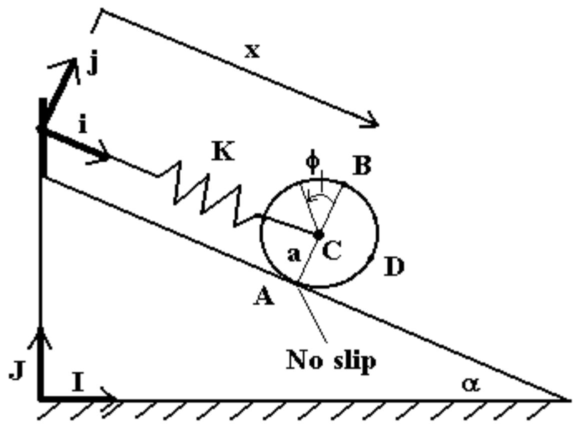
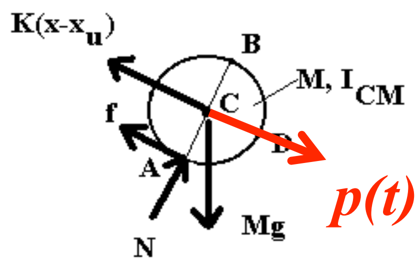
ME 563
Mechanical Vibrations
Lecture #7

Calculating Generalized Forces

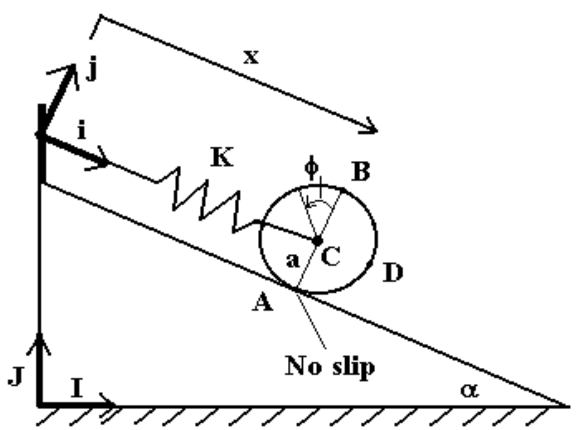
in Lagrange's Equations

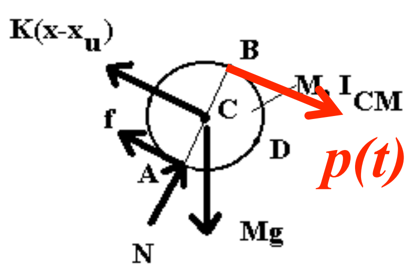
Example
Rolling Disc on Incline





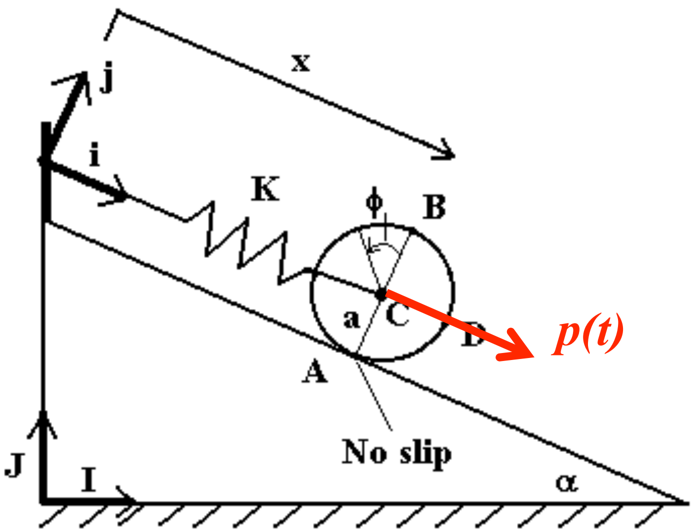
Force, *p(t)*, connected at point C





Force, *p(t)*, connected at point B

Example
Rolling Disc on Incline (Recall)



$$\mathbf{r}_C = x\mathbf{i}$$

$$\frac{\partial \mathbf{r}_C}{\partial x} = \mathbf{i}$$

Calculate generalized forces (for degree of freedom, *x*):

