ECE 63300—Mod. & Sim. of Power System Components

Multi-machine modeling and simulations
Supplement — neglecting subtransient saliency

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Generator terminal equations

For each generator $k = 1, \ldots, n$:

$$\tilde{V}_{as,k} = -(r_{s,k} + jX''_{d,k})\tilde{I}_{as,k} + \frac{1}{\sqrt{2}}\left\{ \left(\frac{X''_{d,k} - X_{ls,k}}{X'_{d,k} - X_{ls,k}} E'_{q,k} + \frac{X'_{d,k} - X''_{d,k}}{X'_{d,k} - X_{ls,k}} \psi'^{r}_{kd,k} \right) \right.$$

$$\left. +j\left[\left(X''_{d,k} - X''_{q,k} \right) i^{r}_{qs,k} - \frac{X''_{q,k} - X_{ls,k}}{X'_{q,k} - X_{ls,k}} E'_{d,k} + \frac{X'_{q,k} - X''_{q,k}}{X'_{q,k} - X_{ls,k}} \psi'^{r}_{kq2,k} \right] \right\} e^{j\hat{\theta}_{r,k}}$$

or

$$\tilde{V}_{as,k} = -(r_{s,k} + jX''_{d,k})\tilde{I}_{as,k} + \frac{1}{\sqrt{2}}j\left(X''_{d,k} - X''_{q,k} \right) i^{r}_{qs,k} e^{j\hat{\theta}_{r,k}} + \tilde{E}''_{as,k}(\mathbf{x}_k)$$

where $\tilde{E}''_{as,k}(\mathbf{x}_k)$ is a state-dependent internal subtransient voltage phasor.

Solving for current, this leads to:

$$\tilde{I}_{as,k} = \frac{1}{r_{s,k} + jX''_{d,k}} \Big[\underbrace{\tfrac{1}{\sqrt{2}} j \left(X''_{d,k} - X''_{q,k} \right) i^r_{qs,k} e^{j\hat{\theta}_{r,k}}}_{\tilde{A}_k} + \tilde{E}''_{as,k}(\mathbf{x}_k) - \tilde{V}_{as,k} \Big]$$

In vector-matrix form, accounting for all generators, we can write:

$$\mathbf{I} = (\mathbf{Z}''_d)^{-1} \left[\mathbf{A} + \mathbf{E}'' - \mathbf{V} \right] \tag{1}$$

where

$$(\mathbf{Z}''_d)^{-1} = \text{diag} \left\{ \frac{1}{r_{s,k} + jX''_{d,k}} \right\}$$

is a diagonal matrix.

Combining with the system admittance matrix ...

The "original" network admittance matrix equation (as seen from the generator terminals) says:

$$\mathbf{I} = \mathbf{Y}\mathbf{V} \tag{2}$$

Equating the right-hand sides of (1) and (2):

$$(\mathbf{Z}_d'')^{-1}(\mathbf{A} + \mathbf{E}'') = \left[\mathbf{Y} + (\mathbf{Z}_d'')^{-1}\right]\mathbf{V} = \mathbf{Y}^*\mathbf{V} \tag{3}$$

The new matrix $\mathbf{Y}^*$ contains the same elements as $\mathbf{Y}$, but with its n diagonal entries increased by $\frac{1}{r_{s,k}+jX_{d,k}''}$, for $k = 1, \ldots, n$.

Equation (3) represents an $n \times n$ linear system that can be solved for $\mathbf{V}$:

$$\mathbf{V} = (\mathbf{Y}^*)^{-1}(\mathbf{Z}_d'')^{-1}(\mathbf{A} + \mathbf{E}'') \tag{4}$$

Equation (4) can be substituted back into (1):

$$\mathbf{I} = (\mathbf{Z}_d'')^{-1} \left[\mathbf{A} + \mathbf{E}'' - (\mathbf{Y}^*)^{-1} (\mathbf{Z}_d'')^{-1} (\mathbf{A} + \mathbf{E}'') \right]$$

