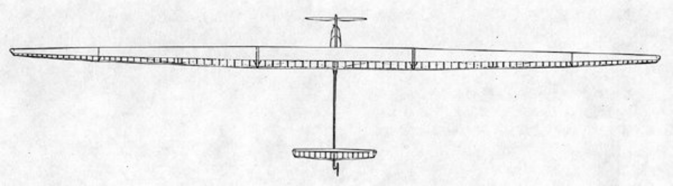


Geometric Programming for Conceptual Aircraft Design Optimization

Woody Hoburg and Pieter Abbeel

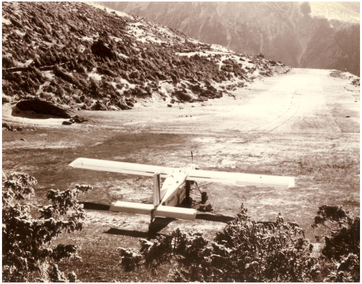
Electrical Engineering and Computer Science Department
University of California, Berkeley
Joint work with Laurent El Ghaoui, Alex Bayen, and Andrew Packard

7^{th} Research Consortium for Multidisciplinary System Design Workshop

July 20, 2012

L/D ≈ 40

endurance ≈ 24 hours



takeoff distance ≈ 640 ft

payload ≈ 85 short tons

Problem

Goal: optimize design parameters; exploit tradeoffs

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Challenges:

- Interacting disciplines
- Expensive function evaluations
- Local optima
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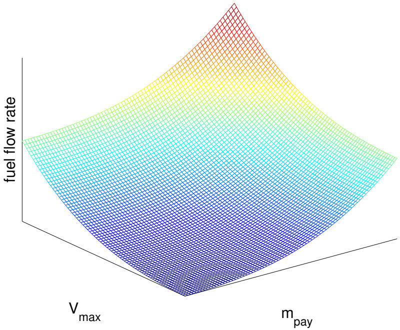

→ Need some way of making this problem tractable

Insight

- Surprisingly, many relationships in aircraft design have an underlying convex structure.

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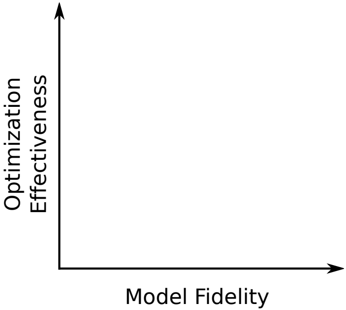

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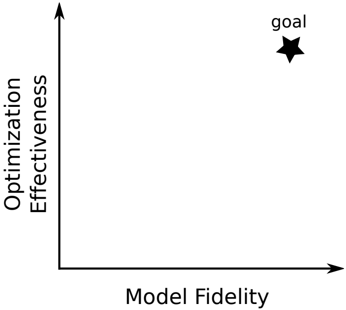

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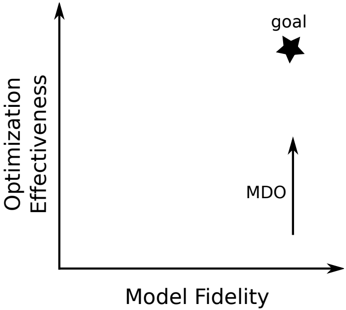

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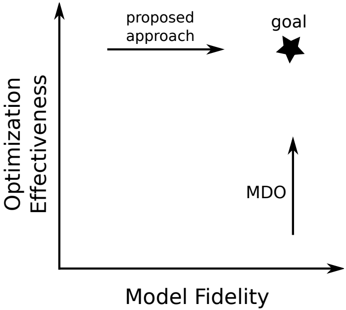

Today's Talk

A New Design Framework

Selected GP-Compatible Models

Example

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Proposed Framework

All-at-once (AAO) formulation [Cramer et.al.]:

Decision Variables: Every unknown quantity
Objective: Tradeoff among performance metrics
Constraints: Physics-based models
Design constraints

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All-at-once (AAO) formulation [Cramer et.al.]:

Decision Variables: Every unknown quantity
Objective: Tradeoff among performance metrics
Constraints: Physics-based models
Design constraints

Additionally,

- Restricted functional forms of objective and constraints
- Emphasis on mathematical properties of physics based models

Geometric Program: Definition

Monomial Function

$$m(x) = c \prod_{i=1}^{n} x_i^{a_i}, \;\; c > 0 \qquad \text{(e.g., } \tfrac{1}{2}\rho V^2 C_L S\text{)}$$

