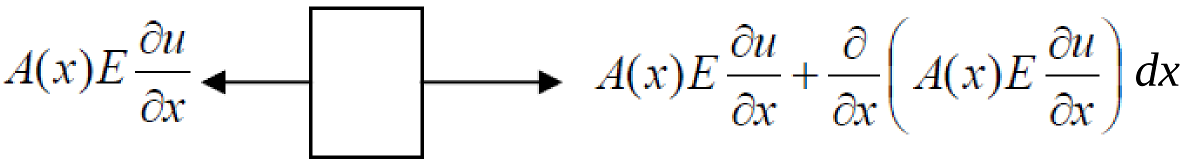
ME 563 **HOMEWORK # 4 SOLUTIONS** **Fall 2008**

Out: September 29, 2008
Due: October 6, 2008 (at 5 p.m. EST)

<u>Problem 1</u>

<u>FBD</u>



Summation of forces on the element:

$$\rho(x)A(x)\frac{\partial^2 u}{\partial t^2}dx = \frac{\partial}{\partial x}(A(x)E\frac{\partial u}{\partial x})dx$$

Response is of the form:

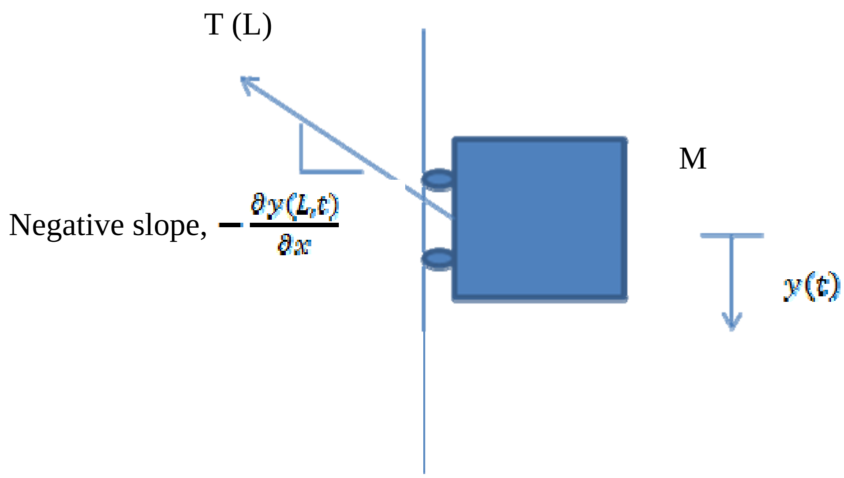
$$u(x,t) = Y(x)G(t)$$

Solve second order differential equations with boundary conditions:

$$A(0)E\frac{\partial u(0,t)}{\partial x} = A(L)E\frac{\partial u(L,t)}{\partial x} = 0$$

Problem 2

FBD



Assumptions

- Constant tension and density
- Small deflections

EOM:

$$\rho \frac{\partial^2 y}{\partial t^2} = T \frac{\partial^2 y}{\partial x^2}$$

Response is of the form:

$$y(x,t) = H(x)G(t)$$

Hence:

$$\frac{\ddot{G}(t)}{G(t)} = \frac{T}{\rho}\frac{H''(x)}{H(x)} = -\omega^2$$

Boundary conditions:

$$y(0,t) = 0$$

$$-T\frac{\partial y(L,t)}{\partial x} = M\frac{\partial^2 y(L,t)}{\partial t^2}$$

Solving differential equation for spatial component and applying the two boundary conditions:

$$H(x) = A\sin\left(\sqrt{\frac{\rho}{T}}\omega x\right) + B\cos\left(\sqrt{\frac{\rho}{T}}\omega x\right)$$

$$H(0) = 0 = B$$

$$H(x) = A\sin\left(\sqrt{\frac{\rho}{T}}\omega x\right)$$

$$-T\frac{\partial y(L,t)}{\partial x} = M\frac{\partial^2 y(L,t)}{\partial t^2}$$

$$-TG(t)\omega\sqrt{\frac{\rho}{T}}A\cos\left(\omega L\sqrt{\frac{\rho}{T}}\right) = MA\sin\left(\omega L\sqrt{\frac{\rho}{T}}\right)\ddot{G}(t)$$

Since:

$$\frac{\ddot{G}(t)}{G(t)} = -\omega^2$$

We get:

$$\tan\left(\omega L\sqrt{\frac{\rho}{T}}\right) = \frac{\sqrt{T\rho}}{M\omega}$$

Problem 3

Using 10 lumped masses in Problem 1, we have density and area as constants. Hence, the EOMs for free free boundary conditions are as follows:

$$\begin{bmatrix} \frac{\rho AL}{10} & 0 & \cdots & 0 \\ 0 & \frac{\rho AL}{10} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\rho AL}{10} \end{bmatrix}_{10x10} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \vdots \\ \ddot{u}_{10} \end{Bmatrix}_{10x1} + \begin{bmatrix} \frac{10AE}{L} & -\frac{10AE}{L} & \cdots & 0 \\ -\frac{10AE}{L} & \frac{20AE}{L} & \cdots & \vdots \\ \vdots & \vdots & \ddots & -\frac{10AE}{L} \\ 0 & \cdots & -\frac{10AE}{L} & \frac{10AE}{L} \end{bmatrix}_{10x10} \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{10} \end{Bmatrix}_{10x1} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}_{10x1}$$

where we have used 1/10th the mass for each DOF and AE/(L/10) for each elemental stiffness because each elemental is of length L/10.

There are only 10 natural frequencies associated with the solution of these EOMs. The continuous rod from problem 1 has an infinite number of natural frequencies. These frequencies are found by solving the eigenvalue problem for the rod, but we do this later in the course.

From intuition, it is clear that the first few modal frequencies of vibration of the rod will be consistent with the frequencies of the lumped parameter system above. As the natural frequencies increase, however, the modal vectors of the lumped parameter system will not be capable of describing the distribution of the mass (kinetic energy) along the rod.

Note that the first natural frequency of the lumped parameter system will be 0 rad/s because the stiffness matrix is singular. This system is called semi-definite for this reason. In this way, the lumped parameter system also matches the continuous rod.