On the Performance of Random Vector Quantization Limited Feedback Beamforming in a MISO System

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***Abstract*— In multiple antenna wireless systems, beamforming is a simple technique for guarding against the negative effects of fading. Unfortunately, beamforming requires the transmitter to have knowledge of the forward-link channel which is often not available a priori. One way of overcoming this problem is to design the beamforming vector using a limited number of feedback bits sent from the receiver to the transmitter. In limited feedback beamforming, the beamforming vector is restricted to lie in a codebook that is known to both the transmitter and receiver. Random vector quantization (RVQ) is a simple approach to codebook design that generates the vectors independently from a uniform distribution on the complex unit sphere. This correspondence presents performance analysis results for RVQ limited feedback beamforming.**

***Index Terms*— limited feedback, multiple antenna system, random beamforming codebook, random vector quantization, transmit diveristy**

I. Introduction

Multiple antenna beamforming is an efficient technique for providing improved performance in fading channels. In beamforming, each transmit antenna is weighted by a properly designed gain and phase shift before transmission. Unfortunately, transmit channel knowledge is required to design the antenna weights.

Limited feedback beamforming, first proposed in [1], [2], can be employed when the transmitter does not have a priori channel knowledge. When implementing limited feedback beamforming, the beamforming vector is restricted to lie in a finite set or codebook that is known to both the transmitter and receiver. The receiver uses its channel estimate to choose the vector from the codebook that maximizes the conditional receive signal-to-noise ratio (SNR). The binary index of the chosen vector is then conveyed to the transmitter over a limited rate feedback channnel.

Recent work has extended the concept of limited feedback beamforming to quantized equal gain transmission [3] and to more general Grassmannian beamforming [4]–[6]. These works used deterministic codebooks that were designed using techniques from Grassmannian line packing. A simple and easily adaptable codebook design is to randomly generate the codebook. Santipach and Honig proposed random vector quantization (RVQ) limited feedback beamforming and analyzed asymptotic performance by keeping the number of feedback bits per transmit antenna constant [7], [8]. In RVQ beamforming, the codebook, which is known to both the transmitter and receiver, is randomly generated each time the channel changes. Other work on limited feedback analysis has been done in [9], [10].

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This correspondence analyzes the performance of RVQ limited feedback beamforming on multiple-input single-output (MISO) channels. The transmitter has access to a low rate, noiseless and zero delay feedback channel from the receiver. Closed-form expressions for the average SNR, outage probability, average bit error probability, and ergodic capacity are derived.

The paper will proceed as follows. Section II provides an overview of the system configuration. Section III analyzes the SNR performance. An outage analysis is performed in Section IV. Section V presents bit error rate results, and Section VI derives the ergodic capacity. We conclude in Section VII.

II. System Model

Consider a MISO system with m transmit antennas and a single receive antenna. The channel is assumed to be frequency flat and block fading. This allows us to model the channel vector as a $1 \times m$ random vector $\mathbf{h} = [h_1, h_2, \ldots, h_m]$. We assume that the entries in $\mathbf{h}$ are independent and identically distributed (i.i.d.) $\mathcal{CN}(0, 1)$. We assume that the transmitter array transmits a single dimensional symbol s chosen from a constellation $\mathcal{S}$. Before transmission on antenna i, the symbol is weighted by a complex number w_i. The weights for all antennas can be collected into an $m \times 1$ beamforming vector $\mathbf{w} = [w_1, w_2, \ldots, w_m]^T$ where $(\cdot)^T$ denotes transposition. The system input-output relationship can be modeled as

$$x = \mathbf{hw}s + n \tag{1}$$

where x is the processed signal at the receiver and n is a $\mathcal{CN}(0, N_0)$ noise term. For power constraint reasons, we will assume that $E[|s|^2] = \mathcal{E}_s$ where $E[\cdot]$ denotes expectation. This yields an average transmit power conditioned on $\mathbf{w}$ of $\|\mathbf{w}\|_2^2\mathcal{E}_s$ where $\|\cdot\|_2$ is the vector two-norm. To limit the total transmit power to $\mathcal{E}_s$, we will require that $\|\mathbf{w}\|_2 = 1$. The receiver decodes by selecting the symbol from the constellation set $\mathcal{S}$ that minimizes its distance with x. Define $\rho = \mathcal{E}_s/N_0$. From (1), the received SNR (averaged with respect to the symbol and noise power) is given by

