Comer

ECE 600 Homework 9

1. Explain why the central limit theorem does not hold if the random variables X_k have a Cauchy density. Find the pdf of the random variable

$$Y = \sum_{k=1}^{n} X_k$$

if the X_k are iid Cauchy random variables with parameters $\mu = 0$ and $\alpha > 0$.

2. The strong law of large numbers states that, with probability 1, the successive arithmetic averages of a sequence of iid random variables converge to their common mean μ. What do the successive geometric averages converge to? That is, find

$$\lim_{n\to\infty}\left(\prod_{k=1}^{n} X_k\right)^{1/n}$$

3. The amount of time that a certain type of component functions before failing is a random variable with pdf

$$f(x) = 2x \quad 0 < x < 1$$

Once the component fails it is immediately replaced by another one of the same type. If we let X_i denote the lifetime of the ith component to be put in use, then $S_n = \sum_{i=1}^{n} X_i$ represents the time of the nth failure. The long-term rate at which failures occur, denoted r, is defined by

$$r = \lim_{n\to\infty}\frac{n}{S_n}$$

Assuming that the random variables X_i, $i \geq 1$, are independent, determine r.

4. The continuous parameter random process $X(t) = e^{At}$ is a family of exponentials depending on the random variable A. Express the mean $\mu(t)$, the autocorrelation function $R(t_1, t_2)$, and the first-order pdf $f(x;t)$ of $X(t)$ in terms of the pdf $f_A(a)$ of A.

5. The random variable C is uniform in the interval $(0,T)$, where T is not random. Find the autocorrelation function of X_t if $X_t = u(t - C)$.

6. A random process has sample functions of the form

$$X(t) = A\sin(\omega_0 t)$$

where A is a random variable uniform on [-1,1], and ω_0 is fixed.

a. Find $E[X(t)]$.

b. Find the pdf of $X(\pi/2\omega_0)$.