ME 563
Mechanical Vibrations
Lecture #1

Derivation of equations of motion
(Newton-Euler Laws)

Derivation of Equation of Motion

Define the vibrations of interest
-Degrees of freedom (translational, rotational, etc.)
-Frequency range (<5 Hz, >15 Hz, etc.)
-Amplitude range (<2 g, >10 g, linear or nonlinear, etc.)

Develop a model representation
-Discrete/lumped elements (springs, dampers, etc.)
-Continuous elements (beams, rods, membranes, plates, etc.)
-Excitation function (ground motion, wind, machinery, etc.)

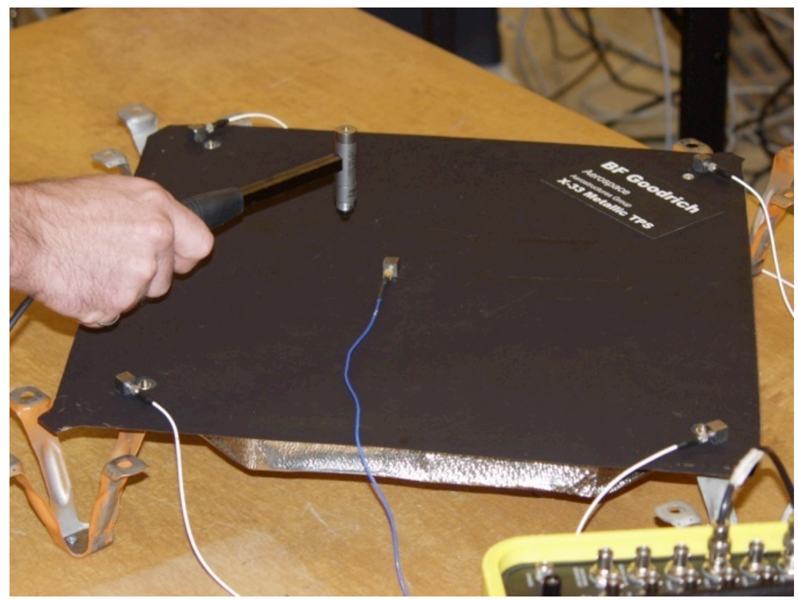
Define motions (kinematics)
-u, v, w, θ, etc.
-Undeformed or deformed datum, direction w/r/t gravitational field
-Constraints on/between the variables and #DOFs (base motion, gears)

Derive equations of motion
-Newton-Euler laws
-Energy/power methods

Calculate system parameters
-Strength of materials or experimentation
-Catalogues from vendors (bushings, mounts, couplings, etc.)

Keep track of assumptions

Define the vibrations of interest



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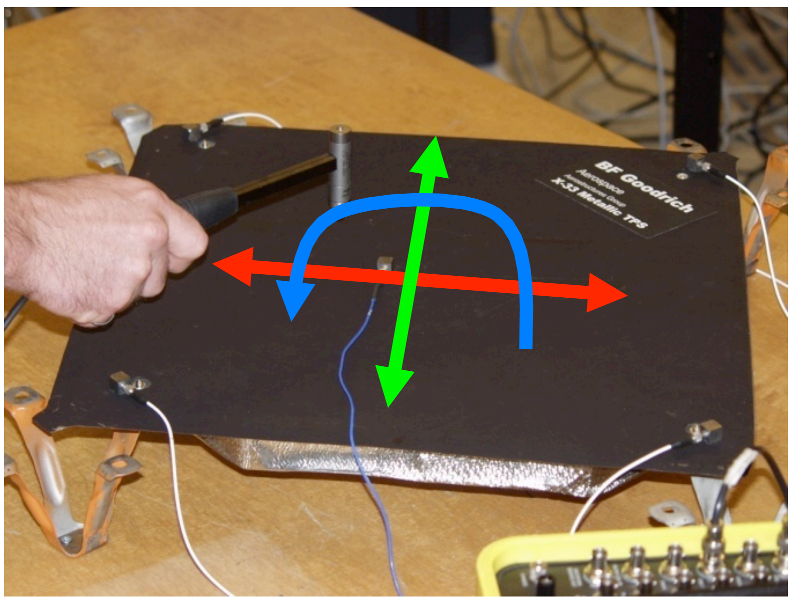
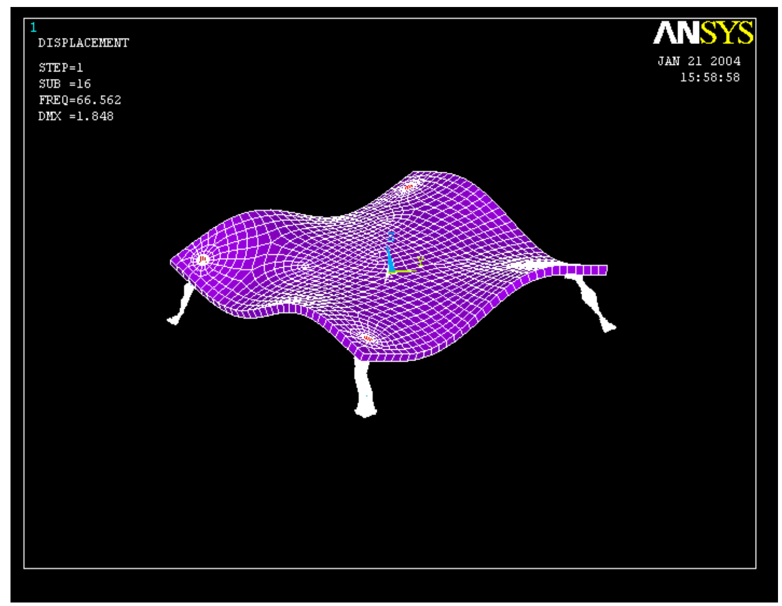
Define the vibrations of interest (Degrees of freedom)