$$\gamma = \rho|\mathbf{hw}|^2. \tag{2}$$

The beamforming vector will be chosen from a randomly generated codebook $\mathcal{F}$ that will be made available to both transmitter and receiver. Let the cardinality of $\mathcal{F}$ be denoted by the positive integer N (i.e. $\mathcal{F} = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_N\}$). The codebook $\mathcal{F}$ will be randomly generated by selecting each of the N vectors independently from the uniform distribution on the complex unit sphere. Using the codebook, the receiver will choose the SNR-maximizing beamforming vector, yielding $\mathbf{w} = \operatorname{argmax}_{\mathbf{f}\in\mathcal{F}} |\mathbf{hf}|^2$. Thus, the conditional SNR at the receive side is

$$\gamma = \rho \max_{\mathbf{w}\in\mathcal{F}} |\mathbf{hw}|^2. \tag{3}$$

In this correspondence, we assume the existence of a low rate, error-free, and zero-delay communication link from the receiver to the transmitter for the purpose of conveying $\mathbf{w}$ to the transmitter. Because the codebook is finite, the beamforming vector $\mathbf{w}$ can be conveyed from the receiver to the transmitter using $\lceil \log_2 N \rceil$ bits of feedback.

III. Signal-to-Noise Ratio Analysis

This section derives the closed-form expressions for the expected SNR for this system. The expected SNR of the system can be found by taking the expectation of (3) with respect to $\mathbf{h}$ and $\mathbf{w}$. Define

$$\nu = \max_{\mathbf{w}\in\mathcal{F}} |\mathbf{hw}|^2 / \|\mathbf{h}\|_2^2.$$

The SNR expression can be rewritten as

$$\gamma = \rho \max_{\mathbf{w}\in\mathcal{F}} |\mathbf{hw}|^2 = \rho\nu \|\mathbf{h}\|_2^2 \tag{4}$$

where ν and $\|\mathbf{h}\|_2^2$ are independent [11]. Using the assumption of spatially uncorrelated Rayleigh fading, $\|\mathbf{h}\|_2^2$ is chi-squared distributed. ν is the squared normalized inner product between $\mathbf{h}$ and $\mathbf{w}$. The density of ν is given by the following lemma:

Lemma 1: The cumulative distribution function (cdf) of $\nu = \max_{\mathbf{w}\in\mathcal{F}} |\mathbf{hw}|^2 / \|\mathbf{h}\|_2^2$ in a MISO system with m transmit antennas using a random N unit vector beamforming codebook is given by

$$\begin{aligned} F_\nu(\nu) &= \left(1 - (1-\nu)^{m-1}\right)^N & (5)\\ &= \sum_{i=0}^{N} \binom{N}{i} (-1)^i (1-\nu)^{i(m-1)} & (6)\\ &= \sum_{i=0}^{N} \sum_{j=0}^{i(m-1)} \binom{N}{i} \binom{i(m-1)}{j} (-1)^{i+j} \nu^j & (7) \end{aligned}$$

for $\nu \in [0,1]$. In addition, for $\nu \in [0,1]$, the probability density function (pdf) is given by

$$\begin{aligned} f_\nu(\nu) &= N(m-1)\left(1 - (1-\nu)^{m-1}\right)^{N-1} (1-\nu)^{m-2} & (8)\\ &= \sum_{i=1}^{N} \binom{N}{i} (-1)^{i+1} i(m-1)(1-\nu)^{i(m-1)-1} & (9)\\ &= \sum_{i=0}^{N} \sum_{j=1}^{i(m-1)} \binom{N}{i} \binom{i(m-1)}{j} (-1)^{i+j} j \nu^{j-1}. & (10) \end{aligned}$$

Proof: Using results from [5, Theorem 1] and [4], the cdf of the squared absolute inner product between two uniformly distributed unit vectors is, for $\nu \in [0,1]$, given by

$$1 - (1-\nu)^{m-1}. \tag{11}$$

To find the cdf of ν, we realize that the cdf of the maximum squared inner product of N independent beamforming vectors is the same as (11) raised to the N-th power, which results in (5). Eqs. (6) and (7) used the binomial series expansion $(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k$ on (5). Eqs. (8), (9), and (10) are derived by taking derivative of (5), (6), and (7), respectively. ∎

The pdf and cdf of ν allow us to find the expectation of ν, which will be used for deriving the expected SNR. The following corollary derives the expectation of ν.