[Boyd 2007]

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Posynomial Function: sum of monomials

$$p(x) = \sum_{k=1}^{K} c_k \prod_{i=1}^{n} x_i^{a_{ik}}, \;\; c_k > 0 \quad (\text{e.g., } \tfrac{w_1}{m_{\text{pay}}} + \tfrac{w_2}{V_{\text{max}}} + w_3 \dot{m}_{\text{fuel}})$$

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Geometric Program (GP)

$$\boxed{\begin{array}{lll} \text{minimize} & p_0(x) & \\ \text{subject to} & p_i(x) \le 1, & i = 1, ..., N_p, \\ & m_j(x) = 1, & j = 1, ..., N_m \end{array}}$$

with p_i posynomial, m_i monomial

$x = (x_1, x_2, ..., x_n) > 0$

[Boyd 2007]

Geometric Program: Convex Formulation

variable change: $y_i := \log x_i$

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$$\log m = b + a^T y \qquad (b = \log c)$$

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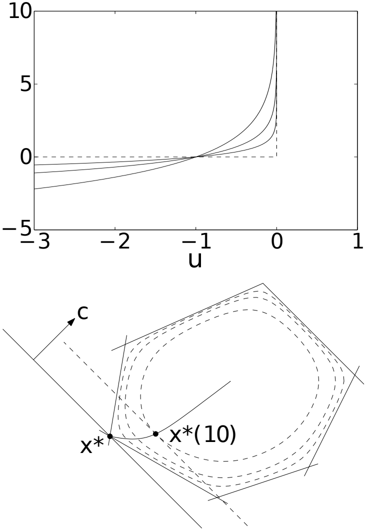
- GP in convex form

$$\boxed{\begin{array}{ll} \text{minimize} & \log \left(\sum_{k=1}^{K} \exp(b_{0k} + a_{0k}^T y) \right) \\ \text{subject to} & \log \left(\sum_{k=1}^{K} \exp(b_{ik} + a_{ik}^T y) \right) \le 0, \quad i = 1, \ldots, N_p \\ & Gy + h = 0 \end{array}}$$

Solution of Geometric Programs

Interior-point methods



[Figures: Boyd 2004]

Benefits:

- Globally optimal solution, guaranteed
- Robust: no need for initial guesses or parameter tuning
- Off-the-shelf solvers

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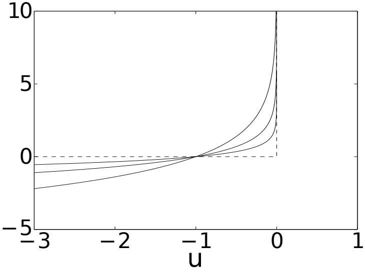

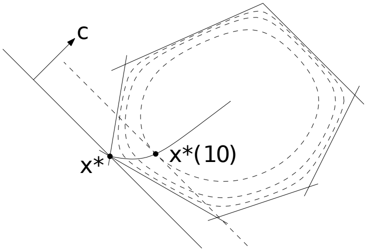

[Figures: Boyd 2004]

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Boyd GP benchmarks (2005) [1]

- dense GP: 1,000 variables; 10,000 constraints; $<$ 1 minute
- sparse GP: 10,000 variables; 1,000,000 constraints; “minutes”

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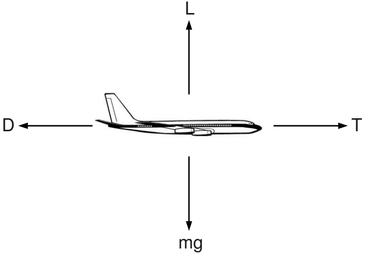
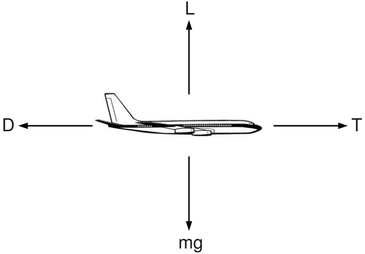
Steady Level Flight Relations



$$mg = L = \frac{1}{2}\rho V^2 C_L S$$

$$T = D = \frac{1}{2}\rho V^2 C_D S$$

Steady Level Flight Relations



$$mg = L = \frac{1}{2}\rho V^2 C_L S$$

$$T = D = \frac{1}{2}\rho V^2 C_D S$$

More models coming:

