection. This result demonstrates that chordal distance precoding allows a tradeoff between the quality of the channel estimation and the rate of the feedback path. At the expense of requiring more feedback, limited feedback chordal distance precoding can obtain a lower

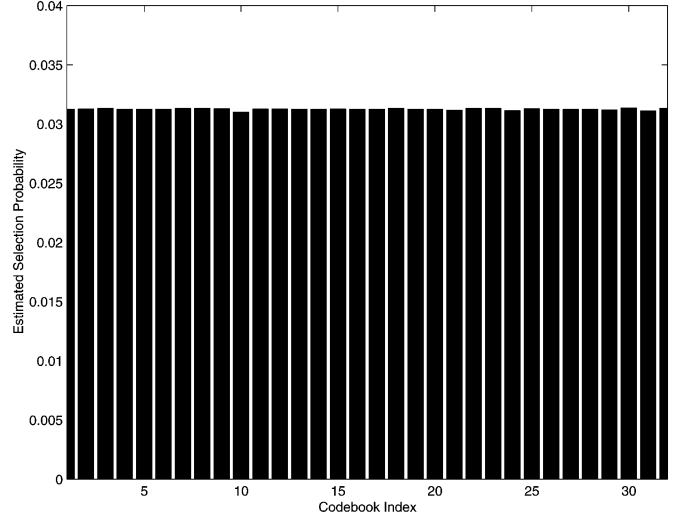


Fig. 7. Probability of selection for each possible codebook precoding matrix for a precoded Alamouti code transmitted on an 8×8 wireless system.

probability of error than antenna selection even in the presence of channel estimation error.

Experiment 6: The final experiment validates that the codebooks achieve the entropy lower bound on codeword bit length [43, pp. 86]. In this experiment, an 8×8 wireless system transmitting a precoded Alamouti code was simulated in order to estimate the probability that each codeword precoder is selected. A five-bit codebook was simulated. The results are shown in Fig. 7. Note that a uniform distribution across the codebook would mean that each matrix is selected with probability $1/2^5 = 0.03125$. All estimated selection probabilities are within 2.5×10^{-4} of the uniform selection probability. Thus, the precoder matrix to be encoded $\mathbf{F} = f(\mathbf{H})$ has an entropy of

$$\sum_{j=1}^{2^5} 2^{-5} \log_2(2^5) = 5 \text{ bits.}$$

An optimal data compression algorithm to encode the precoder matrix $\mathbf{F}$ can *at best* achieve an average codeword bit length of five bits. Because we transmit each precoder codeword with $\log_2 N = 5$ bits, our codebook has been experimentally shown to obtain the optimal codeword bit length lower bound [43].

VII. CONCLUSIONS AND FUTURE WORK

We addressed the problem of limited feedback precoding for OSTBCs. Our approach subsumes the antenna selection problem and allows for a general design and analysis of limited feedback precoding. We proposed a system that uses a codebook of precoding matrices known a priori to the transmitter and receiver. The optimal precoding matrix according to a probability of error selection criterion is chosen from this codebook by the receiver and conveyed to the transmitter over an error-free, zero-delay feedback channel. We addressed the problem of codebook design and showed that it relates to packing subspaces in the Grassmann manifold using the chordal distance.

It is of interest to extend these results to more general space-time codes such as [44]–[48].

