1.) Prove the following. Do not just state that it is true. Go through the formal steps (e.g., u-substitution) to show equality.

i.) sifting: \( \int_{-\infty}^{\infty} x(t) \delta(t - t_0) dt = x(t_0) \).

ii.) \( \int_{-\infty}^{\infty} x(t) \delta(at) dt = \frac{1}{|a|} \int_{-\infty}^{\infty} x(t) dt \)

iii.) \( x(t) \delta(t - a) = x(a) \delta(t - a) \) if \( x(t) \) continuous at \( t = a \). (Hint: Show that this is true using the integral of each side)

iv.) \( \delta(t - a) \ast f(t) = f(t - a) \)

v.) \( A\delta(t - a) \ast B\delta(t - b) = AB\delta(t - a - b) \)

2.) Prob. 2.13, pg. 101 (Do parts (a), (b) and (c) only). Note that \( \Pi(t) = \text{rect}(t) \).

3.) For a real signal \( x(t) \), compute \( \mathcal{F}\left[\mathcal{F}[x(t)]\right] \). How does your answer change for \( \mathcal{F}^{-1}\left[\mathcal{F}^{-1}[x(f)]\right] \)?

4.) Prob. 2.24, pg. 103, For each of the signals find the Fourier Transform and then plot the magnitude and the phase of the Fourier Transform.

5.) Derive \( \mathcal{F}[h(-t) \ast x(t)] \) as a function of \( H(-f) \) and \( X(f) \) (with \( h(t) \leftrightarrow H(f) \) and \( x(t) \leftrightarrow X(f) \)). What happens when \( h(t) \) is real?

6.) In the figure below, \( x(t) = \sin(2\pi f_0 t) \) is an input to a system with impulse response \( h(t) = e^{-\alpha t} u(t), \alpha > 0 \). Compute and sketch the Fourier Transform of both the input \( x(t) \) and output \( y(t) \).
7.) Compute the continuous Fourier transform of \((1/T_s)\Delta(t/T_s)\) where \(T_s\) is a positive constant. Please formally show this. DO NOT SIMPLY CITE \(\mathcal{F}[\Delta(t)]\) or \(\mathcal{F}[\Delta(t/T_s)]\).

8.) Suppose that \(x(t)\) is a real signal and that \(x(-t) = x(t)\). Let \(x(t) \leftrightarrow X(f)\). What is the relation between \(X(f)\) and \(X(-f)\)? What is the imaginary part of \(X(f)\), denoted \(\text{Imag}(X(f))\), equal to?

9.) The signal \(x(t) = \cos(20\pi t)\) is sampled at 15 samples per second. Show that the signal \(\cos(10\pi t)\) impersonates \(x(t)\) as far as sample values are concerned.

10.) Do problem 2.66 on page 108. For part b, a plot (e.g., using Matlab or by hand) and a few sentences of discussion suffice. For part c, remember the discussion in class of convolving the continuous Fourier transform of the original signal with a train of impulses.