$$Q_r^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

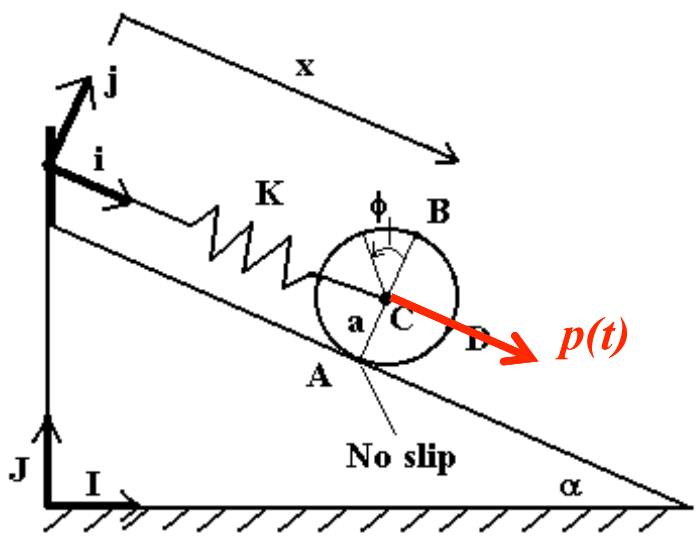
$$= \sum_{i=1}^{N} \left(p(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{r}_C}{\partial x}$$

$$= p(t)\mathbf{i} \cdot \mathbf{i}$$

$$= p(t)$$

$$\left(M + \frac{I_{CM}}{a^2}\right)\ddot{x} + K\left(x - x_u\right) - Mg\sin\alpha = p(t)$$

Example
Rolling Disc on Incline (New)



Calculate generalized forces (for degree of freedom, ϕ):

$$Q_r^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

$$= \sum_{i=1}^{N} \left(p(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{r}_C}{\partial \phi}$$

$$= p(t)\mathbf{i} \cdot -a\mathbf{i}$$

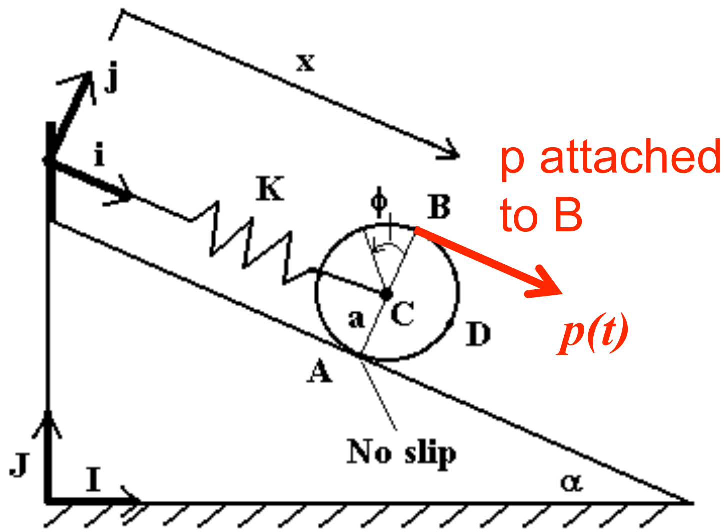
$$= -ap(t)$$

$$\mathbf{r}_C = -a\phi\mathbf{i}$$

$$\frac{\partial \mathbf{r}_C}{\partial \phi} = -a\mathbf{i}$$

$$\left(Ma^2 + I_{CM}\right)\ddot{\phi} + K\left(a^2\phi + ax_u\right) + Mga\sin\alpha = -ap(t)$$

Example
Rolling Disc on Incline (New)



$$\mathbf{r}_B = (x - a\sin\phi)\mathbf{i} + (a\cos\phi)\mathbf{j}$$

$$= \left(x - a\sin\frac{-x}{a}\right)\mathbf{i} + \left(a\cos\frac{-x}{a}\right)\mathbf{j}$$

$$\frac{\partial \mathbf{r}_B}{\partial x} = \left(1 + \cos\frac{-x}{a}\right)\mathbf{i} + \left(\sin\frac{-x}{a}\right)\mathbf{j}$$

Calculate generalized forces (for degree of freedom, *x*):

$$Q_r^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

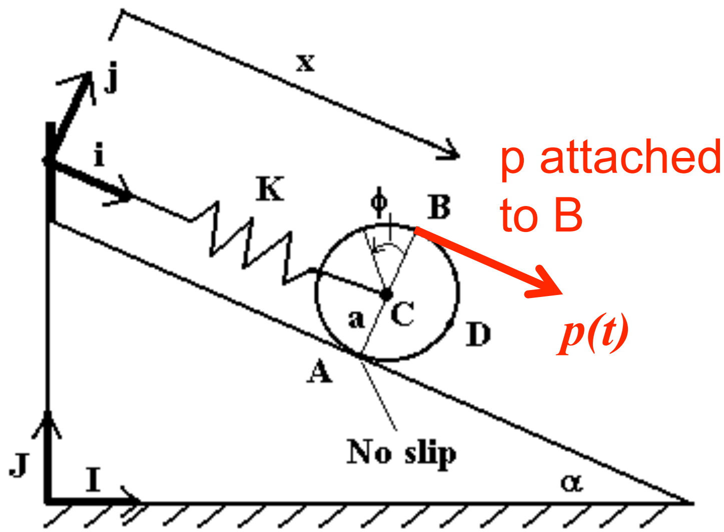
$$= \sum_{i=1}^{N} \left(p(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{r}_B}{\partial x}$$

