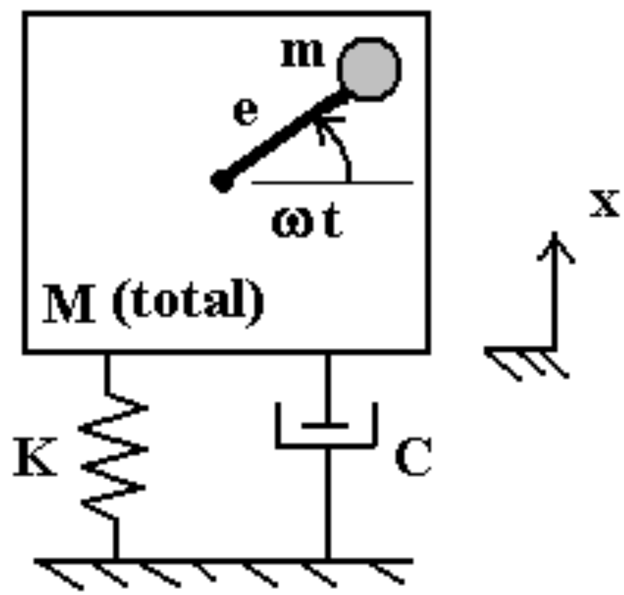
ME 563
Mechanical Vibrations
Lecture #19