- C_D: Constrained by drag model
- m: Constrained by mass models

Drag and Mass Breakdowns

Drag breakdown

$$C_D = \underbrace{\frac{[\mathrm{CDA}]_0}{S}}_{\substack{\text{non-wing}\\ \text{form drag}}} + \underbrace{c_d(C_L, \mathrm{Re}, \tau)}_{\text{wing profile drag}} + \underbrace{\frac{C_L^2}{\pi e A}}_{\substack{\text{induced}\\ \text{drag}}}$$

Drag and Mass Breakdowns

Drag breakdown

$$C_D \geq \underbrace{\frac{[\mathrm{CDA}]_0}{S}}_{\text{non-wing form drag}} + \underbrace{c_d(C_L, \mathrm{Re}, \tau)}_{\text{wing profile drag}} + \underbrace{\frac{C_L^2}{\pi e A}}_{\text{induced drag}}$$

posynomial equality relaxation [Boyd 2007]

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Mass breakdown

$$m_{\mathrm{dry}} \geq m_{\mathrm{fixed}} + m_{\mathrm{pay}} + m_{\mathrm{wing}}$$
$$m_{\mathrm{tot}} \geq m_{\mathrm{dry}}(1 + \theta_{\mathrm{fuel}})$$

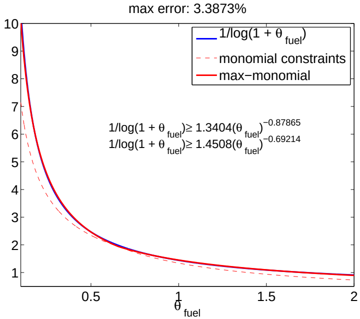
Breguet Range Equation

$$R = \frac{h_f}{g} \eta_0 \frac{L}{D} \log(1 + \theta_{fuel})$$

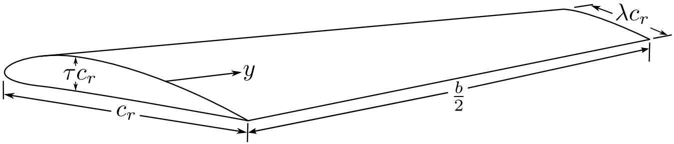
Breguet Range Equation

$$R = \frac{h_f}{g}\eta_0 \frac{L}{D} \log(1 + \theta_{fuel})$$

$$1 \geq \frac{gRD}{h_f \eta_0 L} \frac{1}{\log(1 + \theta_{fuel})}$$



Wing Structural Properties

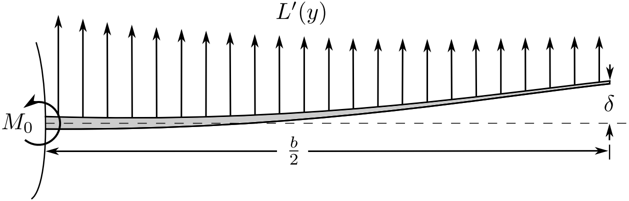


Structural requirement:

$$\frac{M_0}{S_0} \leq \sigma_{\text{safe}}$$

Applied root moment:

$$M_0 \geq 0.115\tilde{m}gb$$



Wing skin section modulus S_0:

$$S_0 \leq \frac{0.81\tau S^2 t_{\text{skin}}}{b^2}$$

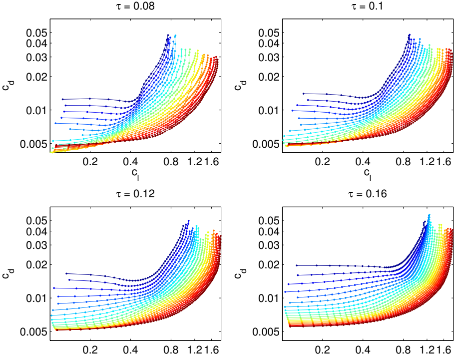
Wing skin mass:

$$m_{\text{skin}} \geq \rho_{\text{skin}} t_{\text{skin}} S(2 + 0.4\tau)$$

Wing Profile Drag

$\approx 10,000$ data points from $c_d(C_L, \text{Re}, \tau)$ for NACA-24xx airfoils, generated using XFOIL [Drela 00]

- C_L ranging from 0 to stall
- Re ranging from 10^5 (seagull) to 10^7 (small jet)
- τ ranging from 8% to 16%



