ME 563 **HOMEWORK # 8 SOLNS** **Fall 2008**

PROBLEM 1: (35%)

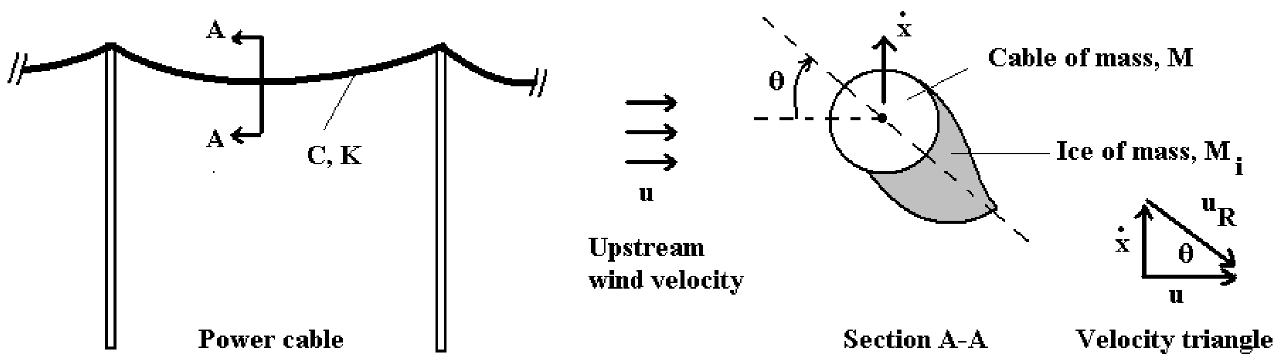
During the winter months, power cables can become coated with ice as shown in the figure below. The ice creates an asymmetric aerodynamic surface overwhich wind blows causing the power cables to 'gallop' and eventually vibrate loose from their supports. 'Galloping' or 'flutter' occurs in this case when the effective damping of the vibrating cables becomes negative. If the angle of attack, θ, which is the angle between the upstream wind velocity, u, and the relative velocity, u_R, is relatively small, then find the maximum wind velocity that will not produce galloping of the cable (mass M, viscous damping C, vertical stiffness K) given that the vertical force on the ice-covered cable (ice mass M_i) is,

$$F = \frac{1}{2}\rho u^2 W C_x$$

where C_x is a force coefficient given by,

$$C_x = \left(\frac{u_R}{u}\right)^2 \left[C_L \cos\theta + C_D \sin\theta\right]$$

with lift coefficient, C_L, and drag coefficient, C_D, and W is the area of the ice-covered cable. Assume all constants are known and that the cable can be modeled as a SDOF system. Comment on the result.



We begin by writing down the SDOF equation of motion for this system including the force due to the wind:

$$M\ddot{x} + C\dot{x} + Kx = \frac{1}{2}\rho u^2 W C_x$$

Our goal is to determine what the equation of motion is for small angles of attack, θ. Since F is a (continuously differentiable) function of the angle of attack, we can expand it into a Taylor series. But first we need to express it entirely as a function of θ. To do this, we note that the ratio of the relative velocity to the wind velocity is $u_R/u=1/\cos\theta$. Thus, F becomes

$$
\begin{aligned}
F &= \frac{1}{2}\rho u^2 W C_x \\
&= \frac{1}{2}\rho u^2 W\left[\left(\frac{u_R}{u}\right)^2 \left(C_L \cos\theta + C_D \sin\theta\right)\right] \\
&= \frac{1}{2}\rho u^2 W\left(C_L \frac{1}{\cos\theta} + C_D \frac{\tan\theta}{\cos\theta}\right)
\end{aligned}
$$

Then the Taylor series expansion is,

$$
\begin{aligned}
F &= F\big|_{\theta=0} + \frac{\partial F}{\partial\theta}\bigg|_{\theta=0}\theta + O(\theta^2) \\
&= \frac{1}{2}\rho u^2 W\left[C_L + C_D\theta + O(\theta^2)\right] \\
&= \frac{1}{2}\rho u^2 W C_L + \frac{1}{2}\rho u^2 W C_D\theta + O(\theta^2)
\end{aligned}
$$

