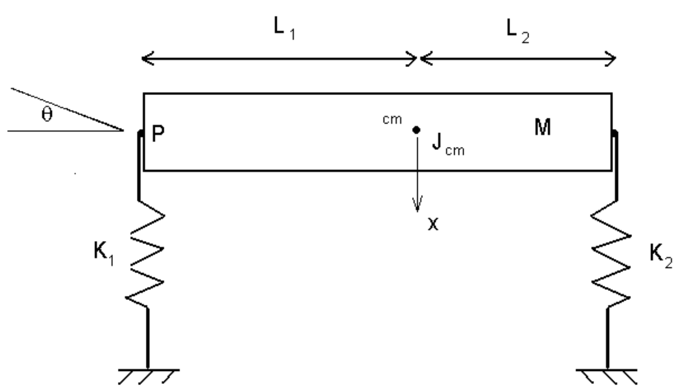
**ME 563** **HOMEWORK # 5** **Fall 2010**

**PROBLEM 1:** (25%)
You are given the lumped parameter dynamic differential equations of motion for a two degree-of-freedom model of an automobile suspension system for small rotations,

$$M\ddot{x} = -K_1\left(x - L_1\theta\right) - K_2\left(x + L_2\theta\right)$$
$$J_{cm}\ddot{\theta} = K_1\left(x - L_1\theta\right)L_1 - K_2\left(x + L_2\theta\right)L_2$$

with $M$=1,800 kg, $L_1+L_2$=3.6 m, $L_1$=1.4 m, $K_1$=42 kN/m, and $K_2$=48 kN/m and a radius of gyration of $R$=1.4 m. Determine the modal frequencies, undamped natural frequencies of oscillation, and the modal vectors. Draw schematics of the two modal vectors. Also give the forms for the two principal modes of vibration. Why are these modes adequate for describing general motions of the suspension system? Re-derive the differential equations of motion using the translational coordinate at point P instead of CM and find the modal frequencies. How are these related to the frequencies you found above and why?




**PROBLEM 2:** (25%)
Find and plot the translational and rotational responses of the suspension system above for initial conditions $x(0)$=0, $dx/dt(0)$=0.01 m/s, $\theta(0)$=0 rad, and $d\theta/dt(0)$=0.2 rad/s. Plot and label the two individual modal responses in addition to the total responses. What should the initial conditions be in order not to excite the body pitch mode of vibration? What about the body bounce mode of vibration?

**PROBLEM 3:** (25%)
How are the eigenvalues and eigenvectors of the system,

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} C_{11} & -C_{12} \\ -C_{12} & C_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2K & -K \\ -K & 2K \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

with proportional damping related to its modal frequencies and modal vectors? You must formulate the eigen-problem for this set of equations in the state-space form. Assume that $M$=10 kg, $C_{ij}=0.1M_{ij}+0.1K_{ij}$, and $K$=10 N/m.

**PROBLEM 4:** (25%)
You are given the lumped parameter linear differential equations of motion for a two degree-of-freedom model of a mechanical system:

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 8 & -5 \\ -5 & 6 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2K & -K \\ -K & 2K \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

with $M$=10 kg and $K$=10 N/m. Determine the modal frequencies, undamped natural frequencies of oscillation, and the modal vectors. Draw schematics of the two modal vectors. How are these modal vectors different when $C$=0 N-s/m? Draw the modes in this case for comparison.