Corollary 1: The expected value of ν is given by the equation

$$E[\nu] = 1 - \sum_{i=0}^{N} \frac{\binom{N}{i}(-1)^i}{i(m-1)+1}. \tag{12}$$

Proof: Integrating $E[\nu] = \int_0^1 \nu f_\nu(\nu) d\nu$ by parts and using (6) yields (12). ∎

Applying $\binom{N}{k} = \frac{(-1)^k (-N)_k}{k!}$ [12, P.6], where $(z)_k = \frac{\Gamma(k+z)}{\Gamma(z)}$ is the Pochmann symbol, and then [12, 6.6.8] gives

$$\sum_{k=0}^{N} \frac{\binom{N}{k}(-1)^k}{k(m-1)+1} = NB\left(N, \frac{m}{m-1}\right) \tag{13}$$

where $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ is the Beta function and $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the Gamma function. Using Corollary 1 and $E[\|\mathbf{h}\|_2^2] = m$, the expected SNR is simply

$$E[\gamma] = m\rho - m\rho N B\left(N, \frac{m}{m-1}\right). \tag{14}$$

Note that the second term in (14) denotes the SNR loss due to quantization. To study the rate of decay of quantization loss due to codebook size, the loss term can be expressed as

$$\begin{aligned} m\rho N B\left(N, \frac{m}{m-1}\right) &= m\rho N \frac{\Gamma(N)\Gamma\left(\frac{m}{m-1}\right)}{\Gamma\left(N + \frac{m}{m-1}\right)} \\ &\sim \frac{\Gamma(N+1)}{\Gamma\left(N+1+\frac{1}{m-1}\right)} \end{aligned} \tag{15}$$

where $\sim$ denotes proportionality. Using Stirling's approximation to the Gamma function, this loss term has the order of $N^{-\frac{1}{m-1}}$. Using $\lim_{x\to\infty} \frac{\Gamma(x)}{\Gamma(x+a)} e^{a \log_2 x} = 1$ [13, 8.328.2],

$$\begin{aligned} &\lim_{N\to\infty} NB\left(N, \frac{m}{m-1}\right) \\ &= \underbrace{\lim_{N\to\infty} \frac{\Gamma(N+1) e^{\frac{1}{m-1}\ln(N+1)}}{\Gamma\left(N+1+\frac{1}{m-1}\right)}}_{=1} \lim_{N\to\infty} \frac{\Gamma\left(\frac{m}{m-1}\right)}{e^{\frac{1}{m-1}\ln(N+1)}} = 0. \end{aligned}$$

Simulation: Figure 1 shows the closed-form expected SNR expression in (14) matched against simulation for the $(m,N) = (2,8)$ and $(m,N) = (3,16)$ cases. The ideally achievable SNR with perfect transmit channel knowledge maximum ratio transmission (MRT) is also included to show quantization loss. Figure 2 shows the effect of increasing the number of beamforming vectors N in the codebook. Clearly the expected SNR approaches the ideal MISO system as $N \to \infty$.

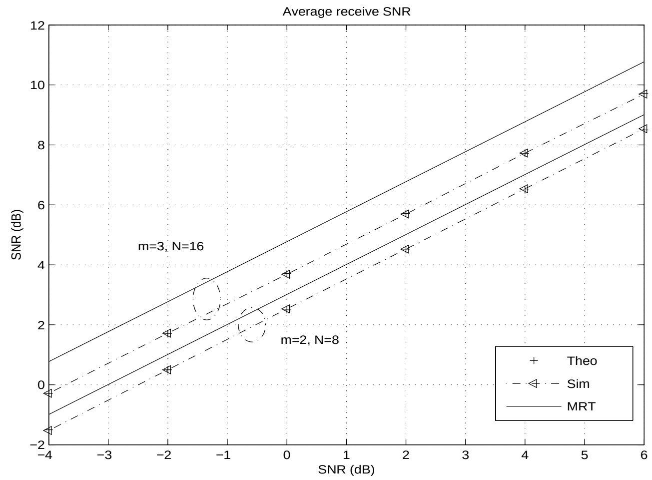

Fig. 1. Closed-form SNR expression matched against simulation. The ideal lossless SNR is also included to measure against the SNR loss due to quantization