Panel rigid body
degrees of freedom



Panel flexible body
degrees of freedom

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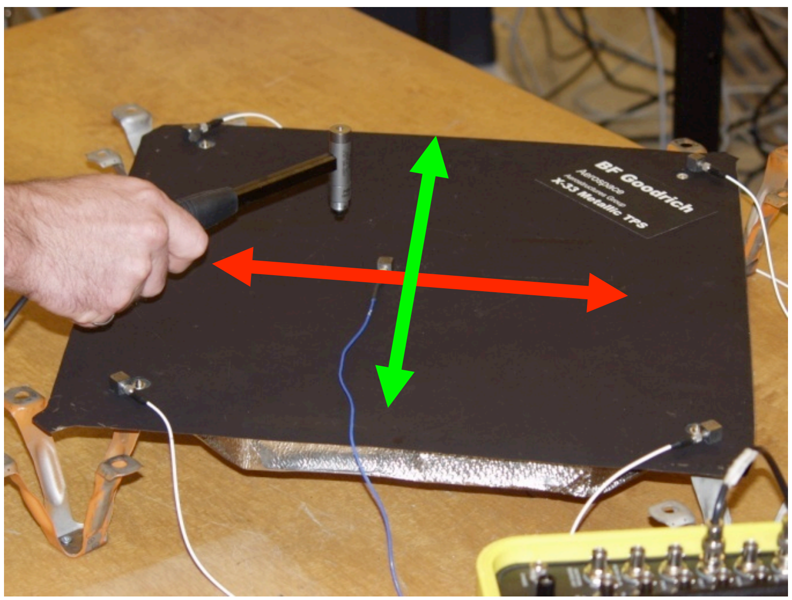
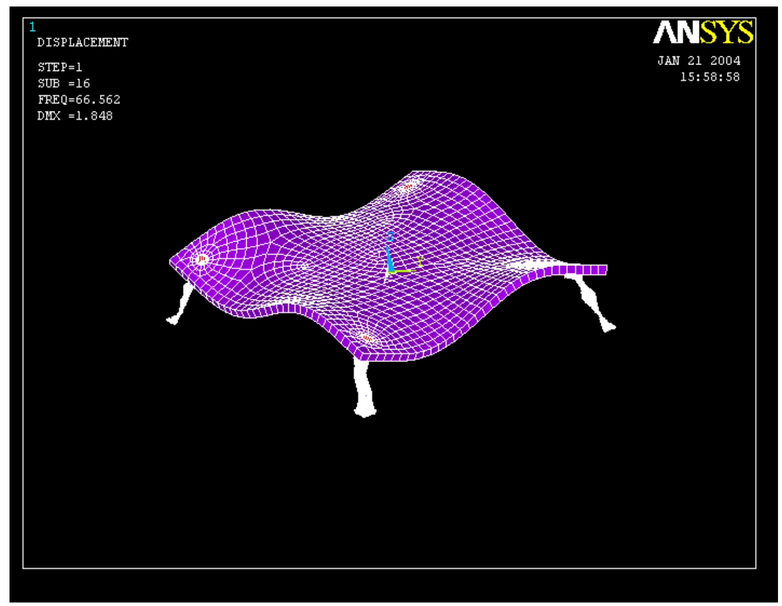
Define the vibrations of interest (Frequency range)





