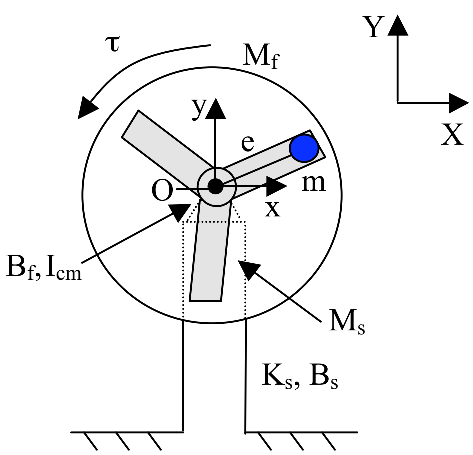
ME 563 Mechanical Vibrations, in-class example problem 2010 DEA

MODELING THE WOBBLE OF AN ELECTRIC FAN

Assumptions:

- The fan motor is 'ideal' in the sense that the applied torque does not depend on the wobbling motion of the fan
- All of the damping in the support and fan bearing is viscous in nature
- The stiffness of the support in the vertical direction is large relative to that in the horizontal direction, so the wobbling motion in the vertical direction can be ignored
- All of the motion of interest takes place in the plane (no wobbling in and out of the page)
- The support can be represented by a set of lumped effective mass, damping and stiffness parameters even though it is a continuous element
- The support stiffness does not permit the fan housing itself to rotate; the only rotation of interest occurs in the fan blades
- The rotation of the fan in counterclockwise
- The forces due to the aerodynamics of the fan blades themselves are negligible at the low speeds of interest

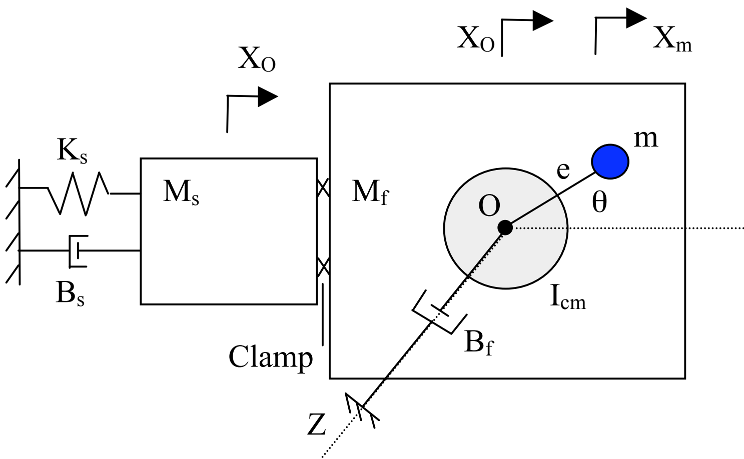
Illustration of fan and parameter labels:



m – Rotating paper clip
M_f – Mass of fan and housing
e – Eccentricity of paper clip
B_f – Viscous damping bearing coefficient
I_{cm} – Mass moment of inertia of rotating portion of fan
K_s – Stiffness of support
B_s – Viscous damping in support
M_s – Effective mass of support
τ - Applied torque of fan motor
O – Center of rotation of fan and support point
X,Y – Global coordinate system
x,y – Local translating coordinate system attached to point O

Schematic diagram:

(Note that the damper representing the fan bearing friction has drawn coming out of the page along the Z axis)

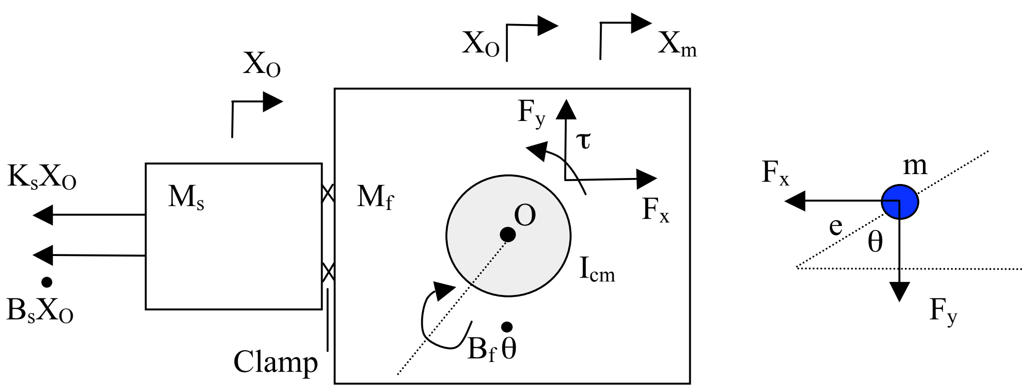


Kinematics:

$X_m = X_O + e\ cos\theta$
(kinematic constr.)