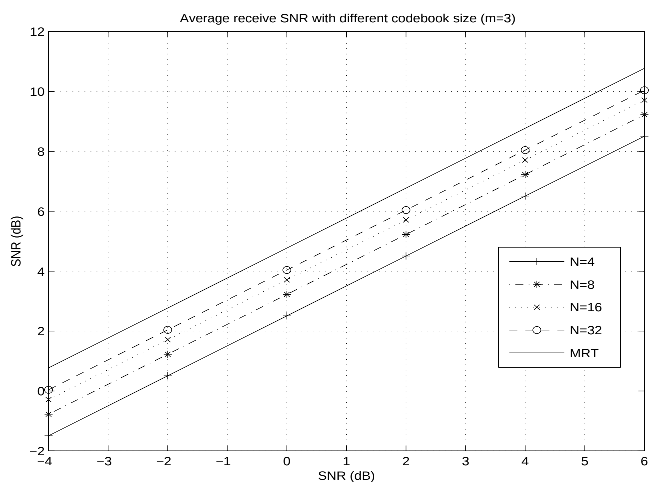

Fig. 2. Average receive SNR for various codebook sizes.

IV. Outage Probability Analysis

This section derives the closed-form expression for the outage probability of the system. The general approach of taking expectation with respect to the channel magnitude and squared absolute correlation will again be employed. The following lemma gives the exact expression for the outage probability of the RVQ limited feedback beamforming system.

Lemma 2: The outage probability $P_{out}(\rho, R)$ for an SNR ρ and rate R in a MISO system with m transmit antennas using a random N vector beamforming codebook is given by

$$P_{out}(\rho,R) = \frac{\gamma\left(m, \frac{2^R-1}{\rho}\right)}{\Gamma(m)} + \frac{\left(\frac{2^R-1}{\rho}\right)^m e^{-\frac{2^R-1}{\rho}}}{\Gamma(m)} \sum_{i=0}^{N} \binom{N}{i} \times (-1)^i \Gamma(i(m-1)+1) U\left(im-i+1, 1+m; \frac{2^R-1}{\rho}\right) \quad (16)$$

where $U(a,b;x) = \frac{1}{\Gamma(a)} \int_0^\infty e^{-xt} t^{a-1}(1+t)^{b-a-1} dt$ is the Kummer U function (also known as the Tricomi function and confluent hypergeometric function of the second kind) and $\gamma(n,x) = \int_0^x t^{a-1} e^{-t} dt$ is the lower incomplete Gamma function.

Proof: The conditional capacity for the beamforming system is given by

$$C(\rho|\mathbf{h}) = \max_{\mathbf{w}\in\mathcal{F}} \log_2\left(1+\rho|\mathbf{hw}|^2\right) = \log_2\left(1+\rho\nu\|\mathbf{h}\|_2^2\right). \quad (17)$$

The outage probability P_{out} is the probability that a realized channel cannot support a given transmission rate R. The current system (1) is equivalent to a single-input single-output (SISO) system with SNR γ. Hence, the outage probability is given as

$$P_{out}(\rho,R) = P\left(C(\rho|\mathbf{h}) < R\right) = P\left(\|\mathbf{h}\|_2^2 \nu < \frac{2^R-1}{\rho}\right). \quad (18)$$

Let $h = \|\mathbf{h}\|_2^2$ and $c = \frac{2^R-1}{\rho}$. Eq. (18) can be evaluated by finding the cdf of h and then taking the expectation with respect to ν. This gives

$$\begin{aligned} &P\left(\nu\|\mathbf{h}\|_2^2 < c\right) \\ &= \frac{1}{\Gamma(m)} \underbrace{\int_0^c e^{-h} h^{m-1} dh}_{=\gamma(m,c)} + \frac{1}{\Gamma(m)} \sum_{i=0}^{N} \binom{N}{i} (-1)^i c \\ &\quad \times \int_0^1 \frac{(1-\nu)^{i(m-1)} e^{-\frac{c}{\nu}} \left(\frac{c}{\nu}\right)^{m-1}}{\nu^2} d\nu \quad (19) \\ &= \frac{\gamma(m,c)}{\Gamma(m)} + \frac{1}{\Gamma(m)} \sum_{i=0}^{N} \binom{N}{i} (-1)^i c^{\frac{m-1}{2}} e^{-\frac{c}{2}} \\ &\quad \Gamma(i(m-1)+1) W_{\frac{(1-2i)(m-1)}{2}, \frac{m}{2}}(c). \quad (20) \end{aligned}$$