Low-frequency (<50 Hz)
response

High-frequency (>100 Hz)
response

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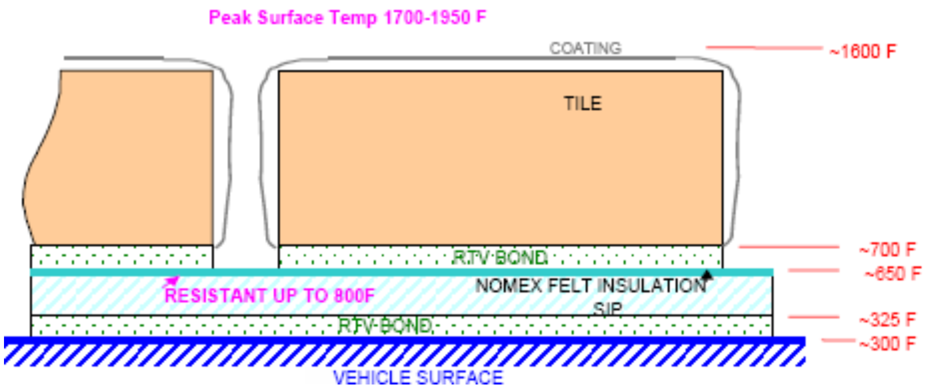
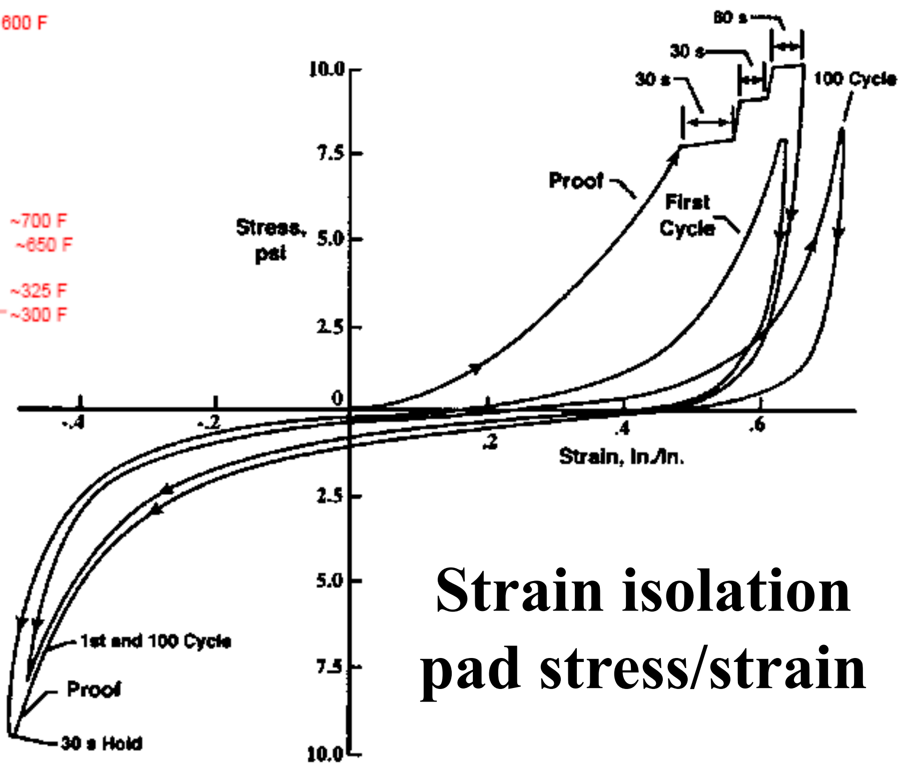
Define the vibrations of interest (Amplitude range)



Ceramic tile attached to orbiter using adhesive bond



Strain isolation pad stress/strain

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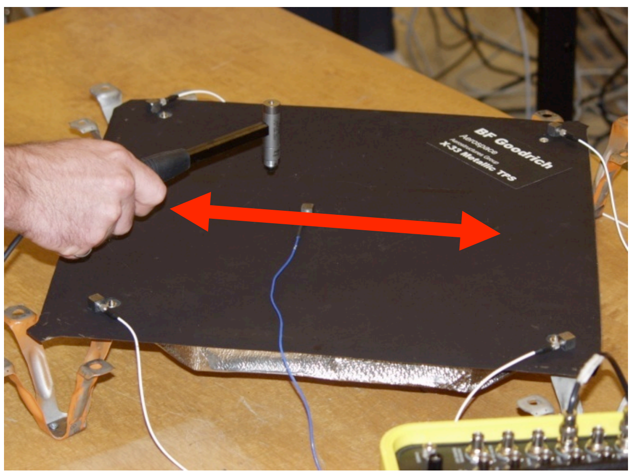
Keep track of assumptions

Develop model representation (Lumped/discrete elements)