Applications: Single Degree of Freedom

Imbalance



$$M\ddot{x} + C\dot{x} + Kx = me\omega^2 \sin(\omega t)$$

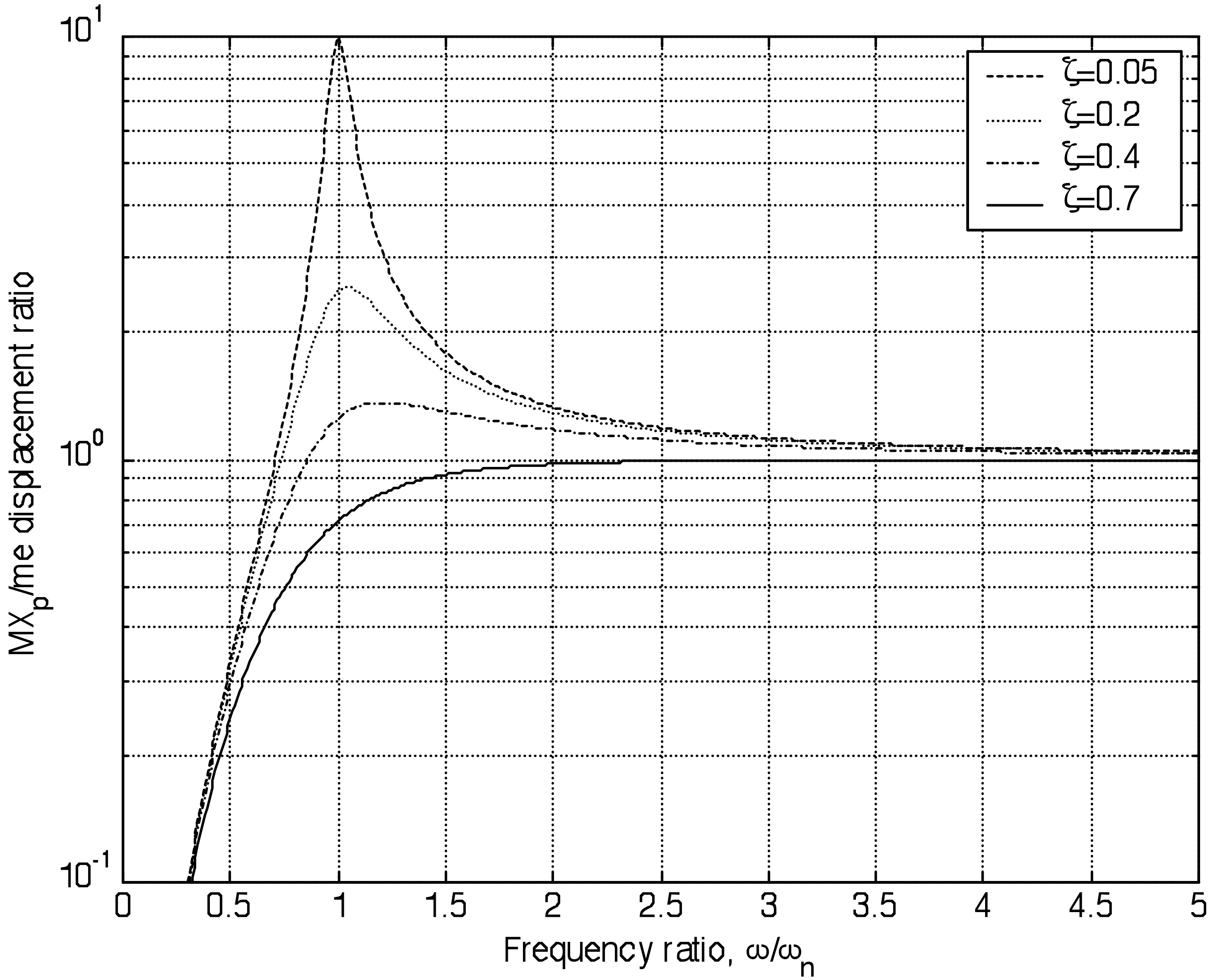
Imbalance

If $f(t) = me\omega^2 \sin(\omega t) = F_i \sin(\omega t)$, then $x_p(t) = X_p \sin(\omega t + \phi_p)$ where

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} = \frac{1/K}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\varsigma\left(\frac{\omega}{\omega_n}\right)}$$ and

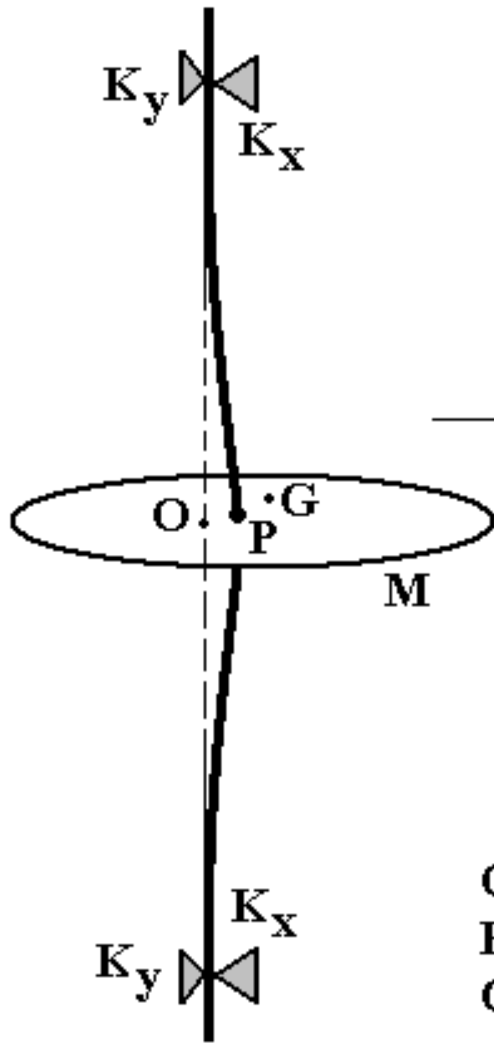
$$\frac{MX_p}{me} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varsigma\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