Eq. (19) is derived using integration by parts, and then substituting (6) and the pdf of a chi-squared random variable. Eq. (20) used the entries [13, 3.471.2] and $W_{\lambda,-\mu}(z) = W_{\lambda,\mu}(z)$ [13, 9.232.1] where $W_{\lambda,\mu}(z)$is the Whittaker function. Expressing the Whittaker function as Kummer U function using [14]

$$W_{k,m}(z) = e^{-z/2} z^{m+1/2} U(m-k+1/2, 1+2m; z)$$

and substituting $c = \frac{2^R-1}{\rho}$ yields the closed-form outage probability expression in (16). ∎

Notice again that the first term of (16) is the ideal outage probability without quantization loss. Therefore, the second term represents the extra outage probability due to quantization.

Simulation: Figure 3 shows the outage probability expression in (16) plotted along with simulated results for the $(m,N) = (2,8)$ and $(m,N) = (3,16)$ cases. Both used a rate of 1.1 bits/sec/Hz. The outage probability in a MRT system is also included to show quantization loss.

V. Bit Error Probability Analysis

This section will develop the bit error probability (BEP) for a RVQ limited feedback MISO beamforming system using binary biorthogonal modulation (ex. BPSK) or orthogonal modulation (ex. BFSK). The following lemma gives the closed-form expression of the BEP.

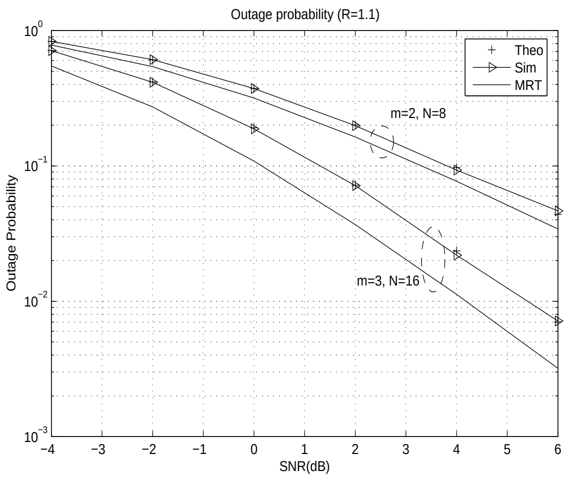

Fig. 3. Closed-form outage probability expression matched against simulation.

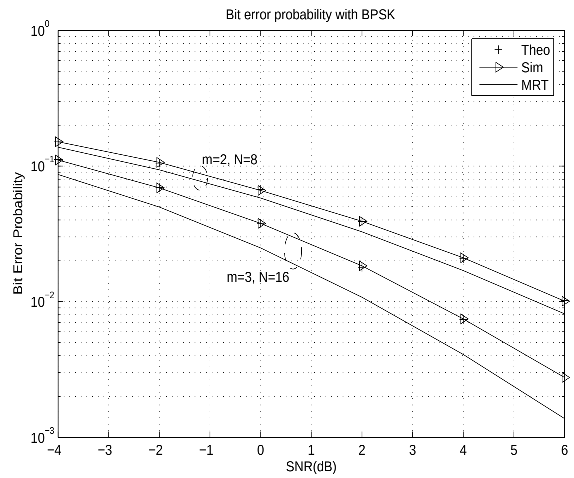

Fig. 4. Closed-form BEP expression matched against simulation with three transmit antennas and a codebook size of $N = 8$.

Lemma 3: The BEP $P_e(\rho)$ for an SNR ρ in a MISO system with m transmit antenas using a random N vector beamforming codebook employing biorthogonal or orthogonal modulation is given by

$$\begin{aligned} P_e(\rho) = &\sum_{k=0}^{m-1}\sum_{a=0}^{m-k}\sum_{i=1}^{N} \frac{(-1)^{a+i+1} i(m-1)(q\rho)^{\frac{a}{2}}}{2^{m+k}} \binom{m-k}{a} \\ &\times \binom{m-1+k}{k}\binom{N}{i} B\left(i(m-1), \frac{a}{2}+1\right) \\ &\times {}_2F_1\left(\frac{a}{2}+k, \frac{a}{2}+1; i(m-1)+\frac{a}{2}+1; -q\rho\right) \end{aligned} \tag{21}$$

where q is a constant related to modulation, $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ is the Beta function and ${}_2F_1(\alpha,\beta;\nu;z)$ is the Gauss hypergeometric function. For biorthogonal signaling, $q = 1$. For orthogonal signaling, $q = 0.5$.