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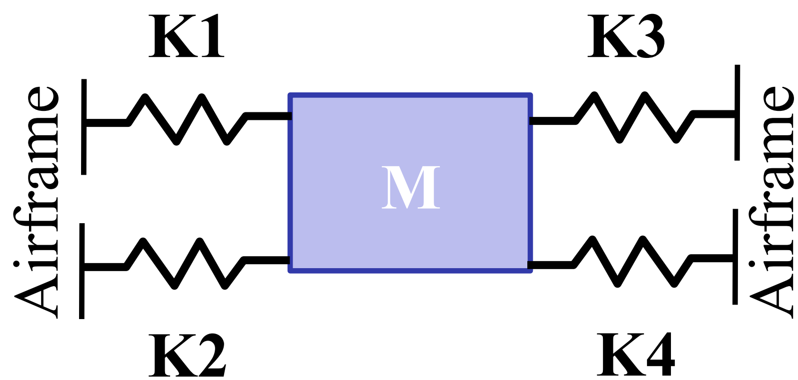

Parallel elements – same motion

Series elements – same force

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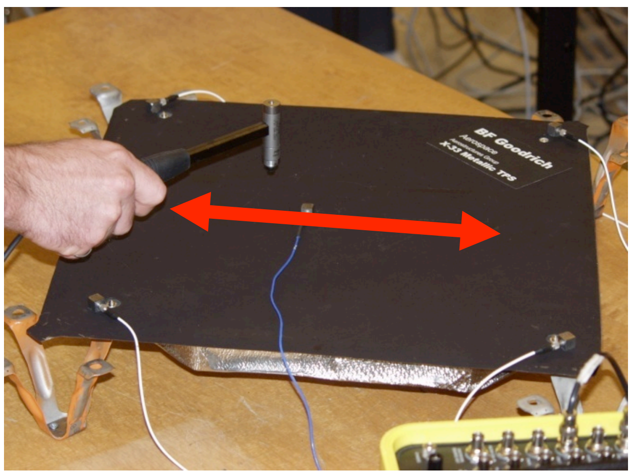
Keep track of assumptions

Develop model representation (Continuous elements)



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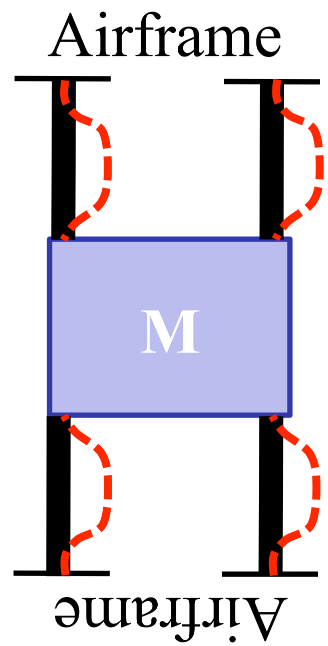

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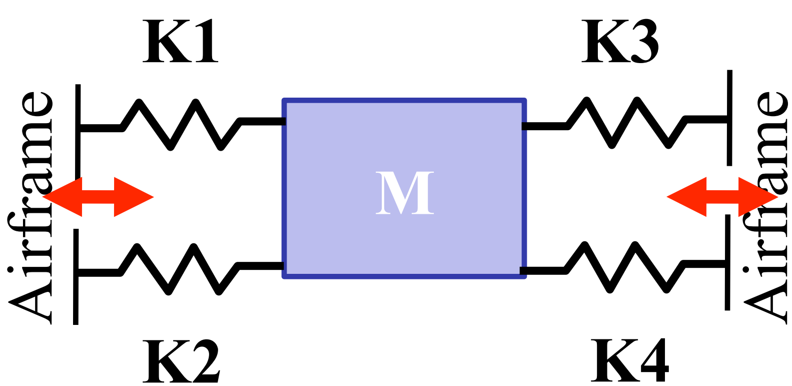
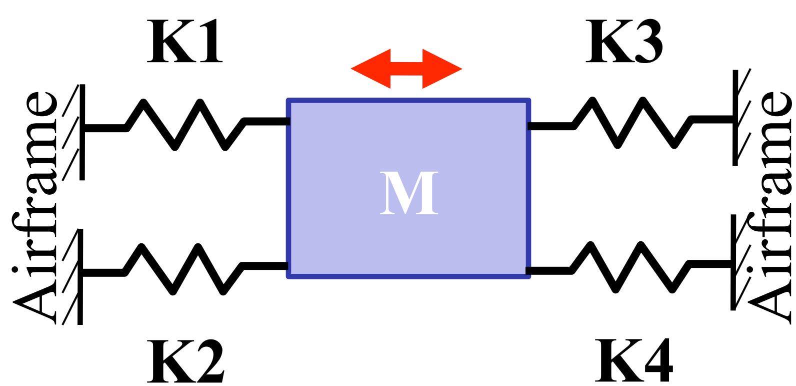
Keep track of assumptions

Develop model representation (Excitation function)