With the velocity triangle given, the second term can be rewritten as:

$$
\begin{aligned}
F &= \frac{1}{2}\rho u^2 W C_L + \frac{1}{2}\rho u W C_D \dot{x} + O(\theta^2) \\
&\approx \frac{1}{2}\rho u^2 W C_L + \frac{1}{2}\rho u W C_D \dot{x}
\end{aligned}
$$

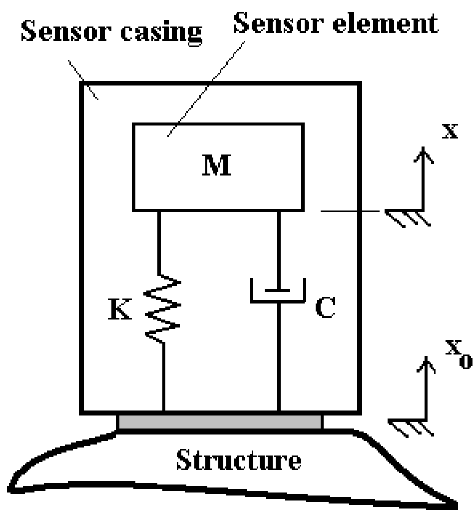
which produces the following equation of motion after grouping velocity terms:

$$
M\ddot{x} + \left(C - \frac{1}{2}\rho u W C_D\right)\dot{x} + Kx = \frac{1}{2}\rho u^2 W C_L
$$

If the wind velocity is less than $2C/\rho W C_D$, then the effective damping is positive and there will not be any galloping of the cable. Note that the lift coefficient produces a steady displacement of the cable (it is a static input) whereas the drag component is responsible for the unstable vibration (‘galloping’).

<u>PROBLEM 2</u> (30%)

You would like to use the sensor below to measure the *displacement* of the structure. Assume that the output measurement from your sensor is proportional to the relative displacement between the base and the sensor element and that the casing is rigidly attached to the structure. Plot and describe the frequency response characteristics of this sensor for different values of the design parameters (M, C, and K). How would you design this sensor to make the best measurements? Can you measure the static displacement of the structure? Explain.



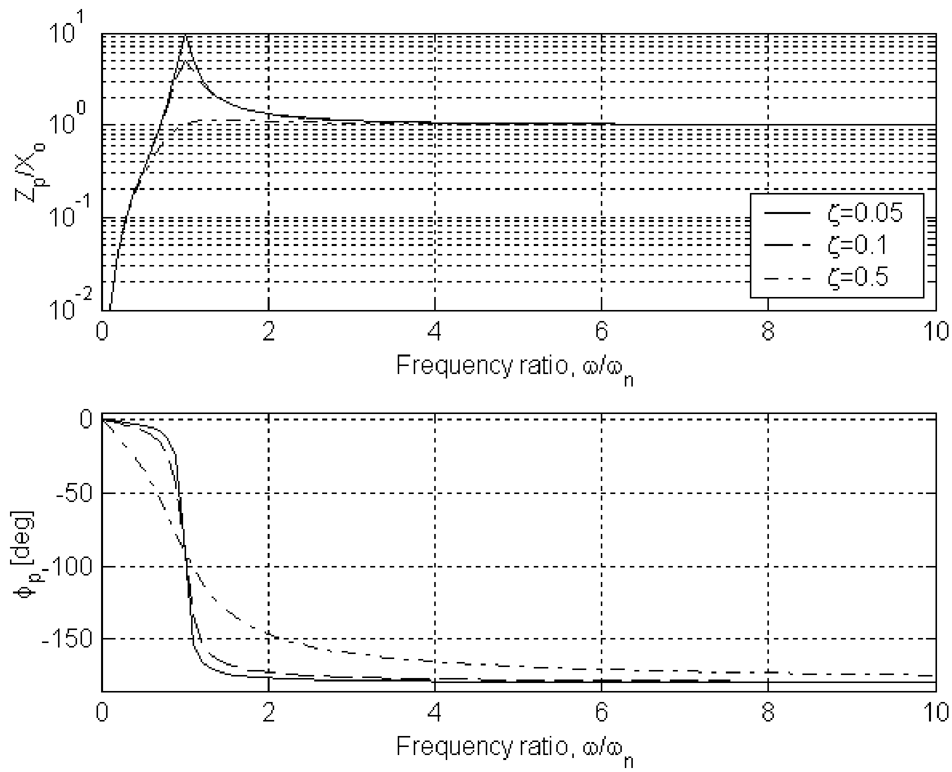
The frequency response function between the base motion and the sensor element motion is,

$$H(\omega) = \frac{Z(\omega)}{X_o(\omega)} = \frac{\omega^2 M}{K - M\omega^2 + j\omega C}$$

where $$\|H(\omega)\| = \frac{M\omega^2}{\sqrt{\left[K - M\omega^2\right]^2 + \left[\omega C\right]^2}} = \frac{\dfrac{\omega^2}{\omega_n^2}}{\sqrt{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\varsigma\dfrac{\omega}{\omega_n}\right]^2}}$$