Proof: The BEP of an additive noise SISO system with SNR ρ and binary signaling has the formula

$$P_e^{SISO}(\rho) = Q\left(\sqrt{2q\rho}\right). \tag{22}$$

The BEP of the current system can be found similarly by

$$\begin{aligned} P_e(\rho) =& E_\nu\left[E_{\|\mathbf{h}\|_2^2}\left[Q\left(\sqrt{2q\rho\nu\|\mathbf{h}\|_2^2}\right)\middle|\,\nu\right]\right] \\ =& E_\nu\left[\int_0^\infty Q\left(\sqrt{2q\rho h\nu}\right) f_{\|\mathbf{h}\|_2^2}(h)dh\right]. \end{aligned} \tag{23}$$

The inner integral in (23) has been computed in closed-form for the error rate analysis of maximum ratio combining systems in [15, 14.4-15]. Using these results define $\mu = \sqrt{\frac{q\rho}{1+q\rho}}$, and expressing $\left(\frac{1-\mu}{2}\right)^m = \left(\frac{1-\mu}{2}\right)^k\left(\frac{1-\mu}{2}\right)^{m-k}$ and $(1-\mu)(1+\mu) = 1-\mu^2 = (1+q\rho)^{-1}$ gives

$$\begin{aligned} &\int_0^\infty Q(\sqrt{2q\rho h}) f_{\|\mathbf{h}\|_2^2}(h)dh \\ &= \sum_{k=0}^{m-1}\frac{1}{2^{m+k}}(1-\mu)^{m-k}\binom{m-1+k}{k}(1+q\rho)^{-k}. \end{aligned} \tag{24}$$

Substituting (24) into (23) and replacing the SNR term ρ in (24) by $\rho\nu$ in (23),

$$P_e(\rho) = E_\nu\left[\sum_{k=0}^{m-1}\frac{(1-\mu)^{m-k}}{2^{m+k}}\binom{m-1+k}{k}(1+q\nu\rho)^{-k}\right] \tag{25}$$

where now $\mu = \sqrt{\frac{q\rho\nu}{1+q\rho\nu}} = \sqrt{\frac{\nu}{\nu+\frac{1}{q\rho}}}$. Expanding $(1-\mu)^{m-k}$ into a binomial series, the BEP can then be computed by taking the expectation with respect to ν as

$$\begin{aligned} P_e(\rho) = &\sum_{k=0}^{m-1}\sum_{a=0}^{m-k}\sum_{i=1}^{N}\frac{(-1)^{a+i+1} i(m-1)}{2^{m+k}(q\rho)^k} \\ &\times\binom{m-k}{a}\binom{m-1+k}{k}\binom{N}{i} \\ &\times\int_0^1 \nu^{\frac{a}{2}}\left(\nu+\frac{1}{q\rho}\right)^{-\left(\frac{a}{2}+k\right)}(1-\nu)^{i(m-1)-1}d\nu. \end{aligned} \tag{26}$$

The closed-form solution for the integral in (26) is given in [13, 3.197.8]. Substituting the equation results in the closed-form BEP expression in (21). ■

Simulation: Figure 4 shows the closed-form BEP expression matched against simulated results for the $(m,N) = (2,8)$ and $(m,N) = (3,16)$ cases. The BEP for an ideal MRT system is included to measure quantization loss.

VI. Ergodic Capacity Analysis

Ergodic capacity is an important performance indicator in limited feedback systems. In this section, we quantify the RVQ limited feedback ergodic capacity into the ideal (i.e. full channel knowlege) ergodic capacity minus a limited feedback penalty term. The following lemma summarizes the result.

Lemma 4: The ergodic capacity $C(\rho)$ for an SNR ρ in a MISO system with m transmit antennas using a random N

vector beamforming codebook is given by

$$C(\rho) = \log_2 e \left(e^{\frac{1}{\rho}} \sum_{k=0}^{m-1} E_{k+1}\left(\frac{1}{\rho}\right) - \int_0^1 \left(1-(1-\nu)^{m-1}\right)^N \frac{m}{\nu} e^{\frac{1}{\rho\nu}} E_{m+1}\left(\frac{1}{\rho\nu}\right) d\nu \right) \tag{27}$$

where $E_n(x) = \int_1^\infty e^{-xt} x^{-n} dt$ is the n-th order exponential integral.