REFERENCES

- [1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, pp. 744–765, Mar. 1998.
- [2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 16, pp. 1451–1458, Oct. 1998.
- [3] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1456–1467, July 1999.
- [4] E. G. Larsson and P. Stoica, *Space-Time Block Coding for Wireless Communications*. New York: Cambridge Univ. Press, 2003.
- [5] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. New York: Cambridge Univ. Press, 2003.
- [6] *WCDMA for UMTS: Radio Access for Third Generation Mobile Communications*, Revised ed., H. Holma and A. Toskala, Eds., Wiley, New York, 2001.
- [7] J. Heiskala and J. Terry, *OFDM Wireless LANs: A Theoretical and Practical Guide*. Indianapolis, IN: SAMS, 2002.
- [8] G. Bauch and J. Hagenauer, "Smart versus dumb antennas-capacities and FEC performance," *IEEE Commun. Lett.*, vol. 6, pp. 55–57, Feb. 2002.
- [9] G. Jöngren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthogonal space-time block coding," *IEEE Trans. Inf. Theory*, vol. 48, pp. 611–627, Mar. 2002.
- [10] G. Jöngren and M. Skoglund, "Improving orthogonal space-time block codes by utilizing quantized feedback information," in *Proc. IEEE Int. Symp. Inf. Theory*, June 2001, p. 220.
- [11] —, "Utilizing quantized feedback information in orthogonal space-time block coding," in *Proc. IEEE Glob. Telecom. Conf.*, vol. 2, Nov.–Dec. 2000, pp. 995–999.
- [12] G. Jöngren, M. Skoglund, and B. Ottersten, "Design of channel estimate dependent space-time block codes," *IEEE Trans. Commun.*, vol. 52, pp. 1191–1203, Jul. 2004.
- [13] E. G. Larsson, G. Ganesan, P. Stoica, and W.-H. Wong, "On the performance of orthogonal space-time block coding with quantized feedback," *IEEE Commun. Lett.*, vol. 6, pp. 487–489, Nov. 2002.
- [14] D. A. Gore and A. J. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Process.*, vol. 50, pp. 2580–2588, Oct. 2002.
- [15] W.-H. Wong and E. G. Larsson, "Orthogonal space-time block coding with antenna selection and power allocation," *Electron. Lett.*, vol. 39, pp. 379–381, Feb. 2003.
- [16] G. Ganesan, P. Stoica, and E. G. Larsson, "Diagonally weighted orthogonal space-time block codes," in *Proc. 36th Asilomar Conf. Signals, Syst. Comput.*, vol. 2, Nov. 2002, pp. 1147–1151.
- [17] T. Xiaofeng, H. Harald, Y. Zhuizhuan, Q. Haiyan, and Z. Ping, "Closed loop space-time block code," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, Oct. 2001, pp. 1093–1096.
- [18] J. Akhtar and D. Gesbert, "Partial feedback based orthogonal block coding," in *Proc. IEEE Veh. Technol. Conf.*, vol. 1, Apr. 2003, pp. 287–291.
- [19] T. Lo, "Adaptive space-time transmission with side information," in *Proc. IEEE Wireless Comm. and Net. Conf.*, vol. 1, Mar. 2003, pp. 601–606.
- [20] S. Zhou and G. B. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Trans. Signal Process.*, vol. 50, pp. 2599–2613, Oct. 2002.
- [21] —, "Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations," *IEEE Trans. Inf. Theory*, vol. 49, pp. 1673–1690, July 2003.
- [22] A. Scaglione, P. Stoica, S. Barbarossa, G. B. Giannakis, and H. Sampath, "Optimal designs for space-time linear precoders and decoders," *IEEE Trans. Signal Process.*, vol. 50, pp. 1051–1064, May 2002.
- [23] R. W. Heath Jr, S. Sandhu, and A. Paulraj, "Antenna selection for spatial multiplexing systems with linear receivers," *IEEE Commun. Lett.*, vol. 5, pp. 142–144, Apr. 2001.
- [24] D. A. Gore, R. W. Heath Jr, and A. Paulraj, "Transmit selection in spatial multiplexing systems," *IEEE Commun. Lett.*, vol. 6, pp. 491–493, Nov. 2002.
- [25] D. Gore, R. Nabar, and A. Paulraj, "Selecting an optimal set of transmit antennas for a low rank matrix channel," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, vol. 5, June 2000, pp. 2785–2788.
- [26] D. J. Love and R. W. Heath Jr, "Limited feedback precoding for spatial multiplexing systems using linear receivers," in *Proc. IEEE Military Commun. Conf.*, vol. 1, Oct. 2003, pp. 627–632.
- [27] —, "Limited feedback precoding for spatial multiplexing systems," in *Proc. IEEE Glob. Telecom. Conf.*, vol. 4, Dec. 2003, pp. 1857–1861.
- [28] R. S. Blum, "MIMO with limited feedback of channel state information," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process.*, vol. 4, Apr. 2003, pp. 89–92.
- [29] D. Agrawal, T. J. Richardson, and R. L. Urbanke, "Multiple-antenna signal constellations for fading channels," *IEEE Trans. Inf. Theory*, vol. 47, pp. 2618–2626, Sep. 2001.
- [30] B. M. Hochwald, T. L. Marzetta, T. J. Richardson, W. Sweldens, and R. Urbanke, "Systematic design of unitary space-time constellations," *IEEE Trans. Inf. Theory*, vol. 46, pp. 1962–1973, Sep. 2000.
- [31] J. H. Conway, R. H. Hardin, and N. J. A. Sloane, "Packing lines, planes, etc.: packings in Grassmannian spaces," *Experimental Math.*, vol. 5, pp. 139–159, 1996.
- [32] A. Gersho and R. M. Gray, *Vector Quantization and Signal Compression*. Boston, MA: Kluwer, 1992.
- [33] R. A. Horn and C. R. Johnson, *Matrix Analysis*. New York: Cambridge Univ. Press, 2001.
- [34] H. Sampath, P. Stoica, and A. Paulraj, "Generalized linear precoder and decoder design for MIMO channels using the weighted MMSE criterion," *IEEE Trans. Commun.*, vol. 49, pp. 2198–2206, Dec. 2001.
- [35] A. Barg and D. Y. Nogin, "Bounds on packings of spheres in the Grassmann manifold," *IEEE Trans. Inf. Theory*, vol. 48, pp. 2450–2454, Sep. 2002.
- [36] A. T. James, "Distributions of matrix variates and latent roots derived from normal samples," *Ann. Math. Statist.*, vol. 35, pp. 475–501, June 1964.
- [37] T. L. Marzetta and B. M. Hochwald, "Capacity of a mobile multiple-antenna communication link in Rayleigh flat fading," *IEEE Trans. Inf. Theory*, vol. 45, pp. 139–157, Jan. 1999.
- [38] T. Strohmer and R. W. Heath Jr, "Grassmannian frames with applications to coding and communications," *Appl. Comput. Harmon. Anal.*, vol. 14, pp. 257–275, May 2003.
- [39] L. Zheng and D. N. C. Tse, "Communication on the Grassmann manifold: A geometric approach to the noncoherent multiple-antenna channel," *IEEE Trans. Inf. Theory*, vol. 48, pp. 359–383, Feb. 2002.
- [40] D. J. Love, R. W. Heath Jr, and T. Strohmer, "Grassmannian beamforming for multiple-input multiple-output wireless systems," *IEEE Trans. Inf. Theory*, vol. 49, pp. 2735–2747, Oct. 2003.
- [41] N. J. A. Sloane. (2003) Tables of Real Grassmannian Packings. [Online]. Available: <http://www.research.att.com/~njas/grass/index.html>
- [42] L. Zheng and D. N. C. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inf. Theory*, vol. 49, pp. 1073–1096, May 2003.
- [43] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1992.
- [44] M. O. Damen, K. Abed-Meraim, and J. C. Belfiore, "Diagonal algebraic space-time block codes," *IEEE Trans. Inf. Theory*, vol. 48, pp. 628–636, Mar. 2002.
- [45] H. El Gamal and A. R. Hammons Jr, "On the design of algebraic space-time codes for MIMO block-fading channels," *IEEE Trans. Inf. Theory*, vol. 49, pp. 151–163, Jan. 2003.
- [46] A. R. Hammons Jr and H. El Gamal, "On the theory of space-time codes for PSK modulation," *IEEE Trans. Inf. Theory*, vol. 46, pp. 524–542, Mar. 2000.
- [47] N. Hassanpour and H. Jafarkhani, "Super-quasi-orthogonal space-time trellis codes," in *Proc. IEEE Int. Conf. Commun.*, vol. 4, May 2003, pp. 2613–2617.
- [48] H. Jafarkhani, "A quasi-orthogonal space-time block code," *IEEE Trans. Commun.*, vol. 49, pp. 1–4, Jan. 2001.