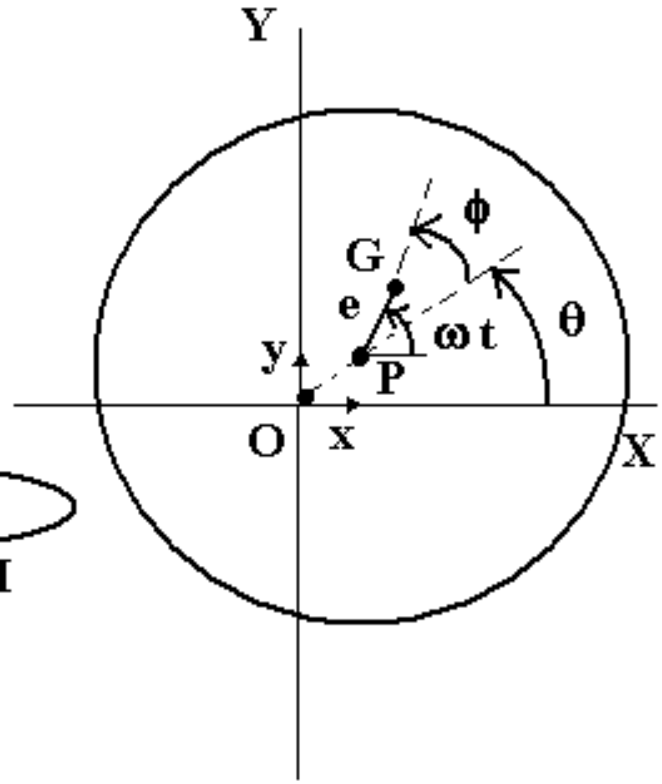
Imbalance



10^1
10^0
10^-1
ζ=0.05
ζ=0.2
ζ=0.4
ζ=0.7
MX_p/me displacement ratio
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency ratio, ω/ω_n