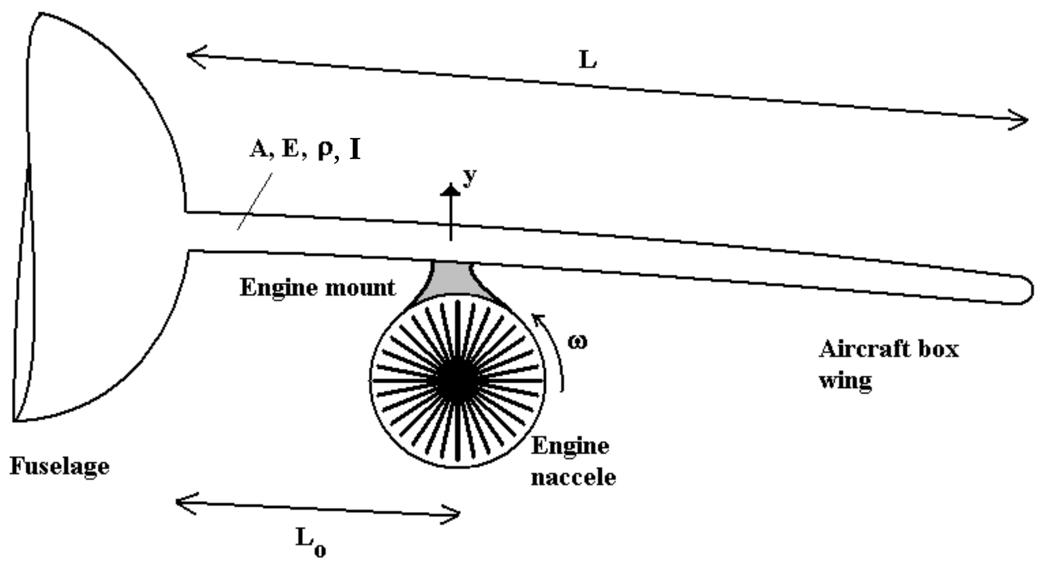
and $$\angle H(\omega) = -\tan^{-1}\frac{\omega C}{K - M\omega^2} = -\tan^{-1}\frac{2\varsigma\dfrac{\omega}{\omega_n}}{1-\left(\dfrac{\omega}{\omega_n}\right)^2}$$

The magnitude and phase of this FRF are shown below. In order to measure the displacement of the surface, we want to make the resonant frequency as low as possible because then for frequencies above ω_n, the magnitude approaches a constant ($||H(\omega>>\omega_n)||\rightarrow 1$) and the phase approaches -180 degrees. This means that the relative displacement approaches the displacement of the surface and is out of phase with the surface as well for frequencies above resonance. For low frequencies, the sensor element moves along with the base and the phase is close to zero; this means that the relative displacement also goes to zero, so we are *not* able to make measurements of the static (DC) displacement of the structure. Also note that we would select the damping ratio to be above 0.5 to minimize the amount of phase distortion in the measurement as discussed in lecture.



<u>PROBLEM 3</u> (35%)

Calculate, plot, and explain the frequency response function between the weighted unbalance mass, *me*, and the dynamic unbalance, MX_p, where X_p is the dynamic steady state displacement of the attachment point of the engine nacelle on the wing below when the rotor is spinning at a speed of *N* RPM, *M* is the total mass of the rotor-blade assembly, *m* is the unbalance mass of the rotor-blade assembly, and *e* is the eccentricity of unbalance. You will also need to calculate the effective mass of the wing as it undergoes vibration from the reciprocating imbalance of the engine. How would you design the nacelle attachment mount to eliminate the vibration amplitude of the wing at a rotational speed of *N* RPM? Show any associated frequency response functions associated with your proposed design.



