



Forms of Euler's equation for a rigid body:

$$\begin{aligned}
 \mathbf{M}_A &= \frac{d\mathbf{H}_A}{dt} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_{CM}}{dt} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_A}{dt} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_A
 \end{aligned}$$

Proof that the first two equations and the first and third equations are the same:

$$\begin{aligned}
 \mathbf{M}_A &= \frac{d\mathbf{H}_A}{dt} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d}{dt} \left( \underbrace{\mathbf{H}_{CM} + \mathbf{r}_{A/CM} \times M\dot{\mathbf{R}}_{CM}}_{\text{This was the substitution I failed to make in my office...}} \right) + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_{CM}}{dt} + \dot{\mathbf{r}}_{A/CM} \times M\dot{\mathbf{R}}_{CM} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_{CM} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_{CM}}{dt} + \dot{\mathbf{r}}_{A/CM} \times M\dot{\mathbf{R}}_{CM} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_{CM}}{dt} + (\dot{\mathbf{R}}_{CM} - \dot{\mathbf{R}}_A) \times M\dot{\mathbf{R}}_{CM} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_{CM}}{dt} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_{CM} \\
 \mathbf{M}_A &= \frac{d\mathbf{H}_A}{dt} + \dot{\mathbf{R}}_A \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_A}{dt} + (\dot{\mathbf{R}}_{CM} - \dot{\mathbf{r}}_{A/CM}) \times M\dot{\mathbf{R}}_{CM} \\
 &= \frac{d\mathbf{H}_A}{dt} + \mathbf{r}_{A/CM} \times M\ddot{\mathbf{R}}_A
 \end{aligned}$$