$$= p(t)\mathbf{i} \cdot \left[\left(1 + \cos\frac{-x}{a}\right)\mathbf{i} + \left(\sin\frac{-x}{a}\right)\mathbf{j}\right]$$

$$= p(t)\left(1 + \cos\frac{-x}{a}\right)$$

$$\left(M + \frac{I_{CM}}{a^2}\right)\ddot{x} + K(x - x_u) - Mg\sin\alpha = p(t)\left(1 + \cos\frac{-x}{a}\right)$$

Example
Rolling Disc on Incline (New)



Calculate generalized forces (for degree of freedom, ϕ):

$$Q_r^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

$$= \sum_{i=1}^{N} \left(p(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{r}_B}{\partial \phi}$$

$$= p(t)\mathbf{i} \cdot \left[\left(-a - a\cos\phi\right)\mathbf{i} + \left(-a\sin\phi\right)\mathbf{j}\right]$$

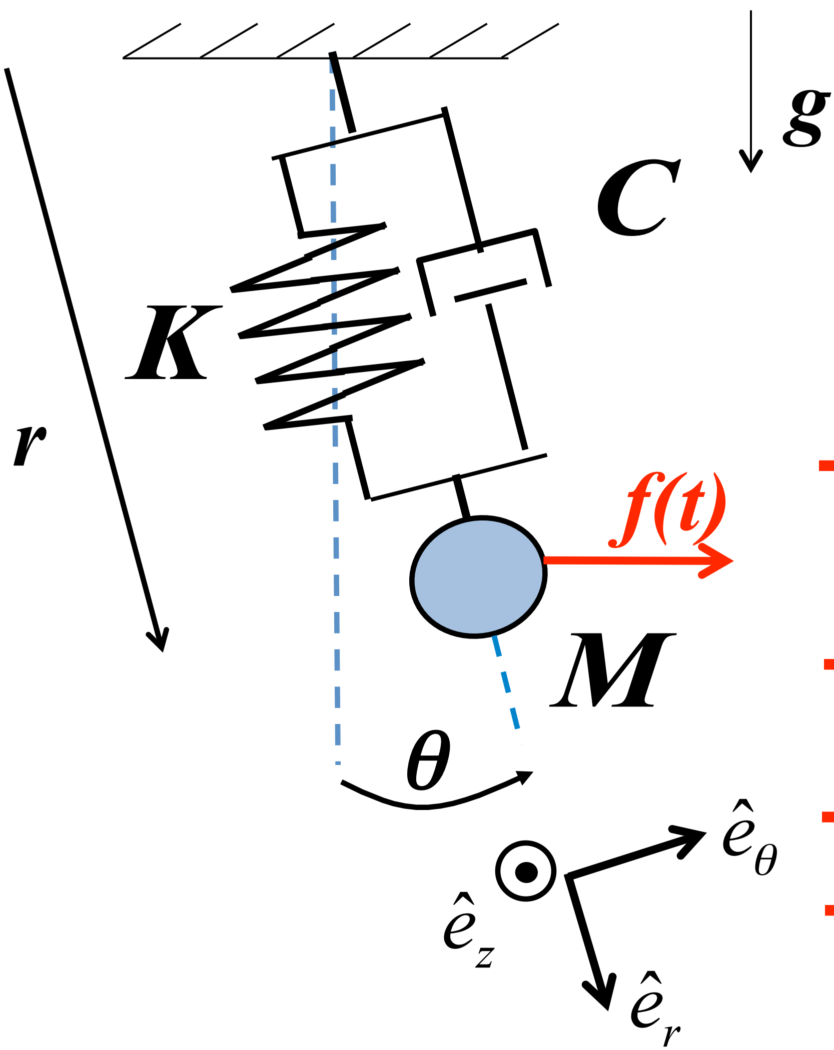
$$= -ap(t)\left(1 + \cos\phi\right)$$

$$\mathbf{r}_B = \left(-a\phi - a\sin\phi\right)\mathbf{i} + \left(a + a\cos\phi\right)\mathbf{j}$$

$$\frac{\partial \mathbf{r}_B}{\partial \phi} = \left(-a - a\cos\phi\right)\mathbf{i} + \left(-a\sin\phi\right)\mathbf{j}$$

$$\left(Ma^2 + I_{CM}\right)\ddot{\phi} + K\left(a^2\phi + ax_u\right) + Mga\sin\alpha = -ap(t)\left(1 + \cos\phi\right)$$