Free Body Diagrams (FBDs):

(Note that the fan blades and the fan housing are treated together as one free body, which is connected rigidly to the support through the clamp on the fan)



Deriving the equations of motion:

First, apply Newton's second law to the translating mass, M_s+M_f

$$\sum F = \left(M_s + M_f\right)\ddot{X}_O = -B_s\dot{X}_O - K_s X_O + F_x$$

then apply Newton's second law to the translating mass, m, to find F_x

$$\sum F = m\ddot{X}_m = -F_x$$
$$= m\left(\ddot{X}_O - e\ddot{\theta}\sin\theta - e\dot{\theta}^2\cos\theta\right)$$

These two equations can be combined to yield one equation with one unknown, X_O

$$\left(M_s + M_f + m\right)\ddot{X}_O + B_s\dot{X}_O + K_s X_O = me\ddot{\theta}\sin\theta + me\dot{\theta}^2\cos\theta \quad (1)$$

Next, apply Euler's law to the rotating fan blade, I_{cm}

$$\begin{aligned}&\sum T = I_{cm}\ddot{\theta} = \tau - B_f\dot{\theta} - eF_x\sin\theta + F_y e\cos\theta \\ &I_{cm}\ddot{\theta} = \tau - B_f\dot{\theta} + e\sin\theta\ \ m\left(\ddot{X}_O - e\ddot{\theta}\sin\theta - e\dot{\theta}^2\cos\theta\right)\end{aligned} \quad (2)$$

Eqs. (1) and (2) are the two equations of motion of the 2 D.O.F. system. Note that they are coupled but only one way, from the rotational coordinate to the translational coordinate due to the assumption of an ideal motor. In many applications, it is sometimes assumed that the speed of rotation is constant, in which case Eq. (1) is all that is needed and the term on the right hand side with the angular acceleration goes to zero as follows:

$$\left(M_s + M_f + m\right)\ddot{X}_O + B_s\dot{X}_O + K_s X_O = me\omega^2\cos\omega t \quad (3)$$

Also note that the kinematics in this simplified case are such that the angle of rotation is now an explicit function of time because the speed is assumed to be constant. Eq. (3) is often used to model imbalance in rotating machinery. The figure below shows the response of the 2 D.O.F. system for a constant applied torque by the motor for arbitrary system parameters. Note how the wobble of the fan contains to different frequencies in the initial portion of the response. We will talk about where these come from later in the course. Also note that the fan doesn't begin to noticeably wobble until the rotational speed reaches a certain value. The second figure shows the block diagram used to simulate the response of this system in SIMULINK.

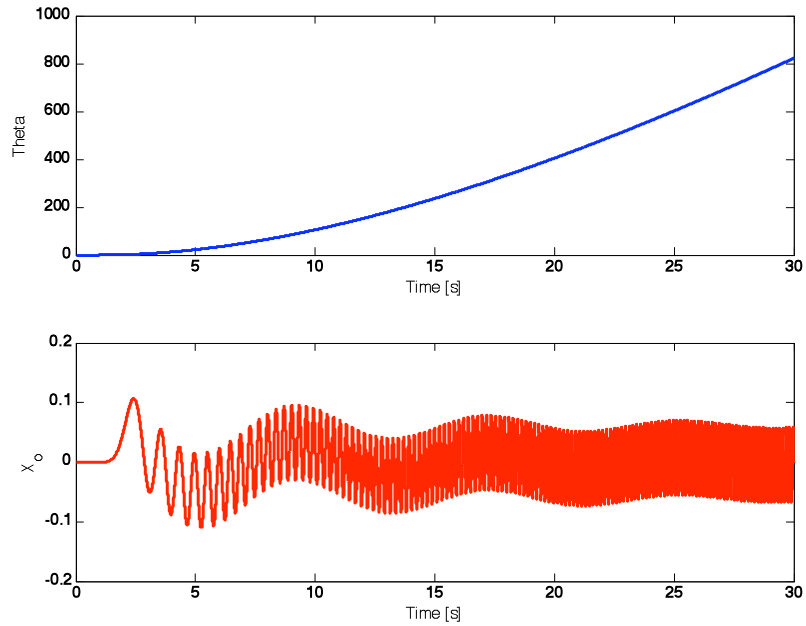


Figure showing (top) change in theta and (bottom) wobble of fan

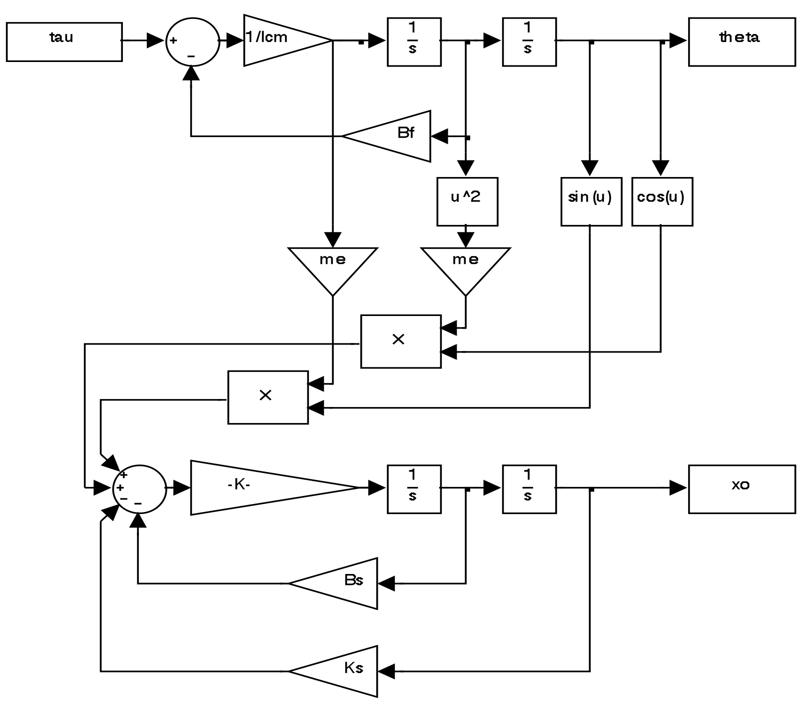


Figure showing SIMULINK model for solving the two D.O.F. equations of motion