COMER

ECE 600 Homework 4

1. The random variable X is $N(10, 1)$. Find $f_X(x|(X-10)^2 < 4)$.
2. Let X be an exponential random variable.
   (a) Find and plot $F_X(x|\{X > t\})$, for t a real number.
   (b) Find and plot $f_X(x|\{X > t\})$.
   (c) Show that $P(\{X > t + x\}|\{X > t\}) = P(\{X > x\})$. Explain why this is called the memoryless property.
3. A random variable X is said to be a geometric random variable if

   $$P(X = k) = pq^{k-1} \quad k = 1, 2, 3, ...$$

   where $p, q > 0$ and $p + q = 1$.
   (a) Show that for any natural numbers m and n,

   $$P(X > m + n|X > m) = P(X > n)$$

   This is known as the memoryless property of a geometric random variable.
   (b) Show that the converse of part a is also true, i.e., if X is a positive integer-valued random variable satisfying the memoryless property for any two natural numbers m and n, then X is in fact a geometric random variable.
4. Let X be the number of customers waiting for a bus. Assume that X is a geometric random variable with parameter p. Suppose that the bus can take M passengers. Find the pmf for $Y = (X - M)^+$, where

   $$x^+ = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

   Note that Y represents the number of customers left behind.
5. Suppose that a voltage X is a zero-mean Gaussian random variable. Find the pdf of the power dissipated by an R-ohm resistor $P = X^2/R$.
6. Let X be uniform on [-1, 1]. Find $g(x)$ such that, if $Y = g(X)$, then $f_Y(y) = 2e^{-2y}u(y)$.
7. Let $Y = e^X$.
   (a) Find the cdf and pdf of Y in terms of the cdf and pdf of X.
   (b) Find the pdf of Y when X is a Gaussian random variable.