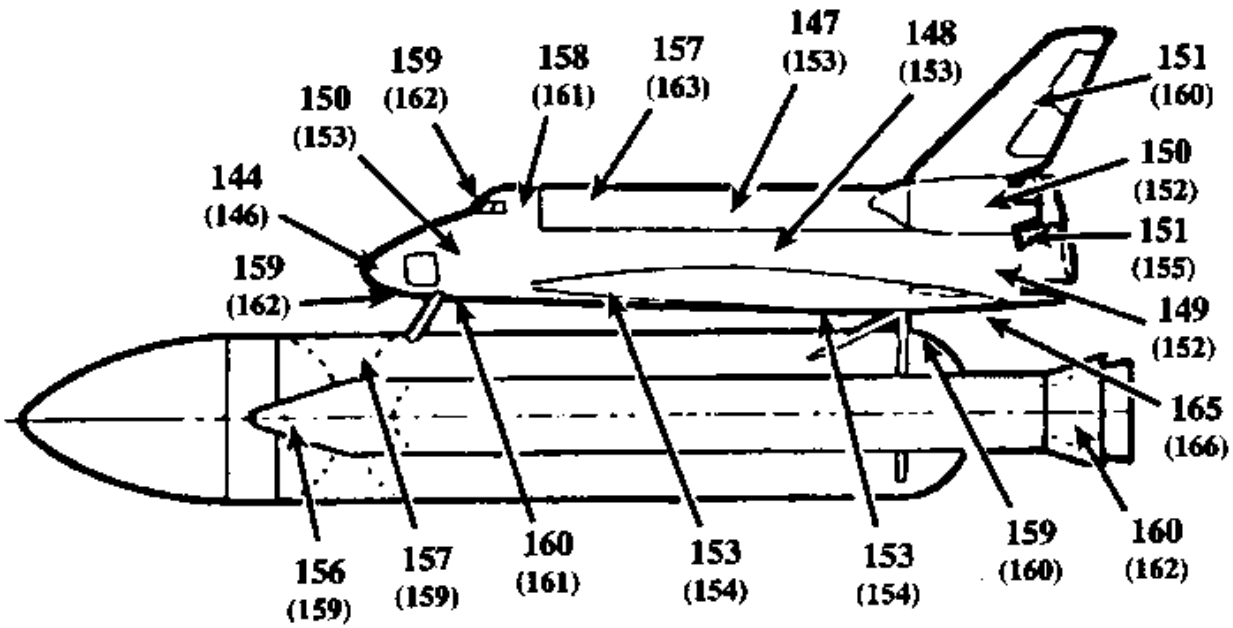


Base motion



Applied force (impact)

Develop model representation (Excitation function)



Ascent aerodynamic noise levels. Maximum space average levels for nominal and wind-dispersed (XXX) vehicle attitudes.

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Derive equations of motion
-Newton-Euler laws
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Calculate system parameters
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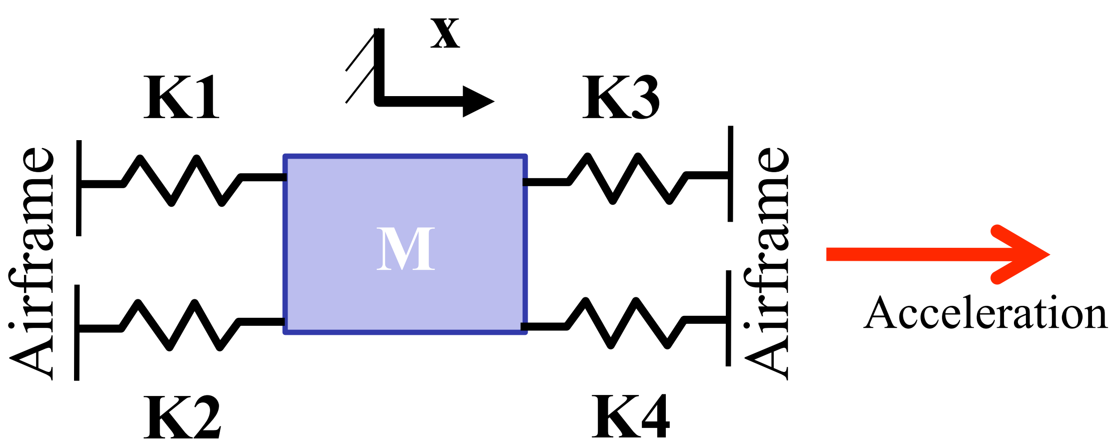
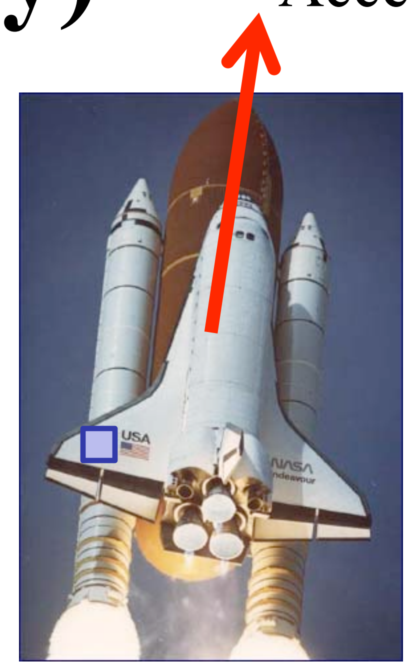
Keep track of assumptions

