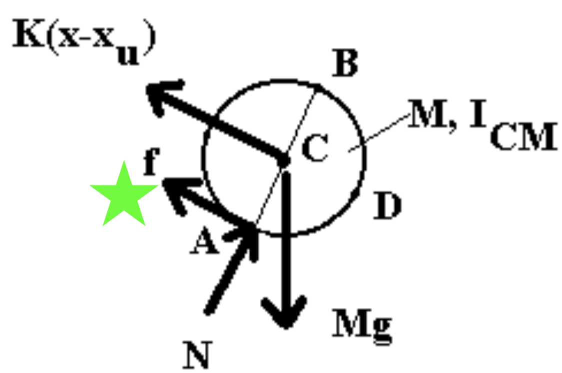
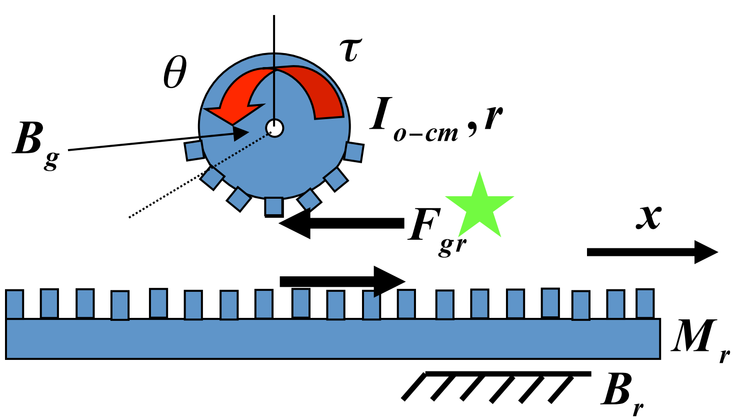
ME 563
Mechanical Vibrations
Lecture #4

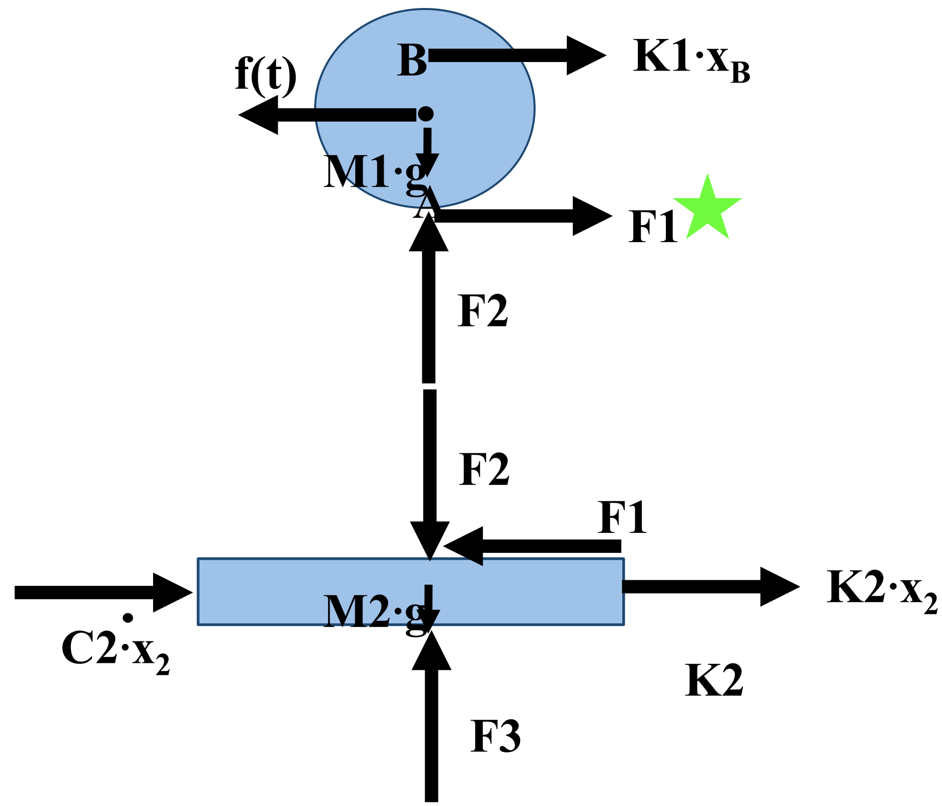
Energy/Power Method for Single Degree of Freedom Systems

Motivation for Energy Method

What did all of these Newton-Euler examples have in common?