Vibrating Shaft





O -- Geometric center

P -- Rotational axis

G -- Mass center of disk

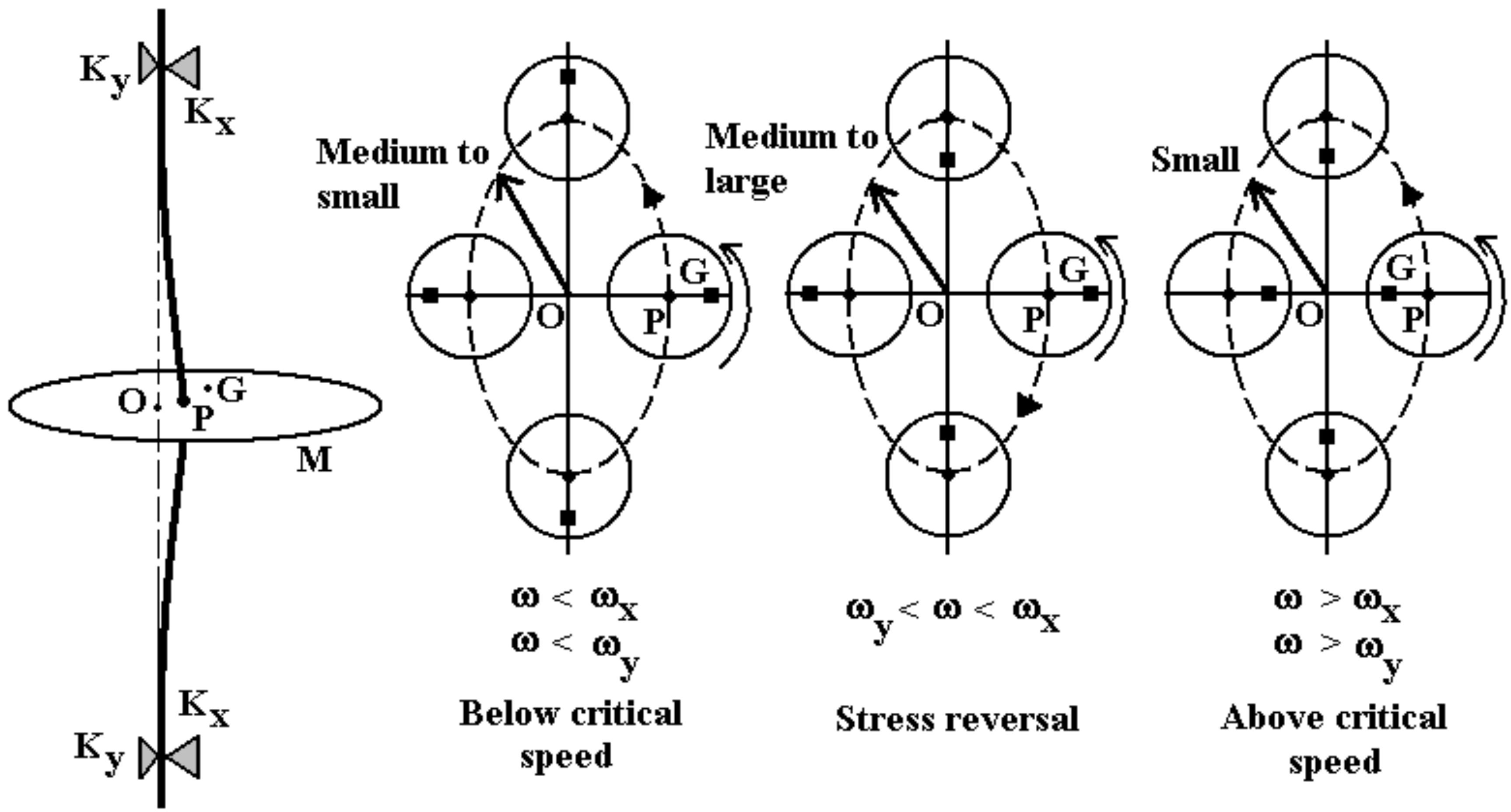
Vibrating Shaft

$$M\ddot{x} + K_x x = Me\omega^2 \cos\omega t$$

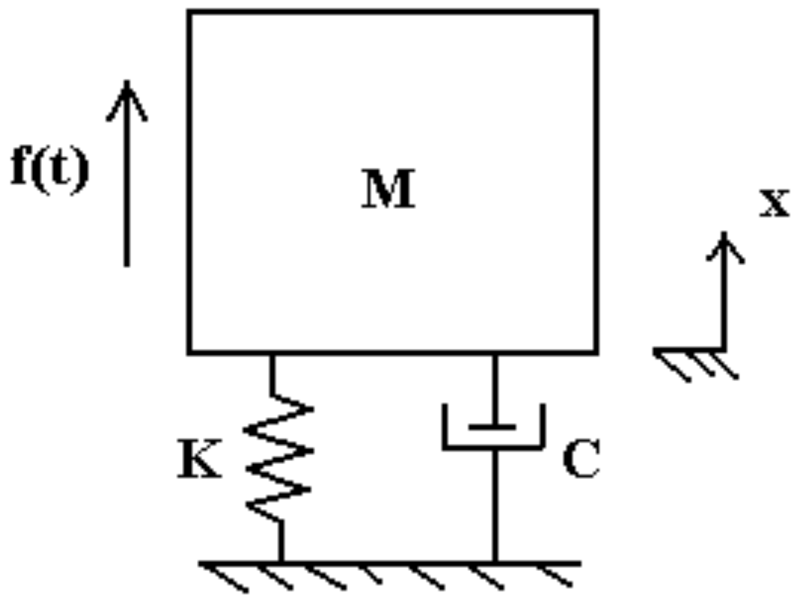
$$M\ddot{y} + K_y y = Me\omega^2 \sin\omega t$$