Sprung Pendulum Example



$$\mathbf{R} = r\mathbf{e_r}, \frac{\partial \mathbf{R}}{\partial r} = \mathbf{e_r}$$

Calculate generalized forces (for degree of freedom, *r*):

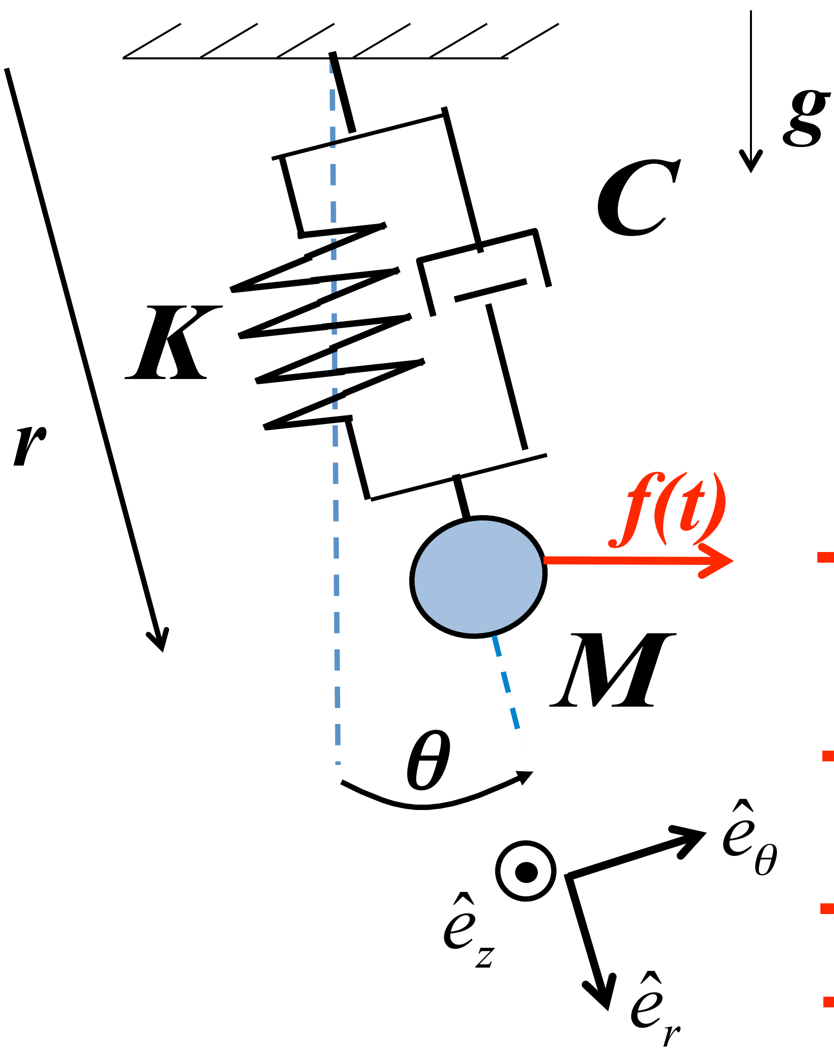
$$Q_r^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

$$= \sum_{i=1}^{N} \left(f(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{R}}{\partial r}$$

$$= f(t)\mathbf{i} \cdot \mathbf{e_r}$$

$$= f(t)\cos\left(\frac{\pi}{2} - \theta\right)$$

Sprung Pendulum Example



$$\mathbf{R} = r\mathbf{e_r}, \frac{\partial \mathbf{R}}{\partial \theta} = r\mathbf{e_\theta}$$

$$\frac{d\mathbf{R}}{dt} = \dot{r}\mathbf{e_r} + r\dot{\theta}\mathbf{e_\theta} = \frac{\partial \mathbf{R}}{\partial r}\frac{dr}{dt} + \frac{\partial \mathbf{R}}{\partial \theta}\frac{d\theta}{dt}$$

Calculate generalized forces (for degree of freedom, θ):

$$Q_\theta^* = \sum_{i=1}^{N} \left(\mathbf{F}_{i,nc}\right) \cdot \frac{\partial \mathbf{r}_i^{(k)}}{\partial q_r^{(k)}}$$

$$= \sum_{i=1}^{N} \left(f(t)\right)\mathbf{i} \cdot \frac{\partial \mathbf{R}}{\partial \theta}$$

$$= f(t)\mathbf{i} \cdot r\mathbf{e_\theta}$$

$$= rf(t)\cos(\theta)$$