ME 563 **HOMEWORK # 9 SOLNS** **Fall 2008**

PROBLEM 1: (50%)

Given the following set of equations for a vibrating system with translational and rotational degrees of freedom,

$$\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{Bmatrix}\ddot{y}\\ \ddot{\theta}\end{Bmatrix}+\begin{bmatrix}0.4 & -0.2\\ -0.2 & 0.2\end{bmatrix}\begin{Bmatrix}\dot{y}\\ \dot{\theta}\end{Bmatrix}+\begin{bmatrix}4 & -2\\ -2 & 2\end{bmatrix}\begin{Bmatrix}y\\ \theta\end{Bmatrix}=\begin{Bmatrix}f(t)\\ \tau(t)\end{Bmatrix}$$

find the analytical expressions for the modal frequency response functions of the system. What combination of forcing functions should be chosen to only excite mode #1? What about for mode #2?

To begin this problem, we need to calculate the modal frequencies and modal vectors by analyzing the free response of the system. By solving the eigenvalue problem in Matlab, we obtain the following information:

$$s_{1,2}=-0.022\pm 0.66j \qquad \{\psi\}_1=\{0.56 \quad 1.0\}^T$$
$$s_{3,4}=-0.228\pm 2.12j \qquad \{\psi\}_2=\{1.0 \quad -0.28\}^T$$

With this information, we can decouple the equations of motion to solve for the modal equations of motion using the approach discussed in class.

$$\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}^T\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}\begin{Bmatrix}\ddot{p}_1\\ \ddot{p}_2\end{Bmatrix}$$
$$+\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}^T\begin{bmatrix}0.4 & -0.2\\ -0.2 & 0.2\end{bmatrix}\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}\begin{Bmatrix}\dot{p}_1\\ \dot{p}_2\end{Bmatrix}$$
$$+\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}^T\begin{bmatrix}4 & -2\\ -2 & 2\end{bmatrix}\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}\begin{Bmatrix}p_1\\ p_2\end{Bmatrix}=\begin{bmatrix}0.56 & 1.0\\ 1.0 & -0.28\end{bmatrix}^T\begin{Bmatrix}f(t)\\ \tau(t)\end{Bmatrix}$$
$$\begin{bmatrix}2.3 & 0\\ 0 & 1.2\end{bmatrix}\begin{Bmatrix}\ddot{p}_1\\ \ddot{p}_2\end{Bmatrix}+\begin{bmatrix}0.10 & 0\\ 0 & 0.53\end{bmatrix}\begin{Bmatrix}\dot{p}_1\\ \dot{p}_2\end{Bmatrix}+\begin{bmatrix}1.01 & 0\\ 0 & 5.28\end{bmatrix}\begin{Bmatrix}p_1\\ p_2\end{Bmatrix}=\begin{Bmatrix}0.56f(t)+\tau(t)\\ f(t)-0.28\tau(t)\end{Bmatrix}$$

Now the modal frequency response functions can be formulated as follows:

$$\begin{Bmatrix} P_1(j\omega) \\ P_2(j\omega) \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{1.01 - 2.3\omega^2 + j\omega 0.10} & 0 \\ 0 & \dfrac{1}{5.28 - 1.2\omega^2 + j\omega 0.53} \end{bmatrix} \begin{Bmatrix} 0.56F(j\omega) + \mathrm{T}(j\omega) \\ F(j\omega) - 0.28\mathrm{T}(j\omega) \end{Bmatrix}$$

where the modal frequency response functions are given by the diagonal entries in the first matrix on the right hand side of the equation above.

To only excite the first mode of vibration, the forcing functions should be chosen such that

$$f(t) - 0.28\tau(t) = 0$$

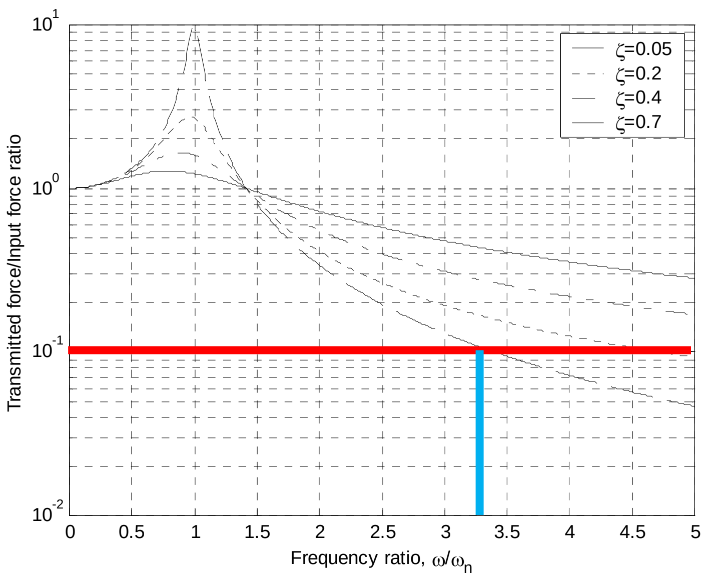
To only excite the second mode of vibration, the forcing functions should be chosen such that

$$0.56f(t) + \tau(t) = 0$$

PROBLEM 2: (50%)

Assume you are designing a vibration isolation system for a machine tool. You must select the mounts to position beneath the tool in order to achieve a 90% reduction in the transmitted cutting forces of the tool. If the machine tool weighs 2000 Newtons and spins at 1000 RPM in the steady state, what mount stiffness do you need to choose if the damping ratio of the system is 0.05? If the measured transmitted force is twice the cutting force for this same machine tool and your selected mount design, what must be the lower speed(s) of operation of the machine?

Given the vibration isolation magnitude plot shown below from the lecture notes,



it is evident that the design target is as shown in the figure in red. To achieve this level of isolation, we must choose a frequency ratio of 3.25 as indicated in blue. Since the rotational speed is 1000 RPM, the natural frequency of the isolation system must be

$$\omega_n = \frac{1000RPM}{3.25} = 308RPM = 5.1Hz$$

To produce this natural frequency, the mount stiffness must be

$$K = M\omega_n^2 = 2000/9.81 \times \left(2\pi \times 5.1\right)^2 = 0.2 \times 10^6 N/m\,.$$

If we choose this mount stiffness and measure a transmissibility of 2, then the machine must be rotating at 0.65 (199 RPM) or 1.25 (383 RPM) times the natural frequency of the isolation system in its lower speeds of operation.