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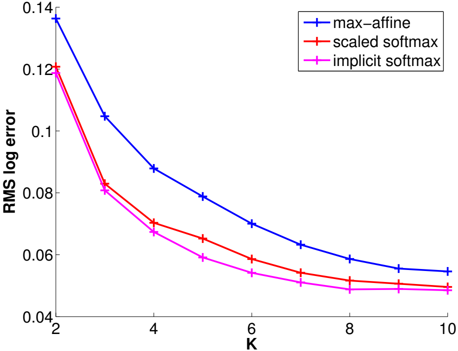

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$$\begin{array}{rl} \text{minimize} & T_1 + \frac{w}{V_2} \\ \text{subject to} & T_2 \leq T_{\max} \\ & \text{Level Flight Relations} \\ & \text{Drag Model} \\ & \text{Wing Structural Models} \end{array}$$

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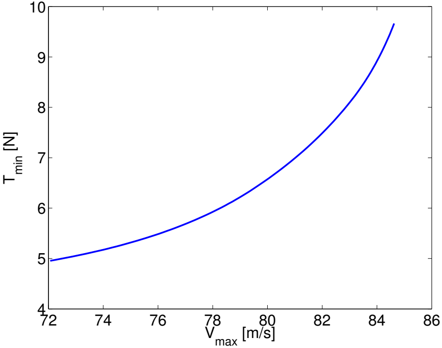

Ease of Incorporating New Models

Example: tail sizing

$$C_D \geq \frac{[\mathrm{CDA}]_0}{S} + c_d(C_L, \mathrm{Re}, \tau) + \frac{C_L^2}{\pi e A}$$

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$$[CDA]_0 \geq [\mathrm{CDA}]_{\mathrm{fuse}} + c_d^{\text{tail, h}} A_{\mathrm{tail,h}} + c_d^{\text{tail, v}} A_{\mathrm{tail,v}}$$

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$$m_{\mathrm{dry}} \geq m_{\mathrm{fixed}} + m_{\mathrm{pay}} + m_{\mathrm{wing}} + m_{\mathrm{tail}}$$

GP as an MDO Framework

Benefits

- Extremely fast
- Globally optimal solutions
- Off-the-shelf solvers

Limitations

- Restricted functional forms
- No disciplinary solvers in the loop

Future work

- GP as a 'discipline'
- Sigmoidal Programming
- Automatic identification of variable transformations

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Prof. Alex Bayen
Prof. Andrew Packard

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Posynomial Equality Relaxation

When can we guarantee $h(x) = 1$ will hold at optimum ?

$$\boxed{\begin{array}{lll} \text{minimize} & p_0(x) & \\ \text{subject to} & p_i(x) \leq 1, & i = 1, ..., N_p, \\ & m_j(x) = 1, & j = 1, ..., N_m \\ & h(x) \leq 1 & \end{array}}$$

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If $\exists\ x_k$ s.t.:

- x_k does not appear in monomial equality constraints, i.e. $\frac{\partial m_j}{\partial x_k} = 0$
- p_0 monotone strictly decreasing in x_k, i.e. $\frac{\partial p_0}{\partial x_k} < 0$
- All p_i monotone decreasing in x_k, i.e. $\frac{\partial p_i}{\partial x_k} \leq 0$
- h is monotone strictly increasing in x_k, i.e. $\frac{\partial h}{\partial x_k} > 0$

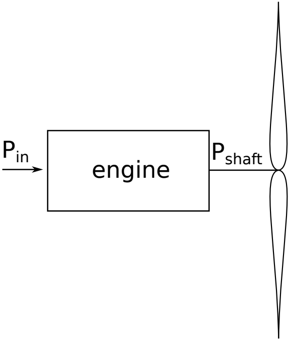
$\rightarrow$ **Conditions satisfied for all relaxations presented today.**

Extensions exist for multiple $h_i(x)$, $\frac{\partial p_0}{\partial x_k} = 0$ case [Boyd et. al., 2007]

Propulsive Efficiency

$$TV = P_{\text{in}}\eta_{\text{eng}}\eta_{\text{prop}}$$

$$\eta_{prop} = \eta_i\eta_v$$



Propulsive Efficiency

$$TV = P_{\text{in}}\eta_{\text{eng}}\eta_{\text{prop}}$$

$$\eta_{prop} = \eta_i\eta_v$$

Use actuator disk theory [**?**]

$$\eta_i \leq \frac{2}{1 + \sqrt{1 + \frac{T}{\frac{1}{2}\rho A_{prop} V^2}}}$$

Introduce helper var z

$$z \geq 1 + \frac{T}{\frac{1}{2}\rho A_{prop} V^2}$$

$$2 \geq \eta_i + \eta_i z^{1/2}$$