Define motions (DOFs) (Generalized coordinates, datum, gravity)





x is defined
w/r/t equilibrium position
(under acceleration of shuttle)

Dynamic response is
large compared to
gravitational response

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Derive equations of motion

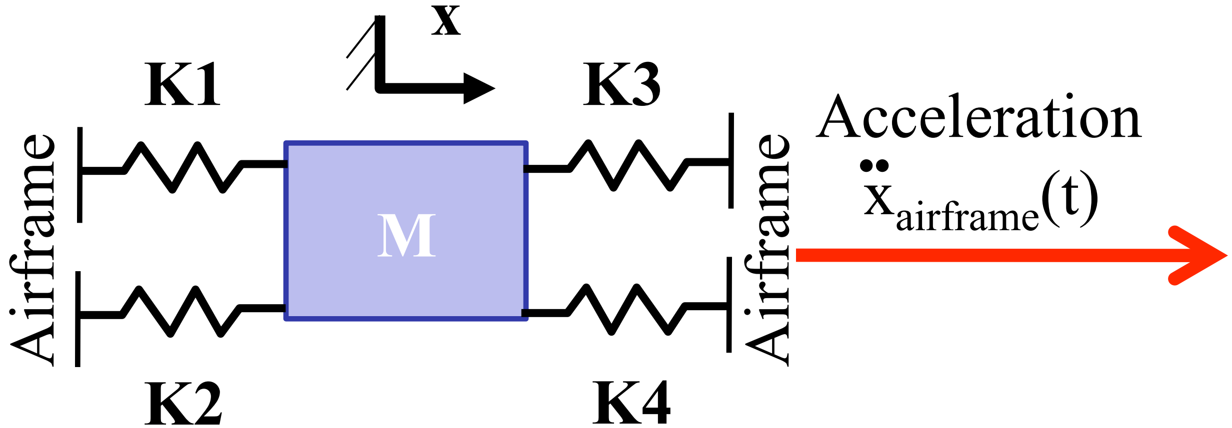
-Newton-Euler laws
-Energy/power methods

Calculate system parameters

-Strength of materials or experimentation
-Catalogues from vendors (bushings, mounts, couplings, etc.)

Keep track of assumptions

Define motions (DOFs) (Constraints on coordinates)



 # D.O.F. = # generalized coordinates - # constraints
= 2 – 1
= 1

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Derive equations of motion

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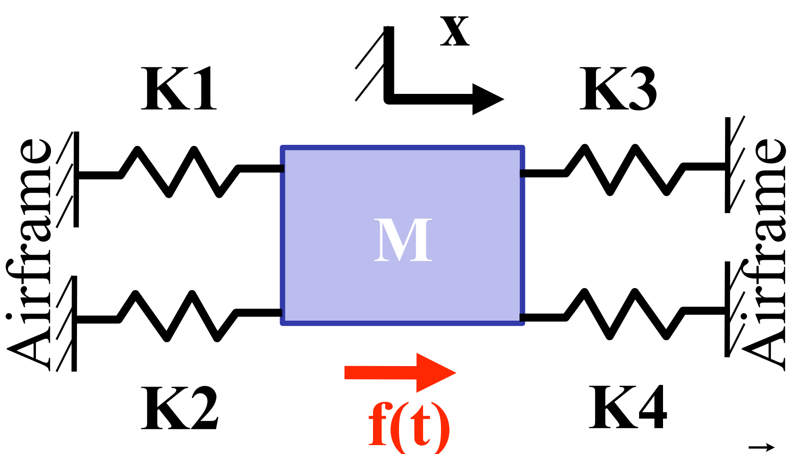
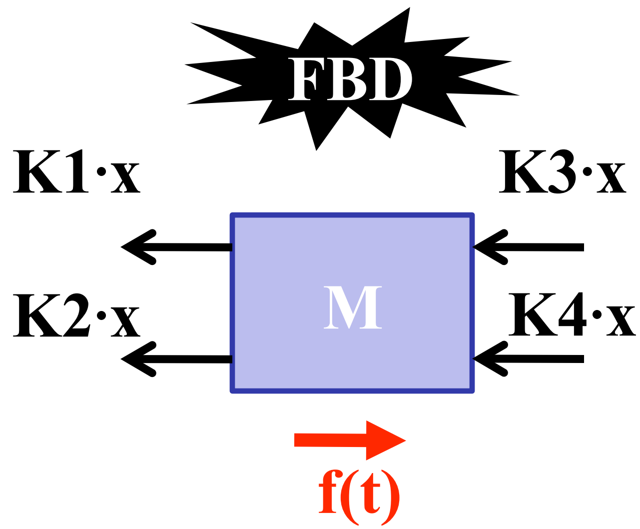
Keep track of assumptions