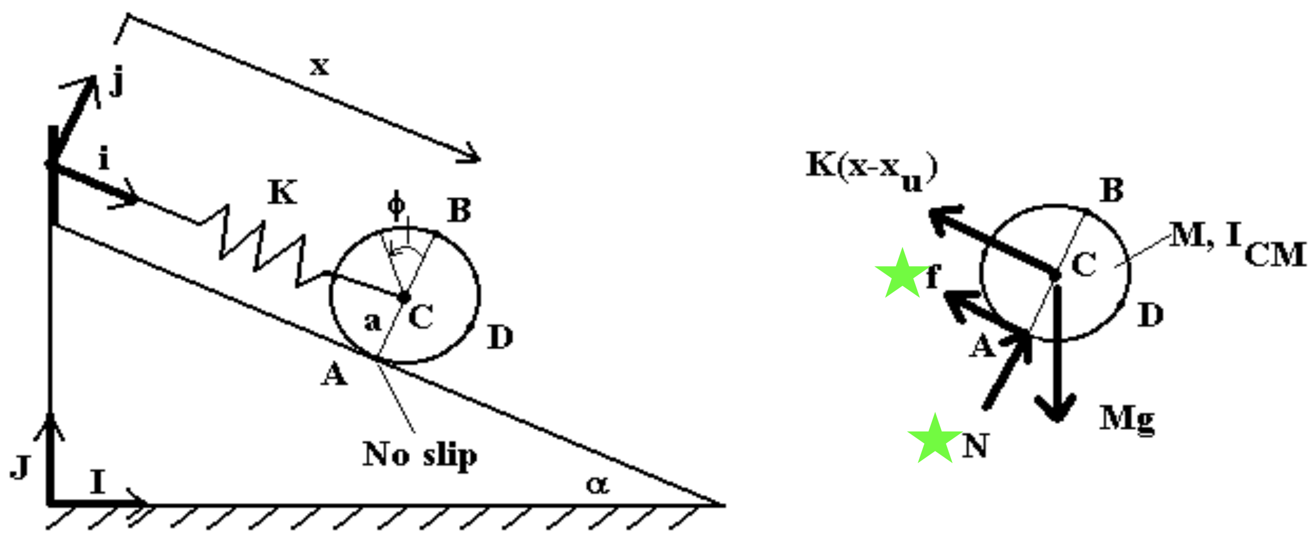






Ideal Forces of Constraint

There are forces of constraint that need not be considered.



What if the disc slips?...Friction force, f, does work

Projection Mechanics

What if we project the equations of motion onto the trajectory of the vibrating system?

$$\rightarrow \left(\mathbf{F}_{CM} - M\mathbf{A}_{CM}\right)\cdot \delta\mathbf{r} = 0$$

$$\rightarrow \left(\mathbf{F}_{active} + \mathbf{F}_{constr} - M\mathbf{A}_{CM}\right)\cdot \delta\mathbf{r} = 0$$

$$\rightarrow \left(\left(N - Mg\cos\alpha\right)\mathbf{j} - f\mathbf{i} + \left(-K\left(x - x_u\right) + Mg\sin\alpha\right)\mathbf{i} - M\ddot{x}\mathbf{i}\right)\cdot \delta\mathbf{r} = 0$$

$$\rightarrow \left(\left(N - Mg\cos\alpha\right)\mathbf{j} + \frac{I_{CM}\ddot{\phi}}{a}\mathbf{i} + \left(-K\left(x - x_u\right) + Mg\sin\alpha\right)\mathbf{i} - M\ddot{x}\mathbf{i}\right)\cdot\left(\delta x\mathbf{i}\right) = 0$$

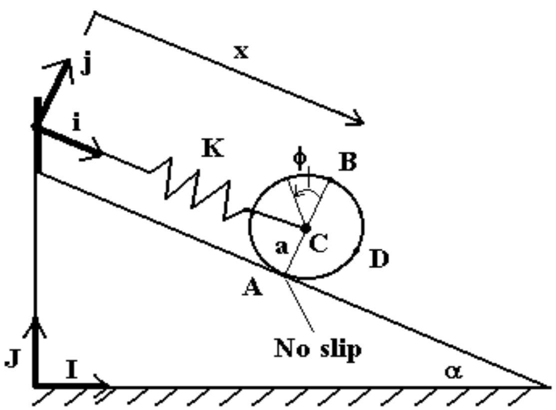
$$\rightarrow \left(\frac{I_{CM}\ddot{\phi}}{a} - M\ddot{x} - K\left(x - x_u\right) + Mg\sin\alpha\right)\delta x = 0$$

$$\rightarrow \left(M + \frac{I_{CM}}{a^2}\right)\ddot{x} + K\left(x - x_u\right) - Mg\sin\alpha = 0 \text{ for } \underline{\text{arbitrary } \delta x}$$

First Law of Thermodynamics

Rate of change in kinetic and potential energy is equal to the rate at which work is done by non-conservative forces (dampers, f(t)):

$$\frac{d}{dt}\left(T+V\right)=\frac{dW_{nc}}{dt}$$



$$T=\frac{1}{2}M\dot{x}^2+\frac{1}{2}I_{cm}\dot{\phi}^2$$

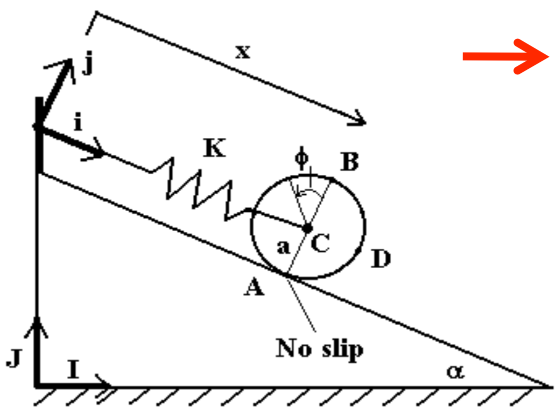
$$V=\frac{1}{2}K\left(x-x_u\right)^2-Mg\left(x-x_u\right)\sin\alpha$$

First Law of Thermodynamics

$$\frac{d}{dt}\left(\frac{1}{2}M\dot{x}^2 + \frac{1}{2}I_{CM}\frac{\dot{x}^2}{a^2} - Mg(x - x_u)\sin\alpha + \frac{1}{2}K(x - x_u)^2\right) = \frac{dW}{dt}$$

$$\left(M\ddot{x} + \frac{I_{CM}}{a^2}\ddot{x} - Mg\sin\alpha + K(x - x_u)\right)\dot{x} = 0$$

$$\left(M + \frac{I_{CM}}{a^2}\right)\ddot{x} - Mg\sin\alpha + K(x - x_u) = 0$$

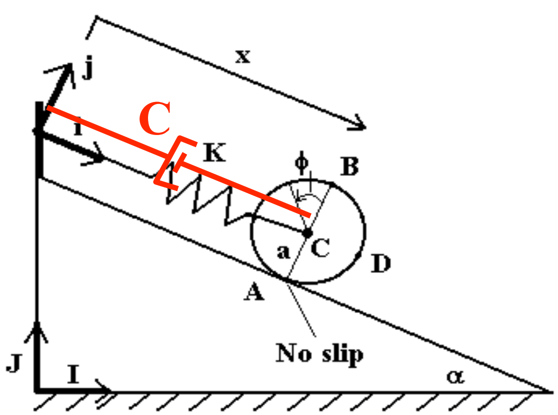


First Law of Thermodynamics

What if a viscous damper is present?

$$dW_{nc} = \mathbf{F}_{damper} \cdot \delta \mathbf{r}_C \qquad \mathbf{r}_C = x\hat{i}$$

$$= -C\dot{x}\mathbf{i} \cdot \delta \mathbf{r}_C \qquad \delta \mathbf{r}_C = \frac{d\mathbf{r}_C}{dx}dx = \frac{dx}{dx}\hat{i}\,dx = dx\hat{i}$$

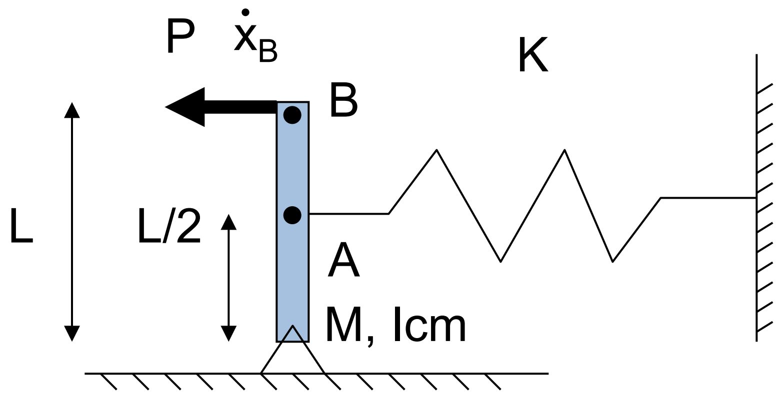


$$\frac{d}{dt}\left(\text{Same as before...}\right) = \frac{dW_{nc}}{dt}$$

$$\frac{d}{dt}\left(\text{Same as before...}\right) = \frac{-C\dot{x}}{dt}dx = -C\dot{x}\frac{dx}{dt} = -C\dot{x} \cdot \dot{x}$$

$$\left(M + \frac{I_{CM}}{a^2}\right)\ddot{x} + C\dot{x} + Kx = Mg\sin\alpha + Kx_u$$

Sprung Lever Example



$$\frac{d}{dt}(T+V)=\frac{dW_{nc}}{dt}$$

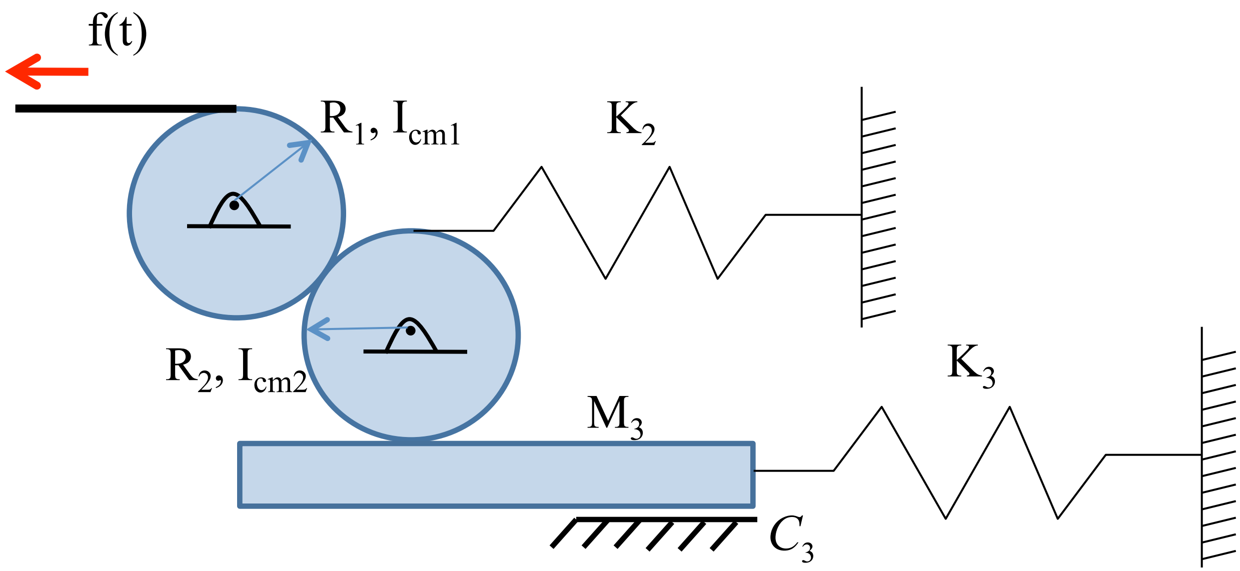
$$T=\frac{1}{2}\left(\frac{M}{4}+\frac{I_{cm}}{L^2}\right)\dot{x}_B^2 \text{ and } V=\frac{1}{2}\left(\frac{K}{4}\right)x_B^2$$

$$\frac{d}{dt}(T+V)=0$$

$$\left(\frac{M}{4}+\frac{I_{cm}}{L^2}\right)\ddot{x}_B+\frac{K}{4}x_B=0$$

Merit of Energy Methods

Energy methods are especially useful when there are many ideal constraints (pins, gears, rolling surfaces, etc.):



Merit of Energy Methods

Applying Newton-Euler methods would require many equations and much algebra to eliminate constraints…but

$$\frac{d}{dt}\left(\frac{1}{2}I_{cm1}\dot{\theta}_1^2+\frac{1}{2}I_{cm2}\dot{\theta}_2^2+\frac{1}{2}M_3\dot{x}_3^2+\frac{1}{2}K_2(R_2\theta_2)^2+\frac{1}{2}K_3x_3^2\right)=-C_3\dot{x}_3\cdot\dot{x}_3$$

$$\frac{d}{dt}\left(\frac{1}{2}I_{cm1}\dot{\theta}_1^2+\frac{1}{2}I_{cm2}\left(\frac{R_1}{R_2}\dot{\theta}_1\right)^2+\frac{1}{2}M_3(R_1\dot{\theta}_1)^2+\frac{1}{2}K_2(R_1\theta_1)^2+\frac{1}{2}K_3(R_1\theta_1)^2\right)=-C_3R_1\dot{\theta}_1\cdot R_1\dot{\theta}_1$$

$$\left(I_{cm1}\ddot{\theta}_1+I_{cm2}\left(\frac{R_1}{R_2}\right)^2\ddot{\theta}_1+M_3R_1^2\ddot{\theta}_1+K_2R_1^2\theta_1+K_3R_1^2\theta_1\right)\dot{\theta}_1=-C_3R_1^{\,2}\dot{\theta}_1\cdot\dot{\theta}_1$$

$$\left(I_{cm1}+I_{cm2}\left(\frac{R_1}{R_2}\right)^2+M_3R_1^2\right)\ddot{\theta}_1+C_3R_1^{\,2}\dot{\theta}_1+(K_2R_1^2+K_3R_1^2)\theta_1=0$$

$f\cdot R_1$