Therefore,

$$\mathbf{I} = (\mathbf{Z}_d'')^{-1} \left[\mathbb{I} - (\mathbf{Y}^*)^{-1} (\mathbf{Z}_d'')^{-1} \right] (\mathbf{A} + \mathbf{E}'') \tag{5}$$

Now we will try to simplify this expression by applying the Woodbury matrix identity to calculate $(\mathbf{Y}^*)^{-1} = (\mathbf{Y} + (\mathbf{Z}_d'')^{-1})^{-1}$. This yields

$$(\mathbf{Y}^*)^{-1} = \mathbf{Z}_d'' - \mathbf{Z}_d'' (\mathbf{Y}^{-1} + \mathbf{Z}_d'')^{-1} \mathbf{Z}_d''$$

Substituting this in (5) above, we obtain

$$\mathbf{I} = (\mathbf{Y}^{-1} + \mathbf{Z}_d'')^{-1} (\mathbf{A} + \mathbf{E}'')$$

The inverse of the original network admittance matrix is the impedance matrix $\mathbf{Z} = \mathbf{Y}^{-1}$. So, the term $\mathbf{Y}^{-1} + \mathbf{Z}_d'' = \mathbf{Z} + \mathbf{Z}_d'' = \mathbf{Z}'' = (\mathbf{Y}'')^{-1}$, represents the original impedance matrix with its diagonal elements incremented by $r_{s,k} + jX_{d,k}''$.

Physically, this "new" network is the same as before, the only change being that we are adding $r_{s,k} + jX_{d,k}''$ to the points of connection of the generators, so that the voltages at these nodes are $\tilde{A}_k + \tilde{E}_k''$.

Therefore,

$$\boxed{\mathbf{I} = \mathbf{Y}''(\mathbf{A} + \mathbf{E}'')} \tag{6}$$

Computing $i^r_{qs,k}$ and $i^r_{ds,k}$

Let's write the vector of phasors $\mathbf{I}$ in terms of its two qd-axes current component vectors, as follows:

$$\mathbf{I} = \tfrac{1}{\sqrt{2}}\mathbf{T}(\mathbf{I}_q - j\mathbf{I}_d)$$

where

$$\mathbf{T} = \mathsf{diag}\left\{e^{j\hat{\theta}_{r,k}}\right\} = \begin{bmatrix} e^{j\hat{\theta}_{r,1}} & 0 & \cdots & 0 \\ 0 & e^{j\hat{\theta}_{r,2}} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & e^{j\hat{\theta}_{r,n}} \end{bmatrix}$$

Also, let's define a diagonal subtransient saliency matrix by

$$\mathbf{S} = \mathsf{diag}\left\{X''_{d,k} - X''_{q,k}\right\}$$

Hence,

$$\mathbf{A} = \tfrac{1}{\sqrt{2}} j \mathbf{STI}_q$$

and (6) can be written as

$$\tfrac{1}{\sqrt{2}} \mathbf{T}(\mathbf{I}_q - j\mathbf{I}_d) = \mathbf{Y}''(\tfrac{1}{\sqrt{2}} j \mathbf{STI}_q + \mathbf{E}'')$$

or after rearranging terms

$$\tfrac{1}{\sqrt{2}} \left[(\mathbb{I} - j\mathbf{Y}''\mathbf{S})\mathbf{TI}_q - j\mathbf{TI}_d \right] = \mathbf{Y}''\mathbf{E}''$$

If we decompose into real and imaginary parts as follows,

$$\tfrac{1}{\sqrt{2}}(\mathbb{I} - j\mathbf{Y}''\mathbf{S})\mathbf{T} = a + jb$$

$$\tfrac{1}{\sqrt{2}}(-j\mathbf{T}) = c + jd$$

$$\mathbf{Y}''\mathbf{E}'' = e + jf$$

then the currents can be found by solving the following
$(2n) \times (2n)$ linear system:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} \mathbf{I}_q \\ \mathbf{I}_d \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$