Derive equations of motion (Newton-Euler, force)





$$\underset{+}{\rightarrow} \sum_{Body\ M} \vec{F} = \frac{d\vec{P}}{dt} = M\ddot{x}$$

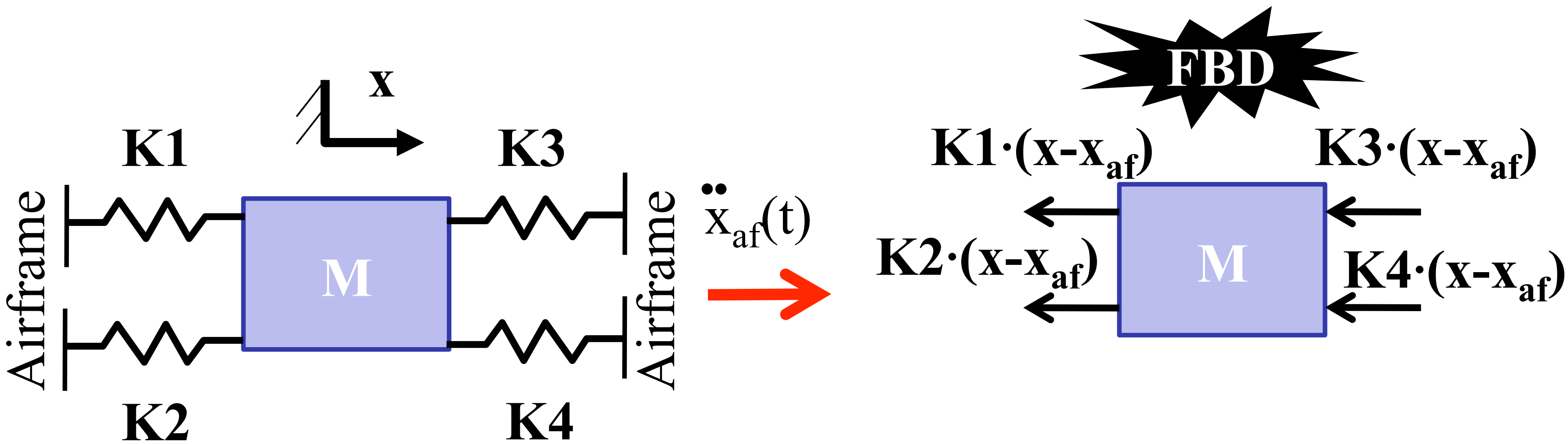
$$= -K_1 x - K_2 x - K_3 x - K_4 x + f(t)$$

$$= -(K_1 + K_2 + K_3 + K_4)x + f(t)$$

$$= -K_{equiv} x + f(t)$$

$$M\ddot{x} + K_{equiv} x = f(t)$$

Derive equations of motion (Newton-Euler, base motion)



$$\xrightarrow{}_{+} \sum_{Body\ M} \vec{F} = \frac{d\vec{P}}{dt} = M\ddot{x}$$

$$= -(K_1 + K_2 + K_3 + K_4)(x - x_{af})$$

$$= -K_{equiv}(x - x_{af})$$

$$M\ddot{x} + K_{equiv}x = K_{equiv}x_{af}$$

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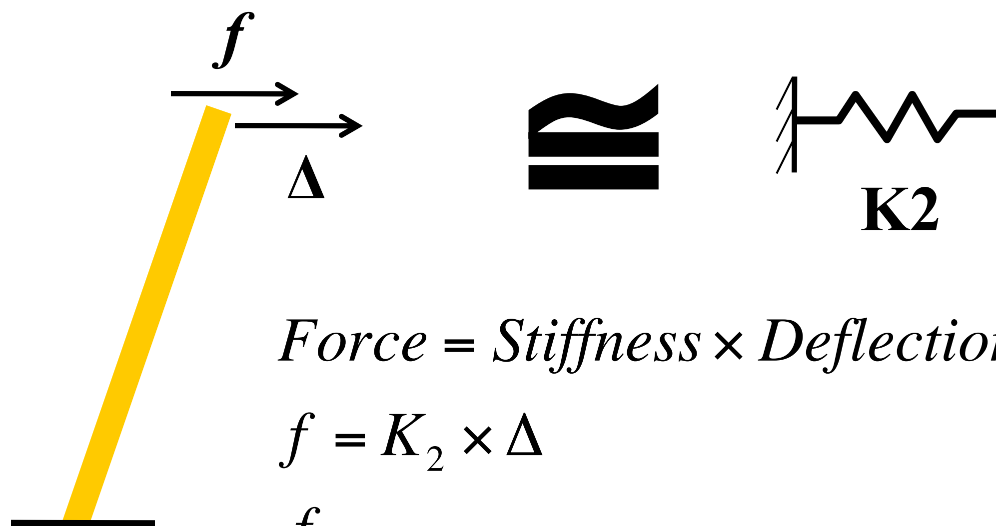
Calculate system parameters (Strength of materials)



