ECE 600 Homework 6

1. Let X and Y be independent random variables with Y uniformly distributed from 0 to 1. Let X have cdf and pdf $F_X(x)$ and $f_X(x)$, respectively. Show that the pdf of the random variable $Z = X + Y$ is given by

$$f_Z(z) = F_X(z) - F_X(z-1)$$

2. The joint density function of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} e^{-(x+y)} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Find the density function of the random variable X/Y.

3. Two fair dice are rolled. Find the joint probability mass function of X and Y when
   (a) X is the larger value rolled and Y is the sum of the two values.
   (b) X is the smaller and Y is the larger value rolled.

4. The number of bytes N in a message has a geometric distribution with parameter p, i.e.,

$$p_N(k) = p(1-p)^k,\ k = 0, 1, 2, 3, \ldots$$

   where $0 \le p \le 1$. Suppose that the messages are broken into packets of length M bytes. Let Q be the number of full packets in a message and let R be the number of bytes left over. Find the joint pmf and the marginal pmfs of Q and R. Are Q and R independent?

5. Let X and Y be continuous random variables and

$$Z = X\cos\phi + Y\sin\phi$$

$$W = -X\sin\phi + Y\cos\phi$$

   where the angle ϕ is not random. Find f_{ZW} in terms of f_{XY}.

6. Let R and Θ be independent random variables such that R has a Rayleigh density

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} u(r)$$

   and Θ is uniformly distributed on $[-\pi, \pi]$. Show that $X = R\cos\Theta$ and $Y = R\sin\Theta$ are independent random variables and that each has a normal density, $N(0, \sigma^2)$.

7. Find the joint pdf of random variables M and V defined by

$$M = \frac{X_1 + X_2}{2}$$

$$V = \frac{(X_1 - M)^2 + (X_2 - M)^2}{2}$$

   in terms of the joint pdf of random variables X_1 and X_2. (Note: M and V are referred to as the sample mean and sample variance of X_1 and X_2.) Evaluate the joint pdf of M and V if X_1 and X_2 are independent exponential random variables with the same parameter.

8. Let X and Y be two independent binomial random variables with pmf

$$p_X(k) = p_Y(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & 0 \leq k \leq n \\ 0 & \text{otherwise} \end{cases}$$

   Let $Z = X + Y$. Find the pmf of Z. Do you find anything special about the pmf of Z? Comment on it.