David J. Love (S'98–M'05) received the B.S. (with highest honors), M.S.E., and Ph.D. degrees in electrical engineering in 2000, 2002, and 2004, respectively, from The University of Texas at Austin.

During the summers of 2000 and 2002, he was a summer researcher at the Texas Instruments DSPS R&D Center, Dallas, TX. At Texas Instruments, he performed research on physical layer design for next-generation wireless systems employing multiple antennas. Since August 2004, he has been with the School of Electrical and Computer Engineering,

Purdue University, West Lafayette, IN, as an Assistant Professor. His research interests are in the design and analysis of communication systems, coding theory, and information theory.

Prof. Love is a member of SIAM, Tau Beta Pi, and Eta Kappa Nu. In 2003, he was the recipient of the IEEE Vehicular Technology Society Daniel Noble Fellowship.



Robert W. Heath, Jr. (S'96–M'01) received the B.S. and M.S. degrees from the University of Virginia, Charlottesville, in 1996 and 1997, respectively, and the Ph.D. from Stanford University, Stanford, CA, in 2002, all in electrical engineering.

From 1998 to 1999, he was a Senior Member of the Technical Staff at Iospan Wireless Inc, San Jose, CA, where he played a key role in the design and implementation of the physical and link layers of the first commercial MIMO-OFDM communication system.

From 1999 to 2001, he served as a Senior Consultant for Iospan Wireless Inc. In 2003, he founded MIMO Wireless Inc., a consulting company dedicated to the advancement of MIMO technology. Since January 2002, he has been with the Department of Electrical and Computer Engineering, The University of Texas at Austin, where he serves as an Assistant Professor as part of the Wireless Networking and Communications Group. His research interests include interference management in wireless networks, sequence design, and all aspects of MIMO communication including antenna design, practical receiver architectures, limited feedback techniques, and scheduling algorithms.

Dr. Heath serves as an Associate Editor for the IEEE TRANSACTIONS ON COMMUNICATIONS and the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY.