ME 563
Mechanical Vibrations
Lecture #20

Common damping models

Viscous Damping

Damping is rarely viscous for the following reasons:

- Force due to dissipation is not an increasing function of frequency (assume $x = \sin\omega t$):

$$F_{viscous} = C\dot{x} = C\omega\cos\omega t$$

- Force is not a linear function of velocity as in the case of fluid drag force:

$$F_{drag} = D\dot{x}^2$$

Equivalent Damping

Damping is rarely viscous but we can often model the appropriate type of damping using *equivalent* viscous damping. To develop this equivalent damping model, we calculate the energy dissipated by the viscous damper per sinusoidal cycle:

$$\text{Given } x(t) = X_p \cos\omega t$$

$$dW_{nc} = F_{viscous} \cdot dx$$

$$\frac{W_{nc}}{cycle} = \int_{cycle} -C_{eq}\dot{x}dx = \int_0^{2\pi/\omega} -C_{eq}\dot{x}^2 dt$$

$$= \int_0^{2\pi/\omega} -C_{eq}X_p{}^2 \cos^2 \omega t dt$$

$$= -\pi\omega C_{eq}X_p^2$$

Hysteretic Damping

For example, structural (or hysteretic) damping is a type of dissipation that is a function of friction within a material. This form of damping is observed to not increase with frequency, so instead of a viscous damping force:

$$\begin{aligned} F_{viscous} &= C\dot{x} \\ &= \omega C X_p \sin(\omega t + \phi) \end{aligned}$$

$$W_{nc} = -\pi \omega C_{eq} X_p^2$$

we have a hysteretic damping force if we set *C*=*h*/*ω*:

$$\begin{aligned} F_{hysteresis} &= C\dot{x} \\ &= \omega C X_p \sin(\omega t + \phi) \\ &= h X_p \sin(\omega t + \phi) \end{aligned}$$

