

PROBLEM 1: (20%)

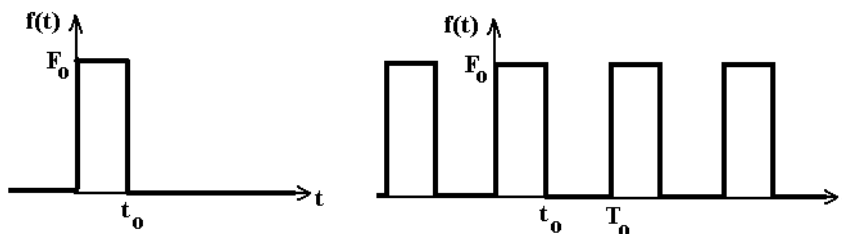
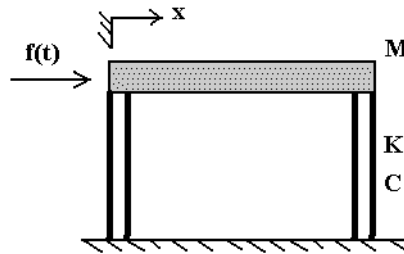
Given the mass matrix and two undamped natural frequencies for a general two degree-of-freedom system with a symmetric stiffness matrix,

$$[M] = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \text{ and } \omega_{n1} = 0 \text{ rad/s}, \omega_{n2} = \sqrt{3} \text{ rad/s}$$

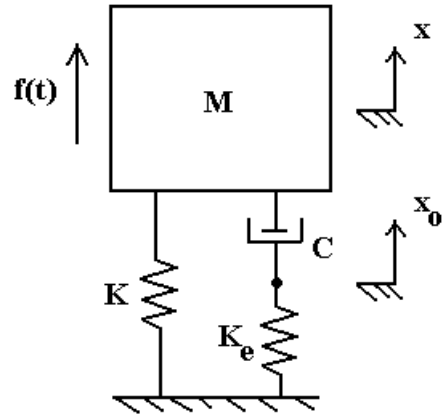
find the stiffness matrix, modal mass and stiffness for each mode of vibration, and the modal vectors (normalize the modal vectors so that the largest coefficient in each vector is of unity magnitude).

PROBLEM 2: (30%)

Calculate, plot (in MATLAB), and describe the response of the single story building below for the two different blast excitations, $f(t)$, shown. Assume $M=100,000 \text{ kg}$, $K=100 \text{ kN/m}$, $C=1000 \text{ N-s/m}$, $t_o=4 \text{ sec}$, $T_o=12 \text{ sec}$, and $F_o=10 \text{ kN}$. What happens to the responses as t_o approaches zero?

**PROBLEM 3** (30%)

Find and graph the frequency response function between the excitation, $f(t)$, and the response, $x(t)$, for the system below. Compare these results qualitatively with the frequency response results for a standard single degree-of-freedom viscously underdamped oscillator. What effect does the series damper-spring have on the frequency response? This system is often used to model elastomer mounts and bushings.



PROBLEM 4: (20%)

Find and plot the approximate frequency response function of the single degree of freedom system below that is subjected to Coulomb friction and a simple harmonic input, $F_i \cos(\omega t)$. Assume the damping force, $\mu N \cdot \text{sgn}(dx/dt)$, is small compared to the excitation amplitude, F_i , so that the steady state response can be assumed to be harmonic at the excitation frequency. Recall that the energy dissipated by Coulomb friction per cycle of harmonic oscillation at a frequency ω is $4\mu N X_p$, where μ is the static coefficient of friction, N is the normal force, and X_p is the steady state response amplitude at ω . Find the analytical condition on the friction force such that this solution is valid.

