COMER

ECE 600 Homework 5

1. Find the mean and variance of the binomial random variable.

2. Show that E$[X]$ for the random variable with cdf

$$F_X(x) = \begin{cases} 1 - \frac{1}{x} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

   does not exist.

3. Let $Y = A\cos(\omega t) + c$, where A is a random variable with mean m and variance σ^2, and ω and c are constants. Find the mean and variance of Y.

4. Let $g(X) = ba^X$, where a and b are positive constants and X is a Poisson random variable. Find E$[g(X)]$.

5. The hat check staff has had a long day, and at the end of a party they decide to return people's hats at random. Suppose that n people have their hat returned at random.
   (a) What is the expected number of people who get the correct hat back?
   (b) In the limit as $n \to \infty$, what is the answer to part (a)? Do you expect anyone to get his/her hat back?

6. There are n bags numbered $1, ..., n$ and an infinite supply of balls. You pick up a ball and it is equally likely that you put it in any of the bags.
   (a) What is the expected number of balls you need to put in the n bags before the i^{th} bag has at least one ball?
   (b) Show that the expected number of balls that you need to put in before every bag has at least one ball is $n\text{H}(n)$, where

$$\text{H}(n) = 1 + \frac{1}{2} + ... + \frac{1}{1-n} + \frac{1}{n}$$

7. Let the joint cumulative distribution function (cdf) of random variables X and Y be $F_{XY}(x, y)$. Show that

$$\text{P}(\{x_1 < X \leq x_2\} \cap \{y_1 < Y \leq y_2\}) = F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1)$$

8. Determine the constant b such that each of the following is a valid probability density function (pdf).

(a)

$$f_{XY}(x,y) = \begin{cases} 3xy & 0 \le x \le 1; 0 \le y \le b \\ 0 & \text{elsewhere} \end{cases}$$

(b)

$$f_{XY}(x,y) = \begin{cases} bx(1-y) & 0 \le x \le 0.5; 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

(c)

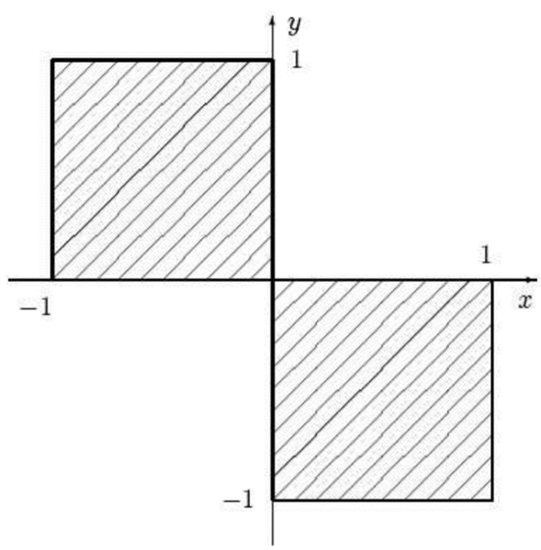
$$f_{XY}(x,y) = \begin{cases} b(x^2+4y^2) & 0 \le |x| < 1; 0 \le y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

9. If $f_{XY}(x,y) = g(x)h(y)$, find the mardinal pdfs $f_X(x)$ and $f_Y(y)$.

10. For two independent continuous random variables X and Y, show that

$$\mathrm{P}(\{Y \le X\}) = \int_{-\infty}^{\infty} F_Y(x) f_X(x) dx = 1 - \int_{-\infty}^{\infty} F_X(x) f_Y(x) dx$$

11. Suppose $f_{XY}(x,y)$ is uniform with region of support shown in the figure below. Find $f_X(x)$ and $f_Y(y)$. Are X and Y independent?



12. The random vector (X,Y) has a joint pdf

$$f_{XY}(x,y) = \begin{cases} 2e^{-x}e^{-2y} & x > 0,\ y > 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability of the following events:

(a) $\{X + Y \leq 8\}$

(b) $\{X < Y\}$

(c) $\{X^2 < Y\}$