$$W_{nc} = -\pi h X_p^2$$

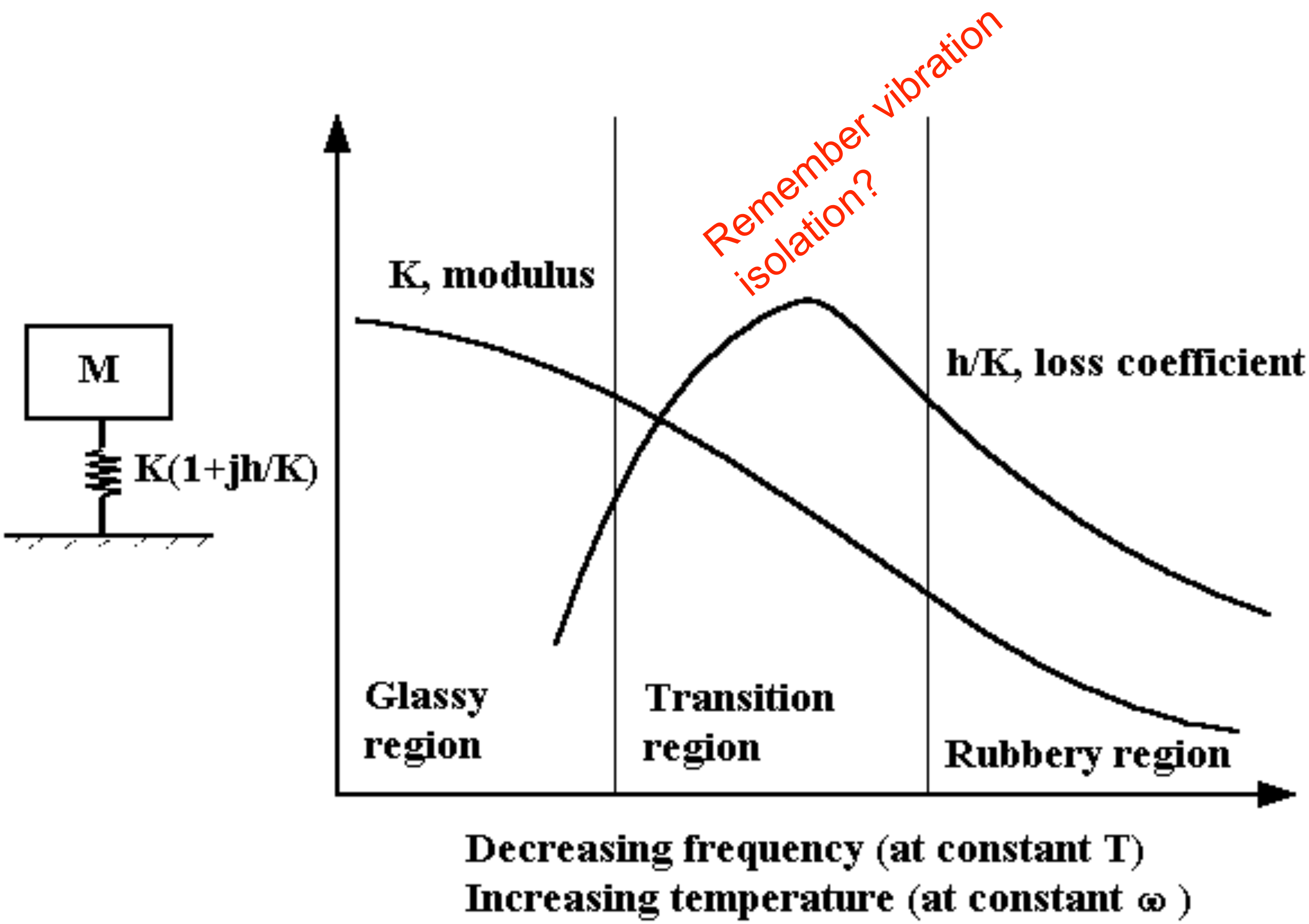
Hysteretic Damping

Structural damping is usually incorporated into models by defining a complex stiffness that is valid when considering the sinusoidal response behavior of the system:

$$M\ddot{x} + K(1 + j\eta)x = F_o \sin \omega t$$

where $\eta = h/K$ is called the loss factor. Note that by using this form for the stiffness, we are enforcing the dependence on the displacement amplitude, phasing (with velocity), and not requiring the force to increase with frequency:

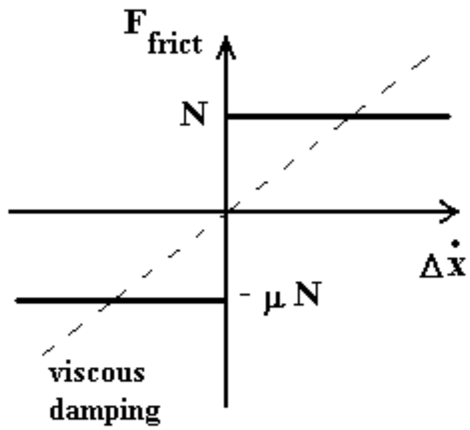
Hysteretic Damping

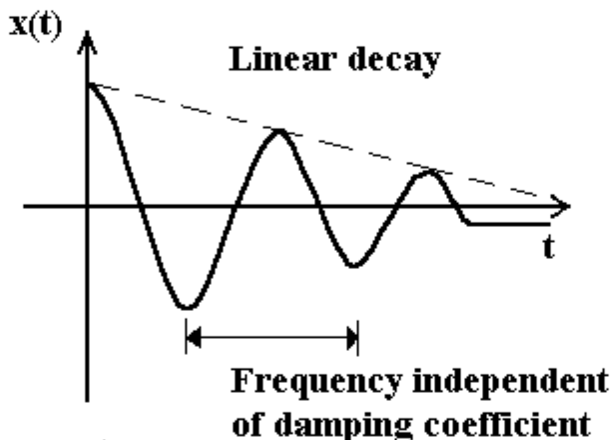


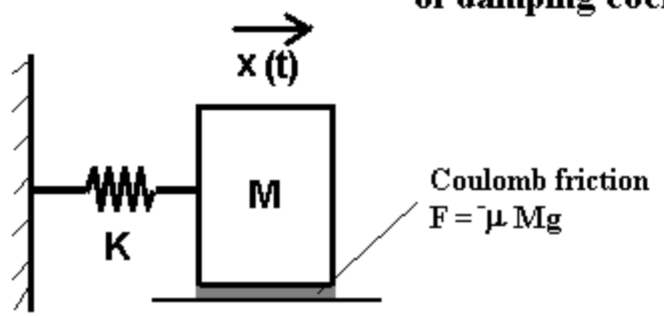
Coulomb Damping

Coulomb friction (damping) produces an interesting type of free decay response:

$$M\ddot{x} + \mu N \operatorname{sgn} \dot{x} + Kx = 0$$







Coulomb Damping

To calculate the equivalent viscous damping coefficient, we equate the energy lost per unit cycle in the case of Coulomb friction to the energy lost due to viscous effects:

$$-\pi C_{eq}\omega X_p^2 = W_{nc} = -4\mu N X_p$$

$$C_{eq} = \frac{4\mu N}{\pi\omega X_p}$$

What does this viscous damping coefficient imply about the amount of damping present in the oscillation?