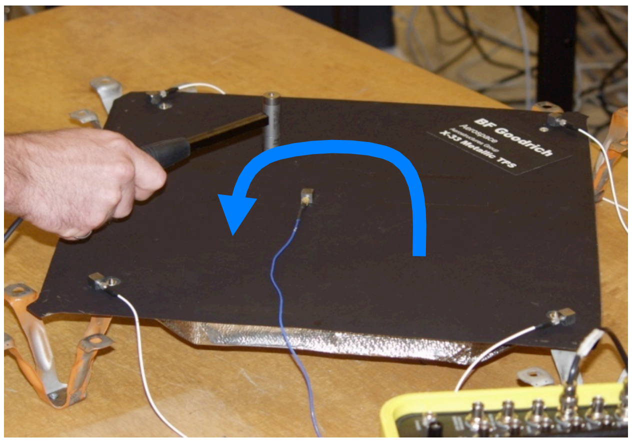
$$Force = Stiffness \times Deflection$$

$$f = K_2 \times \Delta$$

$$\frac{f}{\Delta} = K_2$$

$$\Delta = \frac{fL^3}{3EI}$$

What about the rotational motion?



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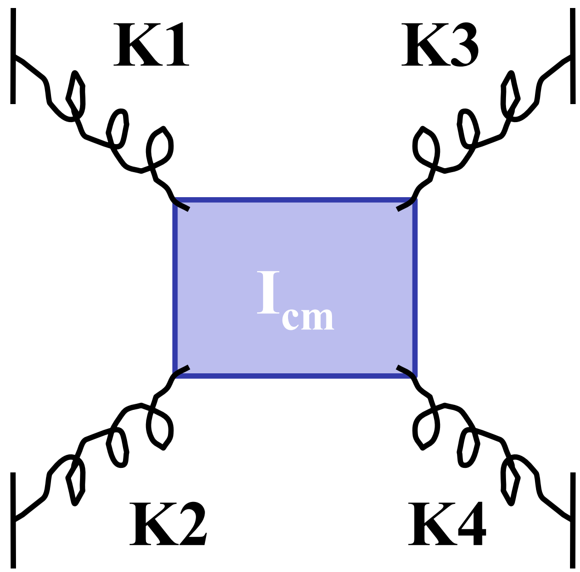
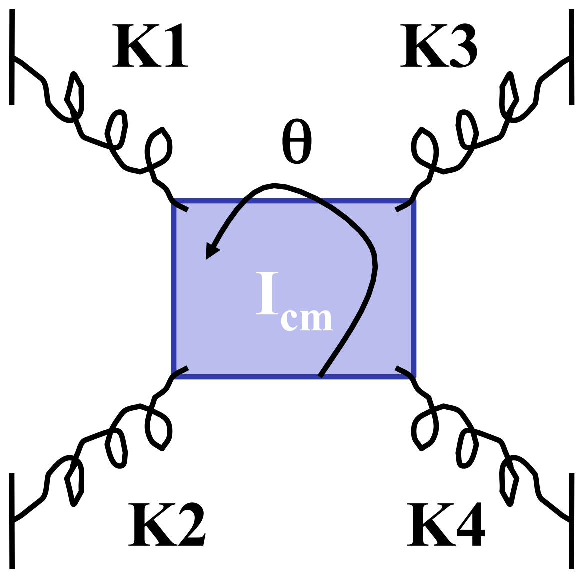
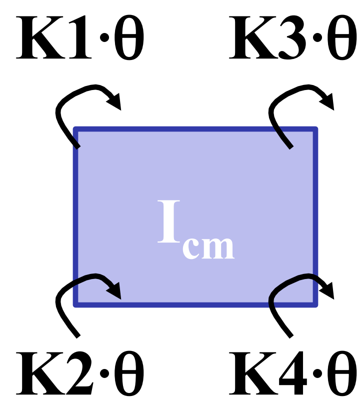


What about the rotational motion?





What is K2?

$$\theta = \frac{TL}{JG}$$

$$\curvearrowleft + \sum_{Body\ I_{cm}} T = I_{cm}\ddot{\theta}$$

$$I_{cm}\ddot{\theta} + K_{equiv}\theta = T_{applied}(t)$$