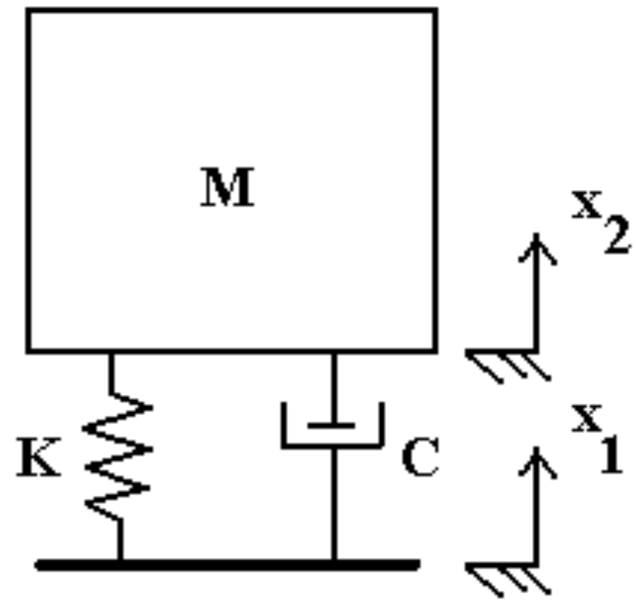
$$\frac{X(j\omega)}{e} = \frac{\left(\frac{\omega}{\omega_{nx}}\right)^2}{1 - \left(\frac{\omega}{\omega_{nx}}\right)^2} \text{ and } \frac{Y(j\omega)}{e} = \frac{\left(\frac{\omega}{\omega_{ny}}\right)^2}{1 - \left(\frac{\omega}{\omega_{ny}}\right)^2} e^{-j\pi/2}$$

Vibrating Shaft



Vibrating Isolation



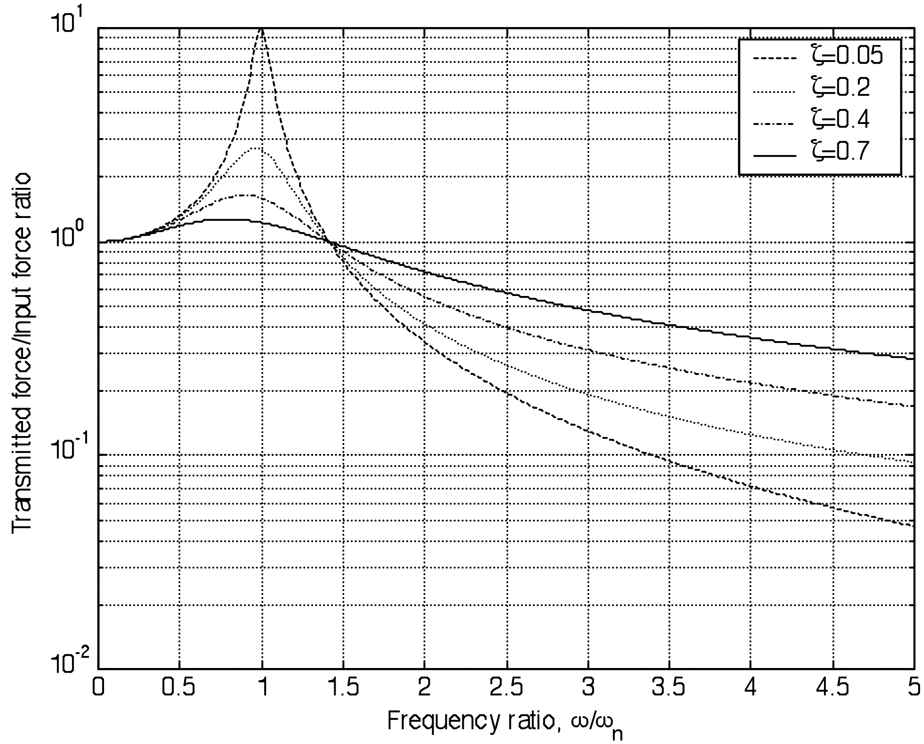


$$M\ddot{x} + C\dot{x} + Kx = f(t)$$

$$f_T(t) = C\dot{x} + Kx$$

$$M\ddot{x}_2 + C\dot{x}_2 + Kx_2 = C\dot{x}_1 + Kx_1$$

Vibrating Isolation



$$\frac{F_T(j\omega)}{F(j\omega)} = \frac{1 + j2\varsigma\left(\dfrac{\omega}{\omega_n}\right)}{1 - \left(\dfrac{\omega}{\omega_n}\right)^2 + j2\varsigma\left(\dfrac{\omega}{\omega_n}\right)} = \frac{X_2(j\omega)}{X_1(j\omega)}$$