Time Domain Analysis

$$\dot{y} + ay = u_s(t)$$

1. Set the right hand side to zero:

$$\dot{y} + ay = 0$$

2. Assume exponential $y_c(t)$:

$$y(t) = Ae^{st} \Rightarrow \left[s + a\right]Ae^{st} = 0$$

3. Solve the characteristic equation:

$$s + a = 0 \Rightarrow s = -a$$

4. Plug roots back into guess:

$$y_c(t) = Ae^{-at}$$

5. Assume $y_p(t)$ with same form as input:

$$y(t) = Bu_s(t)$$

6. Solve for particular solution:

$$0 + aBu_s(t) = u_s(t) \Rightarrow y_p(t) = \frac{1}{a}u_s(t)$$

7. Write the solution as $y_c + y_p$:

$$y(t) = Ae^{-at} + \frac{1}{a}u_s(t)$$

8. Apply I.C.s: $y(t) = \frac{1}{a} + \left(y(0) - \frac{1}{a}\right)e^{-at} \quad \text{for } t > 0$

Frequency Domain Analysis

$$\dot{y} + ay = u_s(t)$$

1. Take the Laplace transform:

$$\left[s + a\right]Y(s) - y(0) = \frac{1}{s}$$

2. Solve for $Y(s)$:

$$Y(s) = \frac{1}{s(s+a)} + \frac{y(0)}{s+a}$$

3. Expand using partial fractions:

$$\frac{1}{s(s+a)} = \frac{A}{s} + \frac{B}{s+a} = \frac{1/a}{s} - \frac{1/a}{s+a}$$

4. Take the inverse Laplace transform:

$$y(t) = \frac{1}{a}u_s(t) - \frac{1}{a}e^{-at} + y(0)e^{-at}$$

$$= \frac{1}{a} + \left(y(0) - \frac{1}{a}\right)e^{-at} \quad \text{for } t > 0$$