A schematic of the two DOF proposed model for this system is shown below. The wing is assumed to vibrate at one mode of vibration. The stiffness is approximated using the equation for a beam, $K_w=3EI/L^3_o$. The effective mass of the wing is found by integrating the infinitesimal kinetic energy along the wing as follows:

$$\frac{1}{2}M_w\dot{x}^2=\int_0^L dT=\frac{1}{2}\int_0^L\left[\dot{x}Y(\eta)\right]^2\rho d\eta \text{ such that } Y(L_o)=1$$

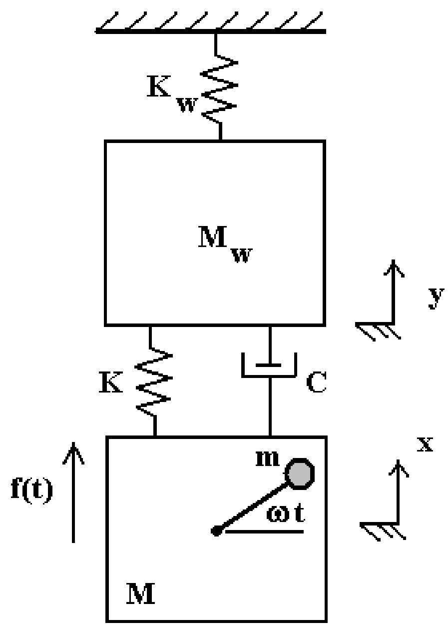
where $Y(x)$ is the first mode of vibration of a fixed-free beam normalized such that $Y(L_o)=1$ and dx/dt is the velocity of the wing at location L_o. This approach is the same as was used earlier in the semester when considering non-ideal springs with inertia. The equations of motion are then given as below after applying Newton's second law:

$$M_w\ddot{y} + C\dot{y} + (K_w + K)y - C\dot{x} - Kx = 0$$
$$M\ddot{x} + C\dot{x} + Kx - C\dot{y} - Ky = f(t) = me\omega^2 \sin\omega t$$

which upon taking the Laplace transform for zero initial conditions yields the transfer function matrix:

$$\begin{bmatrix} M_w s^2 + Cs + (K_w + K) & -Cs - K \\ -Cs - K & Ms^2 + Cs + K \end{bmatrix} \begin{Bmatrix} Y(s) \\ X(s) \end{Bmatrix} = \begin{Bmatrix} 0 \\ F(s) \end{Bmatrix}$$
$$\begin{Bmatrix} Y(s) \\ X(s) \end{Bmatrix} = \frac{1}{\Delta(s)} \begin{bmatrix} Ms^2 + Cs + K & Cs + K \\ Cs + K & M_w s^2 + Cs + (K_w + K) \end{bmatrix} \begin{Bmatrix} 0 \\ F(s) \end{Bmatrix}$$
$$H_{yf}(s) = \frac{Cs + K}{(Ms^2 + Cs + K)(M_w s^2 + Cs + (K_w + K)) - (Cs + K)^2}$$
$$\frac{MY(j\omega)}{me} = \frac{M\omega^2(j\omega C + K)}{(K - M\omega^2 + j\omega C)((K_w + K) - M_w\omega^2 + j\omega C) - (K + j\omega C)^2}$$

where $||Y||$ is the steady state response amplitude of the wing at the operating speed, ω. The magnitude of this function is plotted in the figure below for various values of K and various values of C.



Note that for *C=1 N-s/m*, larger values of K cause large increases in the transmissibility function for higher frequencies, which is where the engine is likely to operate; therefore, it is desirable to select a smaller stiffness for better isolation. Of course, the stiffness cannot be too small or else the nacelle will experience excessive static deflections. Likewise, smaller values of damping result in lower transmissibility at higher frequencies; however, the damping should not be too small or else the resonances during start-up are quite high as illustrated below.

If the response is still too high even after the above considerations are taken into account, then a dynamic absorber could be added to the system to redirect some of the vibration energy from the wing into the absorber. Dynamic absorbers work well as long as the speed of operation is limited to a small range.

It should also be noted that this analysis did not consider any of the higher frequency modes of the wing. These higher frequency modes are likely to be excited more strongly than the lowest frequency mode because the engine operates at frequencies far above the lowest mode. Consequently, a more accurate analysis would treat the wing as a continuous vibrating system instead.