Proof: The capacity of a MISO channel is given by

$$C(\rho) = E_\nu \left[E_{\|\mathbf{h}\|_2^2} \left[\log_2(1+\rho\nu\|\mathbf{h}\|_2^2) \middle| \nu \right] \right]. \tag{28}$$

The inner expectation on $\|\mathbf{h}\|_2^2$ is a special case of the MIMO capacity given by [16], [17]

$$C(\rho|\nu) = (\log_2 e) e^{\frac{1}{\rho\nu}} \sum_{k=0}^{m-1} \frac{\Gamma\left(-k, \frac{1}{\rho\nu}\right)}{(\rho\nu)^k} \tag{29}$$

where $\Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the upper incomplete Gamma function. It can be transformed into an exponential integral function using [14, 37:13:12] $E_n(x) = x^{n-1}\Gamma(1-n; x)$ for nonnegative integer n. Thus, converting (29) into an exponential integration expression and taking the expectation over ν,

$$C(\rho) = \log_2 e \int_0^1 f_\nu(\nu) e^{\frac{1}{\rho\nu}} \sum_{k=0}^{m-1} E_{k+1}\left(\frac{1}{\rho\nu}\right) d\nu. \tag{30}$$

Integrating (30) by parts yields

$$\begin{aligned} u =& e^{\frac{1}{\rho\nu}} \sum_{k=0}^{m-1} E_{k+1}\left(\frac{1}{\rho\nu}\right) \\ du =& \frac{-1}{\rho\nu^2} e^{\frac{1}{\rho\nu}} \left(\sum_{k=0}^{m-1} E_{k+1}\left(\frac{1}{\rho\nu}\right) - \sum_{k=0}^{m-1} E_k\left(\frac{1}{\rho\nu}\right) \right) d\nu \\ d\nu =& f_\nu(\nu) d\nu \\ \nu =& \left(1-(1-\nu)^{m-1}\right)^N = \frac{m}{\nu} e^{\frac{1}{\rho\nu}} d\nu E_{m+1}\left(\frac{1}{\rho\nu}\right) \end{aligned} \tag{31}$$

where the recurrence relation $nE_{n+1}(x) = e^{-x} - xE_n(x)$ was used. Carrying out the integration by parts and multiplying by $\log_2 e$ yields (27). ■

Eq. (27) is the difference of the capacity of an MRT system and loss due to quantization. As $N \to \infty$, the system performance approaches the ideal MRT system performance. This is expected as SNR quantization loss vanishes as $N \to \infty$.

Simulation: Figure 5 shows (27) matched against a simulated ergodic capacity for the $(m,N) = (2,8)$ and $(m,N) = (3,16)$ cases. The ideal MRT ergodic capacity is included to show the loss due to quantization.

VII. Conclusions

In this correspondence, we analyzed a random codebook generation scheme, known as RVQ, to implement limited feedback beamforming. Closed-form expressions for expected SNR, outage probability, and bit error probability were derived in exact form. Ergodic capacity is given in an integral expression as the difference of the ideal MRT capacity and a loss term due to quantization. Simulations were provided to prove the exact match of the expressions derived.

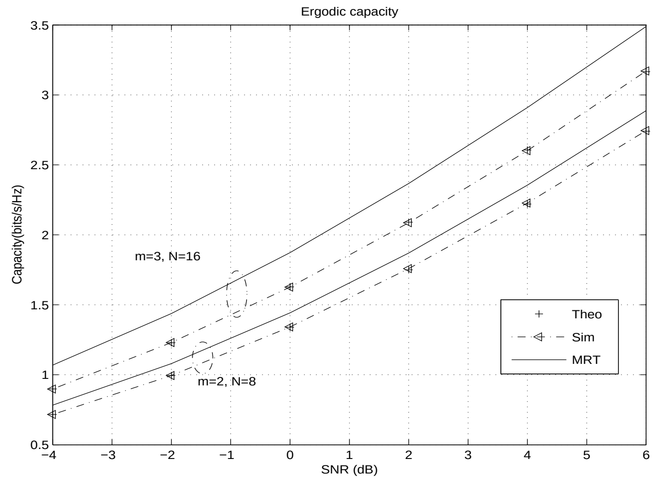

Fig. 5. Ergodic capacity matched against simulation. Ideal ergodic capacity is included to measure loss due to quantization

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